IBM research

Ponder This

IBM research's riddles corner

March 2012

May 1998 = www.businessweek.com/1998/20/b3578130.htm

BIG BLUE'S LITTLE BOOK OF IDEA BOOSTERS

WHAT ARE INVENTORS TO DO when they just plain get stuck? No new ideas, no cries of "Eureka!" from the bathtub? Well, IBM's research scientists now have a little spiral-bound book of 56 tips to try. Sort of like the Tao for people who think about computers all day.

The tips range from the pedestrian to the surreal. In the interest of furthering scientific research, a sampling:

"Clean your desk."
"Go backpacking."
"Shut the door. Practice shooting rubber bands at targets in your office."

"Reread your favorite book from childhood."
"Ponder something else. For example, if a belt were placed around the equator, and then had six meters of length added to it, and you grabbed it at a point and lifted it until all the slack was gone, how high above the earth's surface would you be?" If you figure it out, let IBM's big thinkers know at tips98@us.ibm.com.

By Ira Sager ***EDITED BY HEATHER GREEN***

Aug 1998 = solved Mar 2005

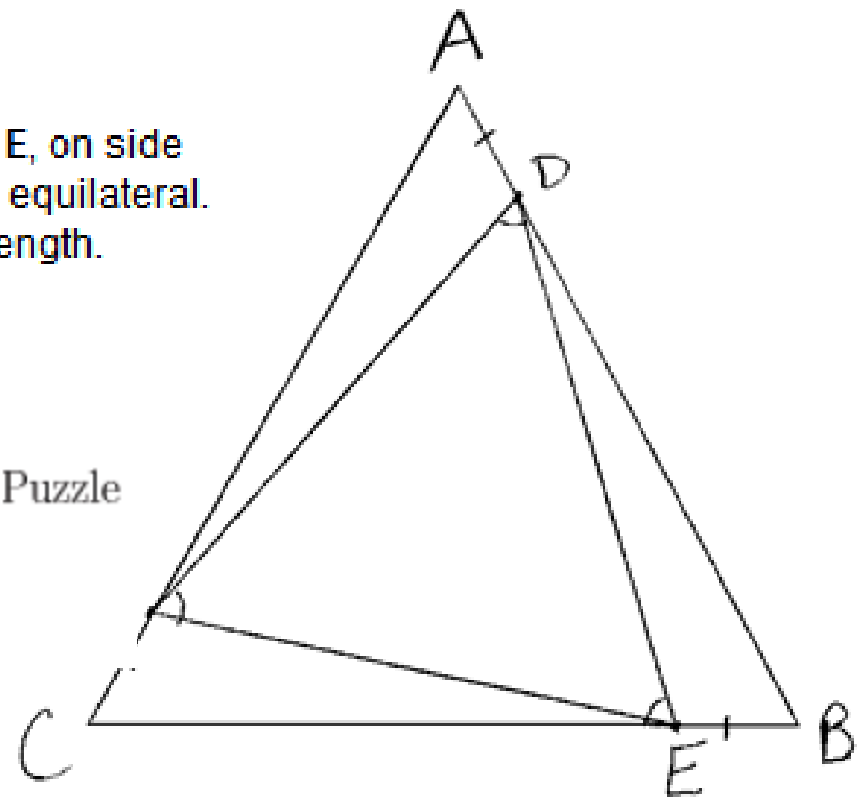
Ponder This Challenge:

We have a triangle **ABC**, with a point **D** on side **AB**, **E**, on side **BC**, and **F** on side **CA**. The smaller triangle, **DEF**, is equilateral. The line segments **AD**, **BE**, and **CF** all have equal length.
Problem: Prove that **ABC** is also equilateral.

An Elementary Solution to the IBM August 1998 Puzzle

Fuxiang (Sean) Yu (fuxiang@cs.sunysb.edu)

March 1, 2005

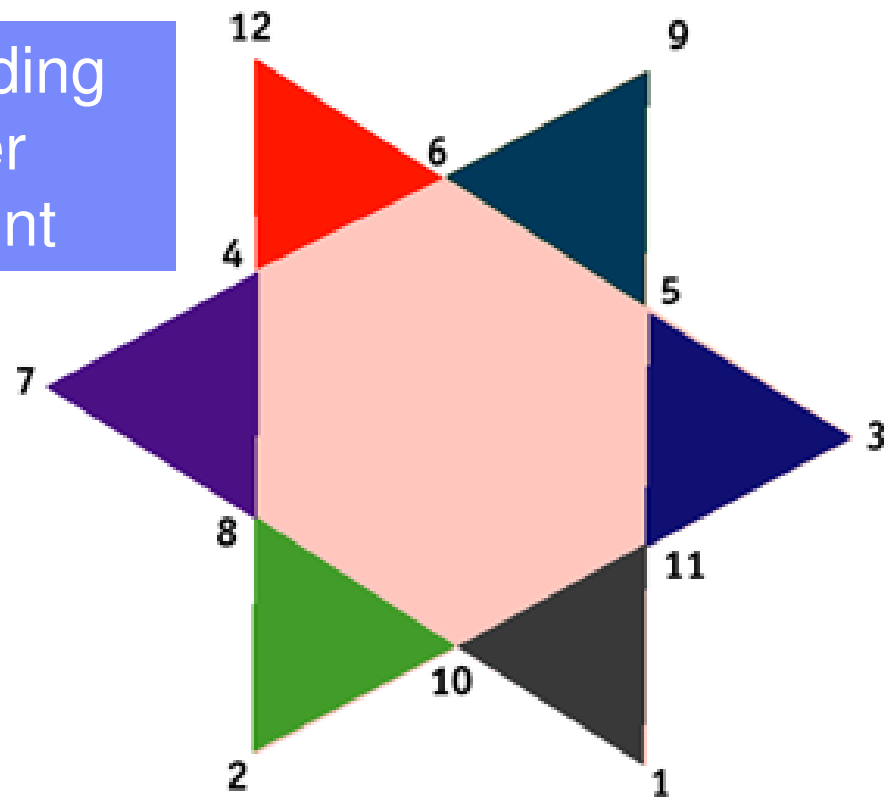


Comment: Although we have a solution for this problem, it is not a simple one.

