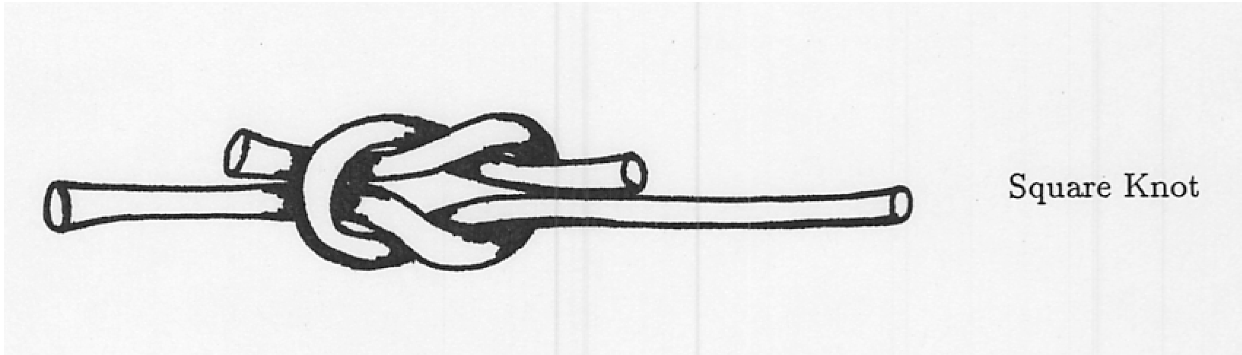
The Granny Knot, the Square Knot and the Analysis of Hitches
by Louis H. Kauffman

Here is the square knot.

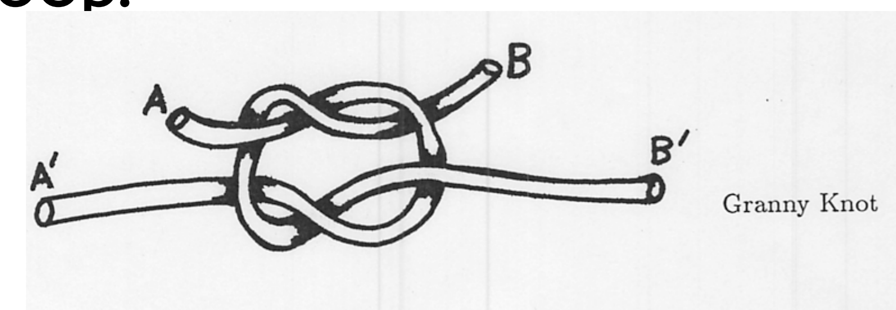
Square Knot

The square knot makes an excellent splice. That is, you can reliably join two lengths of rope by using the square knot as shown above.



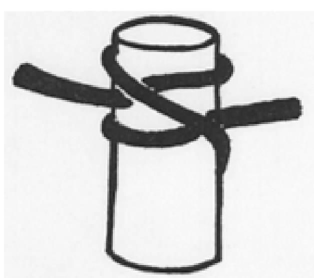
The reason the square knot is so good as a splice is that forces applied to the two lengths of rope cause each of the two loops in the splice to constrict the base of the other loop.

Here is the granny knot.

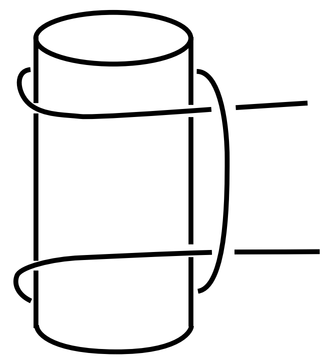


You might attempt to use the granny knot as a splice, but this is a dangerous idea. Forces applied at A' and B' will not constrict the bases of the loops, and the whole assemblage can slip.

These properties of the square knot and the granny knot are well known. What is not so well known is that each splice can be converted to a ‘hitch’ by making one of the lengths of rope the axis (post) for the hitch. When we make the conversion, it turns out that the granny knot becomes the well-known clove hitch, a very reliable hitch! The very dangerous granny knot converts to a very reliable hitch. Conversely, the very reliable square knot converts to a less reliable but still workable hitch that we call the square hitch.



