IBM Research challenge corner – Ponder-This

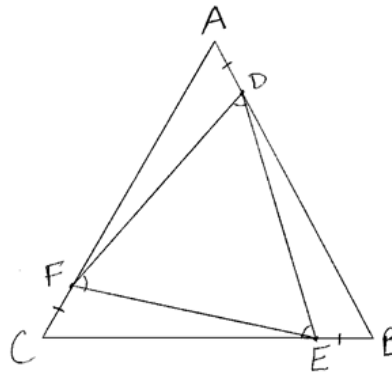Oded Margalit

Over the years we asked (www.research.ibm.com/ponder) 168 monthly challenges.

Here are 010 (Octal) open questions about them.

1. August 1998:
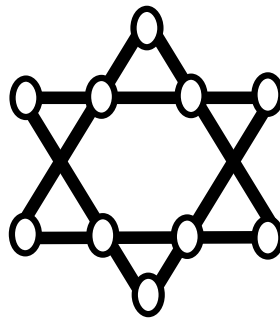   If ΔABC is equilateral and AD=BE=CF then ΔDEF is equilateral too.

   Prove that is ΔDEF is equilateral and AD=BE=DF then ΔABC is too.
   We have a solution but looking for a simpler one.

2. September 1998:
   Arrange the numbers 1,2,...,12 in the vertices of the shield of David such that the sum over all four integers on each line and the sum over the internal hexagon are all the same

3. October 2010:
   In the following story, can you fill in the values a, b, c, d, and e with integer numbers so as to make the story true? And what would be X?

   P: What is the area of the lake?
   T: I can not tell since I do not know anything about it.
   P: It has the shape of a convex pentagon; with integer edges.
   T: Can you give me more details?
   P: Here are the sorted integer lengths: a, b, c, d and e.
   T: Now we are getting somewhere, but I still don't know.
   P: Last hint: Two of the angles are right (90 degrees).
   T: Thank you. The lake area **MUST** be X square kilometers.

   The smallest area we found is ~33.99 and the smallest integer area is 70.
   Can you do better?

4. April 2011 (Vi Hart):
   We asked how many 20-segments long snakes are they. Can you solve it for general N?

5. July 2011:
   There are 80 students in a school. Each of them eats fruit for dessert every day, and the available fruits are apples, bananas, and cherries.
   Find a possible setting of desserts for X days such that for every set of three students, there exists at least one day in which they all ate different desserts.

   We know of a solution for X=14, and a lower bound of 10. Can you close that gap?

6. September 2011:
   A computer program, named 6to2, gets a sequence of purely random independent and fair dice tosses and outputs a sequence of uniformly and independent bits. It generates as many output bits as it can from the dice inputs -- as long as it can ensure that the output is indeed completely random.
   The problem is that our program actually gets its input from a similar program named 2to6, which generates random dice

tosses from random bits.
Our question was: What is the efficiency of the above process? Starting from 27 bits, converting them to dice tosses, and then back to bits, how many bits will we get on average?

The answer is surprisingly close to an integer. Why?

7. October 2011:
   We call a real number R "interesting" if the product of every 8 consecutive decimal digits is 40,320.
   Build a network from no more than twenty 1-ohm resistors such that their overall resistance R would be interesting.

   It can be done with 12 resistors, even with 11 (but not parallel-series).

   Is 11 minimal?

8. March 2012:
   Arrange the numbers 1, 2, 3,..., N on the nodes of a tree such that if we write the difference of the edges on each edge – no two edges would be the same.

   We have a solution for a balanced binary tree, but it is an open question for arbitrary tree.
   Can you find a simple tree for which you do not have such numbering?