Sep 1998 = Can be solved by a child

We found the following solution using trial and error. Computers are good for that.

When adding another constraint



Jul 2003 = open question

This is an old problem due to Roy Adler, Wayne Goodwyn and Benjamin Weiss. To our knowledge, it remains unsolved.

We are given a collection of N cities and 2N one-way roads. (For example, we are told that the two roads leading out of Pittsburgh lead to Buffalo and Erie, respectively.) Each road leads from one city to another (or possibly back to the same city). Each city has exactly two roads leading out of it, and at least one leading in. We are assured that it is possible to proceed from any city to any other city by legal moves along the roads (that is, in the proper direction).

We must impose one other technical condition:
Define a "cycle" to be a route leading from one city through some other cities and back to the starting point (following the roads legally, and without visiting any city twice), and the "length" of a cycle to be the number of roads traversed. We must assume that for each prime number p there is a cycle whose length is not divisible by p. (For example, not all cycles have length divisible by 3.)

Our task is to color each road red or green, so that for each city the two roads exiting that city have different colors, and so that "universal directions" can be given: namely, if a friend calls up and says "I don't know where I am; how do I get to Pittsburgh?", we can respond: "Take the red road out of your present city, then the green road, then the next green road, then red, then green, and then red; then you will be in Pittsburgh."

The Road Coloring Problem

A.N. Trahtman*†

Israel Journal of Mathematics

Abstract

A synchronizing word of a deterministic automaton is a word in the alphabet of colors (considered as letters) of its edges that maps the automaton to a single state. A coloring of edges of a directed graph is synchronizing if the coloring turns the graph into a deterministic finite automaton possessing a synchronizing word.

The road coloring problem is the problem of synchronizing coloring of a directed finite strongly connected graph with constant outdegree of all its vertices if the greatest common divisor of lengths of all its cycles is one. The problem was posed by Adler, Goodwyn and Weiss over 30 years ago and evoked noticeable interest among the specialists in the theory of graphs, deterministic automata and symbolic dynamics.

The positive solution of the road coloring problem is presented.

Dec 2006 = There is a better way to ask it

Ponder This Challenge:

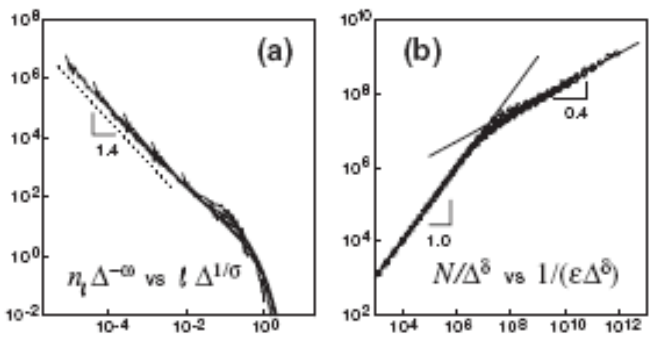
Puzzle for December 2006.

Consider a random permutation, P, on n elements. P can be decomposed into cycles. Let x be a fraction between .5 and 1. Let f(x,n) be the probability that all the cycles of P have size less than x*n. This month's problem is to find the asymptotic behavior of f(x,n) for fixed x as n --> infinity.

Apr 2007 = PRL paper

This month's puzzle concerns a frog who is hopping on the integers from minus infinity to plus infinity. Each hop is chosen at random (with equal probability) to be either +2 or -1. So the frog will make steady but irregular progress in the positive direction. The frog will hit some integers more than once and miss others entirely. What fraction of the integers will the frog miss entirely? Please find an exact answer.

PRL 99, 180602 (2007) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2007



We acknowledge Brazilian agencies CNPq and Faperj for partial financial support and IBM research "Ponder This" for having drawn attention to this model.

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Aug 2007 = My first challenge

Define f(0)=1 and f(n) to be the number of different ways n can be expressed as a sum of integer powers of 2 using each power no more than twice.
For example, f(10)=5 since there are five different ways to express 10: 1+1+8, 1+1+4+4, 1+1+2+2+4, 2+4+4 and 2+8.
Describe, in a single sentence, the multiset {f(n)/f(n-1)} for positive integer n.
Show your proof to this sentence.