The clove hitch.



The square knot hitch

We will give illustrations of the conversion of the square and granny into corresponding hitches after the next section. In this next section we give an excerpt from the Author's book "Knots and Physics" (World Scientific, 1991-2012) where we give an exposition of the analysis of hitches due to Bayman:

Benjamin Bayman, Theory of Hitches, Amer. J. Physics, Vol. 45, No. 2, Feb. 1977.

After the excerpt, we show the conversions and we analyse the square hitch in the same fashion as we shall have already analysed the granny hitch aka the clove hitch.

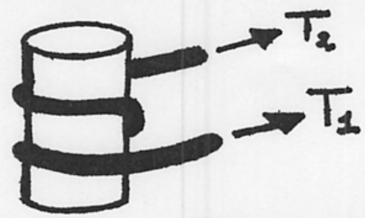
Excerpt from “Knots and Physics” by LK.

1^0. Theory of Hitches.

This section is based on the article [BAY].

We give a mathematical analysis of the properties of **hitches**. A **hitch** is a mode of wrapping a rope around a post so that, with the help of a little friction, the rope holds to the post. And your horse does not get away.

First consider simple wrapping of the rope around the post in coil-form:



Assume that there are an integral number of windings. Let tensions T_1 and T_2 be applied at the ends of the rope. Depending upon the magnitudes (and relative magnitudes) of these tensions, the rope may slip against the post.

We assume that there is some friction between rope and post. It is worth experimenting with this aspect. Take a bit of cord and a wooden or plastic rod. Wind the cord one or two times around the rod. Observe how easily it slips, and how much tension is transmitted from one end of the rope to the other. Now wind the cord ten or more times and observe how little slippage is obtained – practically no counter-tension is required to keep the rope from slipping.

In general, there will be no slippage in the T_2-direction so long as

$$T_2 \leq \kappa T_1$$

for an appropriate constant κ. This constant κ will depend on the number of windings. The more windings, the larger the constant κ.

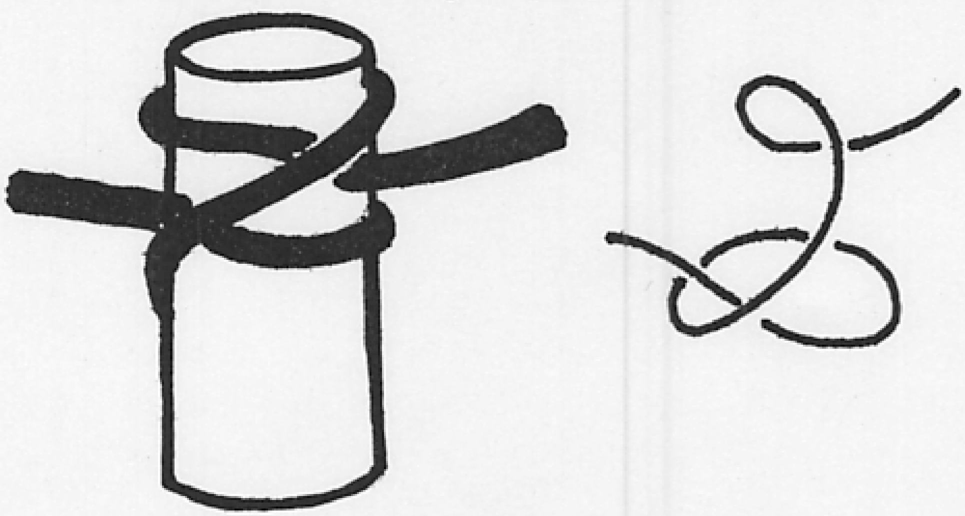
A good model is to take κ to be an exponential function of the angle (in radians) that the cord is wrapped around the rod, multiplied by the coefficient of friction between cord and rod. For simplicity, take the coefficient of friction to be unity so that

$$\kappa = e^{\theta/2\pi}$$

where θ is the total angle of rope-turn about the rod.