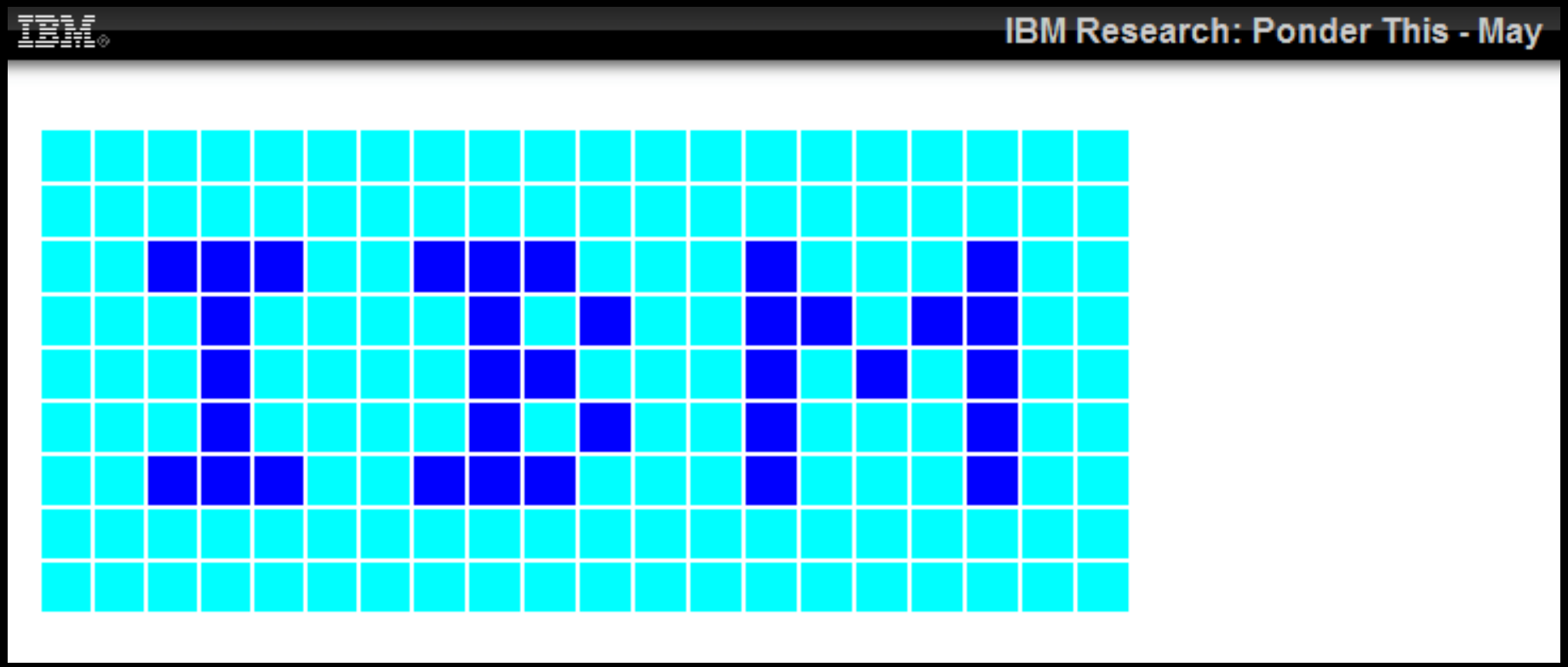
As usual, we ask that you only submit your original work.

UPDATE, 8/2/07: The solution is more elegant than just a recursion formula. You'll recognize it once you find it.

Feb 2008 = Unknown game

Parameter	Sequence
1	1 2 4 8 16 32 64 128 256 512 1024 2048 4096 8192 16384 ...
2	1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 ...
2.5	1 2 3 5 7 10 15 22 32 47 69 101 148 217 318 ...
3	...
3.5	1 2 3 4 6 8 11 15 21 27 35 46 61 82 109 ...
4	...
4.5	1 2 3 4 5 7 9 11 14 18 22 27 34 43 54 ...
5	1 2 3 4 5 6 8 10 12 15 18 22 27 33 41 ...
5.5	1 2 3 4 5 6 8 10 12 15 18 22 26 31 37 ...
6	1 2 3 4 5 6 7 9 11 13 16 19 23 27 32 ...
6.5	1 2 3 4 5 6 7 9 11 13 15 18 21 25 29 ...
7	1 2 3 4 5 6 7 8 10 12 14 16 19 22 26 ...

May 2008 = efil fo emaG



Aug 2008 = GEC

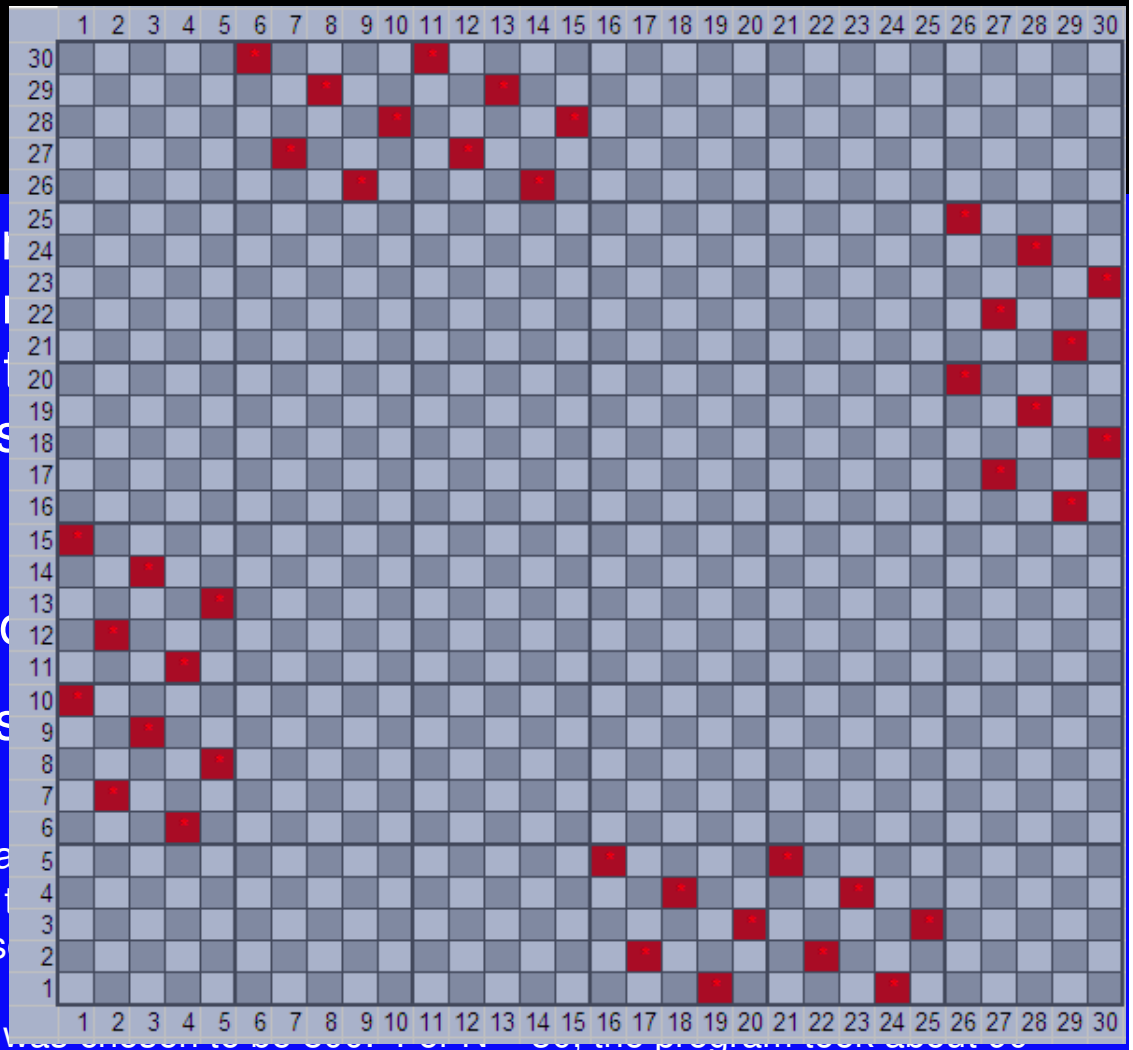
The first construction was fou
solving an MILP optimization
trivially translates to MILP, bu
a solution fast on large board

Best regards,

Graduate stu

Rutgers Center for Operation

to store in memory or process in a reasona
sense of not having more than one queen
number of cases sufficiently, they were als
threatened by any queen. The cases with
on to the next stage. This number of sets
minutes to run on my PC and resulted in 61 solutions, one of which is shown below.



- **Back-of-envelope (new!) solution**

Nov 2008 = ibmresearchnews.blogspot.com/2008/10/ibm-researchs-ponder-this-puzzle.html

FRIDAY, OCTOBER 31, 2008

IBM Research's "Ponder This" puzzle reaches milestone, inspires research paper

Guest blogger: Oded Margalit, Machine Learning Researcher, IBM Global Research, Haifa.

"Ponder This" is IBM Research's monthly brain-twister where you can match wits with some of the best minds in IBM Research, run by James B. Shearer and Oded. Oded wanted to share the following milestone with all of you.

Our Ponder-this corner reached its 127th month, which is the maximal number that fits into signed byte. To celebrate this, we will praise you - our devoted solvers - by sharing some of your flattering emails, and, of course, give you a special ponder-this question.

First, a salute to your persistence. Here's an example of one devoted follower who sent in solutions from the Intensive Care Unit:

"Let me just take this time to thank you and IBM for this wonderful web site. It has provided me with many hours of puzzle solving pleasure, and also some interesting conversations with my family.

The first three weeks of June this year I spent in the hospital. One of the nights in the ICU my family and I passed some of the time by talking about Ponder This. My daughter read the DNA problem to me at that time, and jokingly I told her I could be the first person to ever solve and answer a Ponder This while in the ICU. I actually sent my entry from my hospital bed about a week later.

Again thanks. Looking forward to next month's puzzle."

Many thanks to all of those who sent in great original and innovative riddles.

Of course it is heartwarming to get compliments like:

"PS.: Thank you 10^10 times for maintaining this extremely enlightening challenge. Each time I participate, I discover a new world.

That's just great! ;-)"

ponder^{this} = ponder...