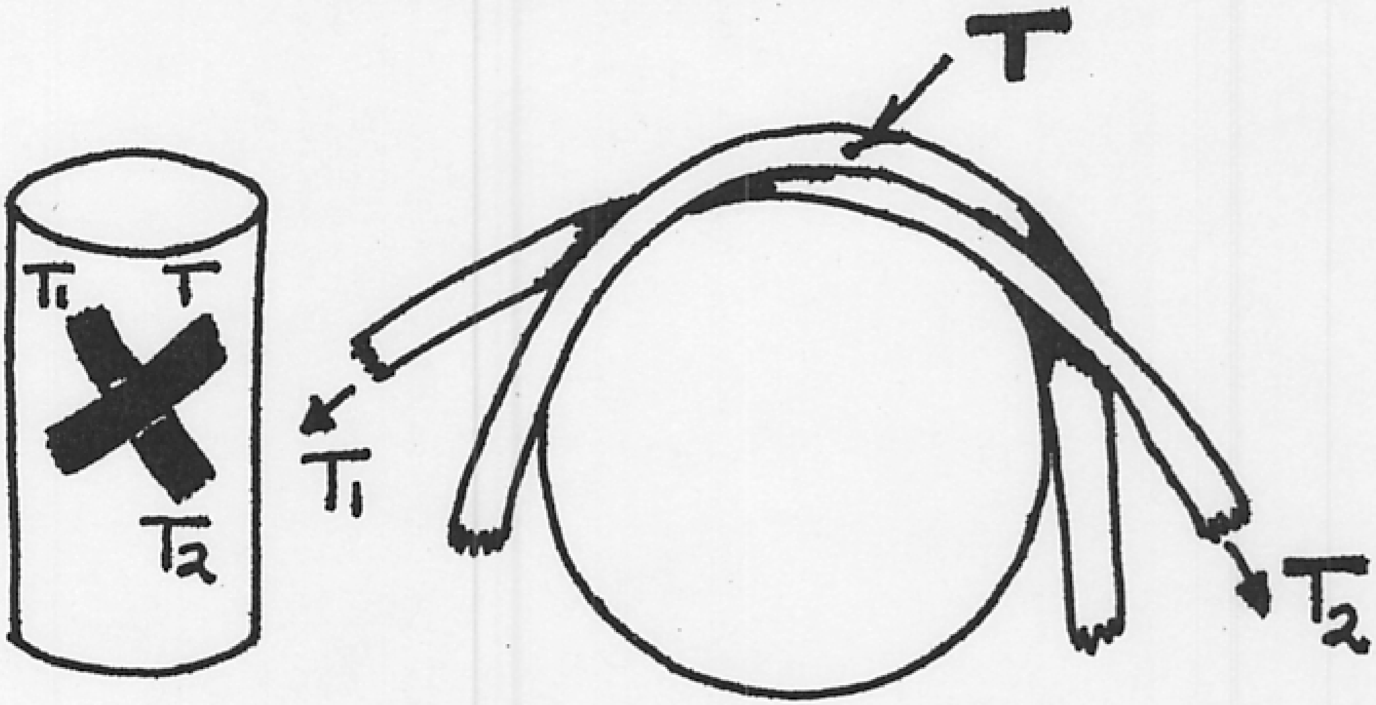
Thus, for a single revolution we need $T_2 \leq eT_1$ and for an integral number n of revolutions we need $T_2 \leq e^n T_1$ to avoid slippage.

A real hitch has "wrap-overs" as well as windings:



Clove Hitch

Here, for example, is the pattern of the clove hitch. In a wrap-over, under tension, the top part squeezes the bottom part against the rod.