.poor*.heron .distorted*.terrier

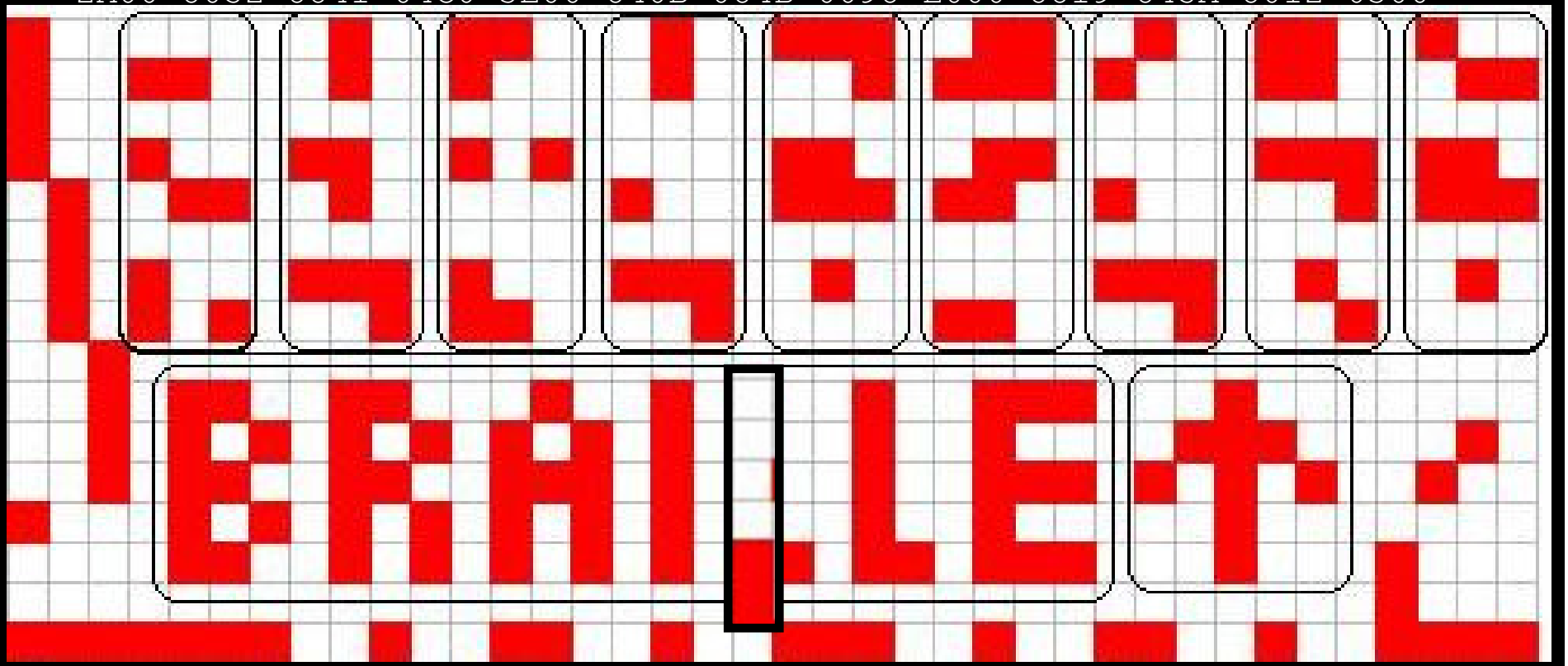
---------- + ----------

|.dr-.psi+.norton| |.ret-.en+.pier|

msb 0:found 1:question 2 out of 15 in 7 questions

Feb 2009 = Cant see

- **Find the error in the following**
- 900F 80F0 8F00 80CA BE12 AA90 9400 0048 3E5B 8AC0 3400 00CB BC81
  8A08 3C00 0050 BE43 00C0 3E00 A019 8059 BE13 2000 0092 BE9B 2A0B
  2A00 8052 8841 04C0 3E00 840B 084B 0098 E000 8819 845A 8012 0300



Apr 2009 = efficient erasure code

- **f: $2^{24} \rightarrow 2^{8*4}$**
  - Recover 2 erasures
  - Implement in 5 ops

$$f(x) = ((((x*c_1)\&c_2)*c_3)\&c_4)\%c_5,$$

where

$$c_1 = (2^{792}-1)/(2^{33}-1)$$
$$c_2 = (2^{816}-1)/(2^{34}-1)$$
$$c_3 = 1+2^{34}+2^{3*34}+2^{4*34}+2^{8*34}$$
$$c_4 = (2^{1088}-1)/(2^{34}-1)$$
$$c_5 = 2^{33}-1$$

- **Michael: general table**
- **Eli: One instruction (PCLMULQDQ)**

May 2009 = find the time

- **Find**
  - Palindromic digital
  - Seconds bisects the min/hour hands
- **Round or floor?**
- **00:55:00, 04:00:40, 04:55:40, 10:11:01**
- **14:11:41, 20:22:02 and 24:22:42**

June 2009 = permutation

$$\left(\sum_{\pi \in S_{14}} 4^{\prod_{i=1}^{14} (2+(Ni \bmod 2))} \right) \bmod \quad 1299$$

$f_i = 4^{2^i 3^{14-i}}$ $\quad C_i$ = # Combinations with i even, $14-i$ odd

$f_0 = 256 \quad f_2 = f_4 = \ldots = f_{12} = 1 \quad f_{14} = 1045$

$$\begin{aligned} \text{ans} &= C_0 f_0 + C_2 f_2 + \ldots + C_{14} f_{14} \\ &= C_0 (256 + 1045) + C_2 + C_4 + \ldots + C_{12} \\ &= C_0 (256 + 1045 - 2) + 14! \\ &= (14!)|_{1299} = 648 \end{aligned}$$

From Johns Hopkins University Applied Physics Laboratory

ANS = 648

July 2009 = BackGammon

HASKELL program

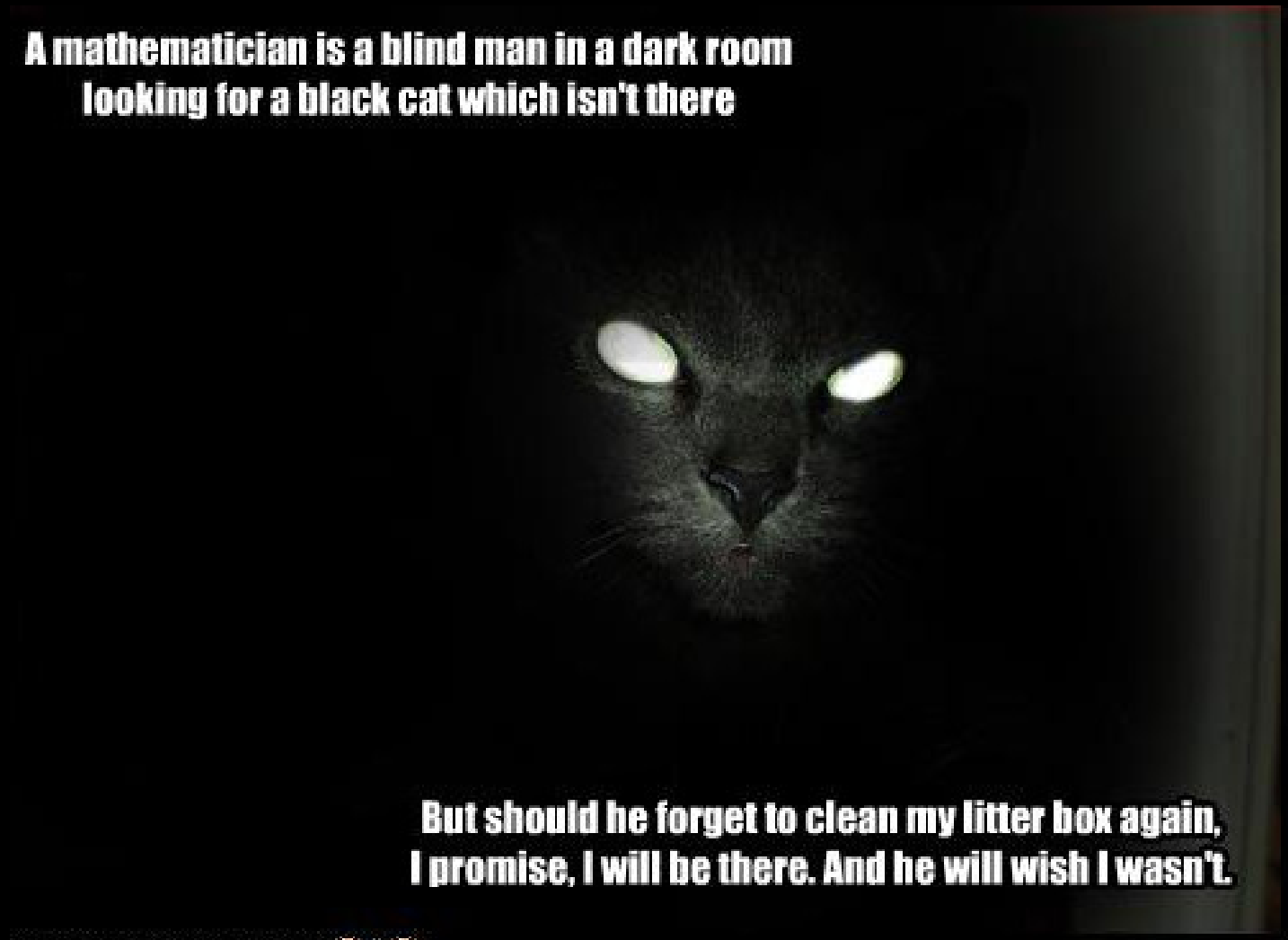
```
import Data.Ratio

main = putStr $ show (f 8 8) ++ "\n"


f :: Int -> Int -> Rational

f 1 b = 1%6

f 2 b = 1%6 + 5%6 * f b 1

f a b = 1%6 * f b (a-2) + 5%6 * f b (a-1)
```

August 2009 = UYHIP Boolean NOT

- **How to compute 3 NOTs using 2?**
- **(I found this a fascinating problem by the way). Ideally this explanation would be given in the form of a poem with rhyming scheme:**
  `baabeccefddfhIjgIKKIdIfIolnlemcmsqatbtvpuprq`

October 2009 = different kind

- A mathematician is a blind man in a dark room looking for a black cat which isn't there
- about
- ill be
- und
- ‹



January 2010 = 2+2=5

gradians

$$\sqrt{\operatorname{acos}\left(\frac{d}{dx}2\right)}$$

$$Fib(2+2)$$

$$\operatorname{Ackerman}(2,\operatorname{sign}(2))$$

$$\left|\sqrt{22}\right| - \left\lfloor -\sqrt{(2+2)!}\right\rfloor$$

$$e^{-\ln(.2)}$$

$$Mirror(2) = 5$$

$$\frac{(X \cdot X - X - X - X)}{X} = 2$$

$$2\,M + \ldots MR$$

$$\ldots \cos(2)$$

Some people think that 2+2 **is** 5

$$\frac{\operatorname{CastAsDecimal}(\operatorname{ConvertToBinary}(2))}{2}$$

$$\coth \ldots (2+2)$$

$$\coth(\ln(\sec(arc\csc(2)))) - 2$$

$$\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log\left(\frac{1}{\log(2)}\right)}\right)}\right)}\right)}\right)}\right)}\right)}\right)} = 5.00083$$

$$\frac{\operatorname{arcsec}(-2)}{2!!}$$

degrees

$$\sqrt{2\Sigma}$$

$$\cos(\arctan(2))^{-2}$$

$$\frac{2}{\sqrt{-\arctan''(2)}}$$

$$\theta(\varphi(\varphi(\theta(\varphi((2+2)!)!)))))$$

April 2010 = (Hexa)Decimal

- **Find a prime number which stays the same (21 digits truncated)**
- **Easy Lattice problem**

$$\sum_{i=0}^{n} d_i 10^i = \sum_{i=0}^{n-21} d_{i+21} 16^i$$

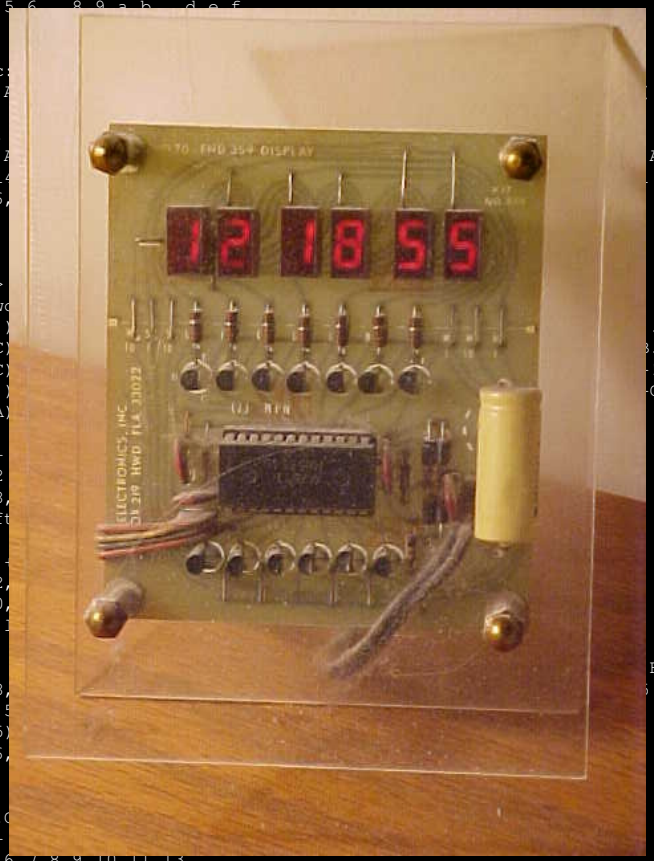
- **2280762253152254361009441410610677210864968 3164642972183286269214619099352157014522058 0989963852549032815456958871896032672 01**
- **Solver: iterate Hex->Dec->Delete21 till converges**

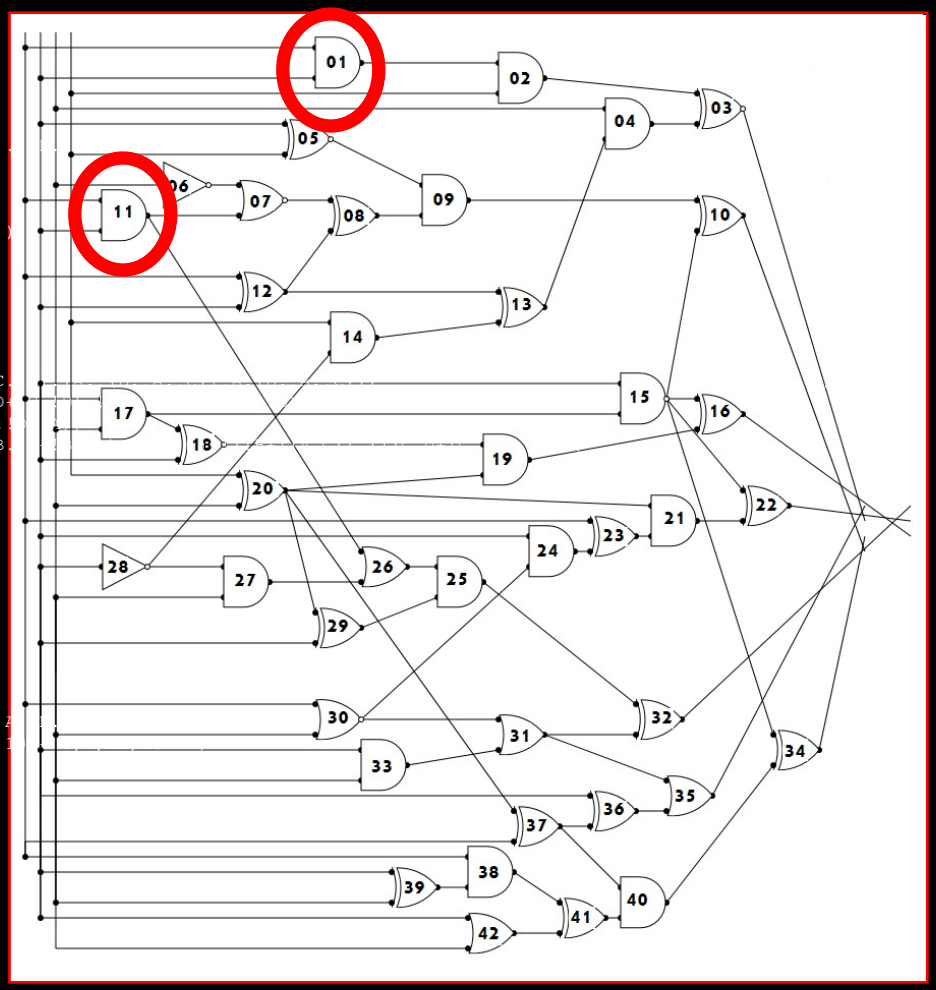
May 2010 = Prob' vector Quantization

- **Quantize a 10-long prob' vector to [0,0.25) [0.25,0.5) [0.5,0.75) [0.75,1]**
- **A. How many bins are realizable?**
- **B. If we change the boundaries, what is the maximum #?**
- **Nice combinatoric argument for A**
- **Interesting reservation for B**

June 2010 = Boolean function

```
For c and d to both be affected, the problem gate is number 20.

Try c again. replace 20: XOR with AND
c: ! ( A.B.C.!D  +  !A.!B.C.D + A.B.C.D  )  +   A.B.C.D
C:! ( 7 + 12 + 15   ) + 15
c:0,1,2,3,4,5,6,  8,9,a,b,  d,e,f
c == ???

refactored c:
c:(  !A.B + A                                          A.B.C )

Try c again.
c:(  !A.B + A                                          A.B.C.D )
c:( 2,6,10,14                                            + 15
c: 2,3,4,5,6,
c == middle
20 == XNOR

D.!C+!D.C =>
c thru g rewo
d:(!B+!(A.C)                                           .((C.D+!C
e:((A.B+!B.C)                                          .C).((C.D
f:(!A.!C+B.C)                                          !(C.D+!C.
g:(!B+!(A.C))                                          C)+!(A.(B
C.D+!C.!D).A)

d:( !A.!B  +
d:( 0,4,8,12
d: 0,4,5,6,8,
d == top left

e: A.B.!C.D +
e: 11  +  12,
e:0,2,6,8,10,
e == bottom

f: !( !B  +                                            B.!C.D + A
f:!( 0,1,4,8,                                          + 0,2,8,
f:!(0,1,2,4,5
f:(3,6,10,15)
f:(0,2,3,5,6,
f == top

g: !(   A.B.C
g: !( 7,15 +
g:0,1,3,4,5,6,7,8,9,10,11,13
g == bottom right

bingo!
```





July 2010 = Fibonacci (Comb' seminar)

- **Find n such that**
  $10^9 \mid \mathrm{round}(1+2\cos(20º))^n)$
- **$X_{t+1} = 3x_t - x_{t-2}$**

October 2010 = Pentagon lake

- **What is the area of 2,3,4,5 trapezoid?**
- **Find a convex pentagon shaped lake with integer edges lengths and two right angles**
- **Minimal area 33.99, integer area 70**
- **What is the density?**

November 2010 = Benoît Mandelbrot

```
f(x,y,m){return      x&m&&m&y?x>m?f(
x&m,m      &y,m/      2):f(      (y>m?
-y:y)      &m,x&m,m/2):x-m     -2&&--
y&&y-                           m-m;}
main(z){for(z=N       *N;z--;printf(
           "%c%c"     ,64>>f
(1+z%N,1+z/N,N),    " \n"[1>z%N]));}
```

December 2010 = Elections

- In an election, Charles came in last and Bob received 24.8% of the votes.
- After counting two additional votes, he overtook Bob with 25.1% of the votes.
- Assuming there were no ties and all the results are rounded to the nearest promille (one-tenth of a percent), how many votes did Alice get?

January 2011 = Nim like game

- **Playing on a word**
  - Start with N beans
  - Alternately take 1 for A; 2 for B; …; 26 for Z
  - If you can not play – you lose
- **Winning place is when the n-th digit of 204193/178481 + 2**-40, in octal, is 1**
- **Someone guessed the word**

February 2011 = Strange palindrome

- **Find N such that**
  - N=Flip(N) in seven-segments
  - N*N is flippable
  - N (mod 2011) = 0
- **Bonus – N (mod 2011) = 100**
- **Smallest solution by brute force**
- **Simple solution with almost no calculation**
- **1 is not exactly flippable…**

April 2011 = Vi Hart's snakes

- **How many 20 segments snakes are they?**
- **Starts with Fibonacci, till 12-segments**
- **Subtracting 26 gives the # of IBM patents on 2010**
- And also 2010 and 5896 have the same Heegner number (67) as divisor ☺

May 2011 = Bar Code

- What is the mean distance between bad (at least 20 black bits long) bars?
- The Math Factor paradox

June 2011 = Parking cars

- What is the efficiency of random car parking?
- Can be solved analytically
- Handwritten or printed font?

PROBLEM (*Ponder This*, June 2011).

A circular road is divided into 100 sectors. One by one, cars park across a pair of adjacent unoccupied sectors, chosen uniformly at random among all such pairs. Eventually, no such pairs remain, so no more cars can park. On average, how many cars find a space?

SOLUTION ( ).

July 2011 = Minimal perfect hash

- Find a schedule for 80 people to eat 3 deserts for 14 days such that
- For any three there exists a day in which they ate different desert
- Lower bound – 10
- 80 = punched card

August 2011 = Cooperation

- Circumference: I don’t know
- Area: Me neither
- Circumference: Now I know
- Area: Me too

A proof that there are only 3 solutions

September 2011 = random bits & dice

- Take 27 bits
  Convert it to dice
  Convert back to bits
- How many bits (on average) you get?
- ~23.99978
  Why is it so close to an integer?

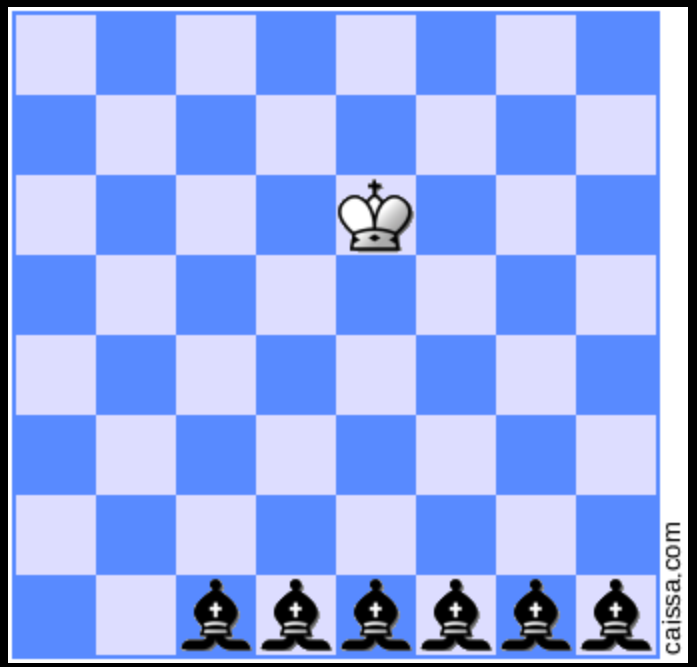
October 2011 = Resistors network

- *Interesting* = prod of every 8 decimal digits is 40,320
- Find a circuit with < 20 resistors whose resistance R is *interesting.*
- We can do it with 12 (even 11, but not parallel-series)
- And with 53 to get all 10 digits

November 2011 = Volleyball Tournament

December 2011 = Infinite chess

- How many white Queens are needed to checkmate a black King?
  - **Rooks?**
  - **kNights?**
  - **Bishops?**
- 1/Q + 1/R + 1/B + 1/N = ?



January 2012 = Exploding eggs

- After 2^{-i} seconds, the ith dwarf toggles every i^{th} lamp
- The red dragon lay eggs on n(n-1)/2
- If an egg near 576 mod 1000 explodes, what time is it (day, hour, minute, second)?

The answer is 6 ponies.

Just kidding, I have no clue what this problems means,

but my boyfriend is having trouble solving it right now and I would love to get my name on the board just to drive him nuts.

In case you sympathize with my situation, my name is Ariane Huddleston.

If you don't care about my situation (honestly I wouldn't either), then thanks for your time and have a fantastic day!

February 2012 = Biased coin

- I win if the first head is
  - **The 1st, 14th, 15th, 18th, 19th, or 23rd**
- What is the best approx for p with <10 digits denominator?

March 2012 =?

- ?

Epilog – Beware, it might be addictive

Thanks Oded,

In the evening, I got into the riddles corner, just for a minute - and my husband found me still there at 1am.

Thanks again,

β