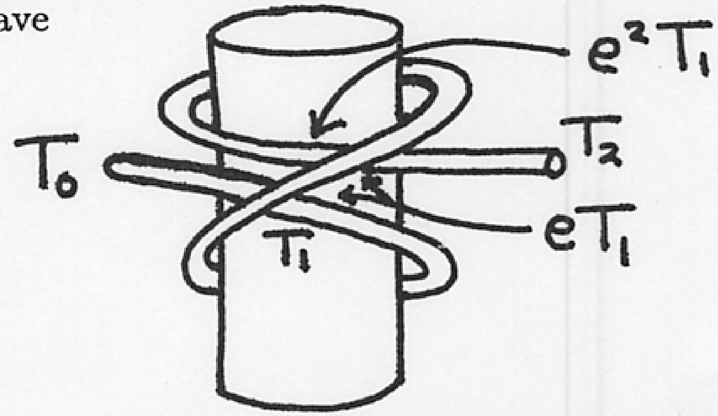


Hold Fast

$$T_2 \leq T_1 + uT$$

This squeezing produces extra protection against slippage. If, at such a wrap-over point, the tension in the overcrossing cord is T, then the undercrossing cord will hold-fast so long as $T_2 \leq T_1 + uT$ where u is a certain constant involving the friction of rope-to-rope, and T_2 and T_1 are the tensions on the ends of the undercrossing rope at its ends.

With these points in mind, we can write down a series of inequalities related to the crossings and loopings of a hitch. For example, in the case of the clove hitch we have



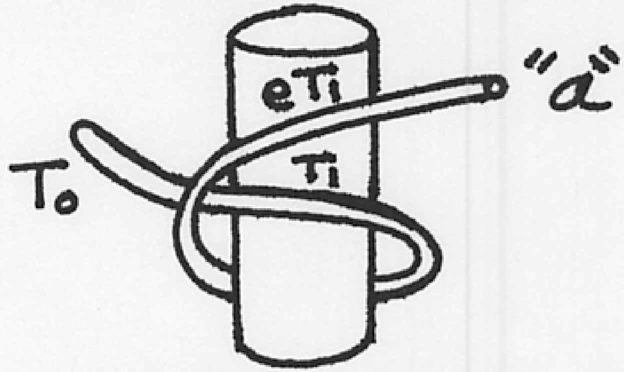
$$\boxed{\begin{array}{lcrcr} T_1 & \leq & T_0 & + & ueT_1 \\ T_2 & \leq & e^2T_1 & + & ueT_1 \end{array}}$$

that the equations necessary to avoid slippage are:

$$T_1 \leq \quad T_0 + ueT_1$$
$$T_2 \leq e^2T_1 + ueT_1.$$

Since **the first inequality holds whenever** $ue > 1$ or $u > 1/e$, we see that **the clove hitch will not slip** no matter how much tension occurs at T_2 just so long as the rope is sufficiently rough to allow $u > 1/e$.

Remark. Let's go back to the even simpler "hitch":



$$\boxed{T_1 \leq T_0 + ueT_1}$$

Our abstract analysis would suggest that this will hold if $ue > 1$. However, there is no stability here. A pull at "a" will cause the loop to rotate and then the "u-factor" disappears, and slippage happens. A pull on the clove hitch actually tightens the joint.

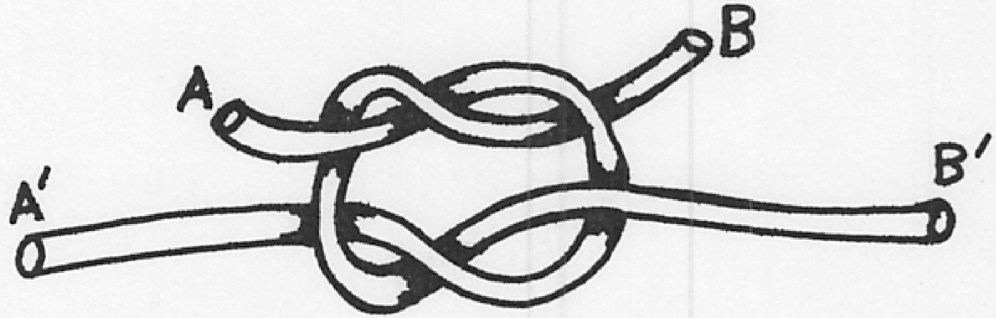
This shows that in analyzing a hitch, we are actually taking into account some properties of an already-determined-stable mechanical mechanism that happens to be made of rope. [See also Sci. Amer., Amateur Sci., Aug. 1983.]

There is obviously much to be done in understanding the frictional properties of knots and links. These properties go far beyond the hitch to the ways that ropes interplay with one another. The simplest and most fascinating examples are the **square knot** and the **granny knot**. The square knot pulls in under tension, each loop constricting itself and the other - providing good grip:

Square Knot

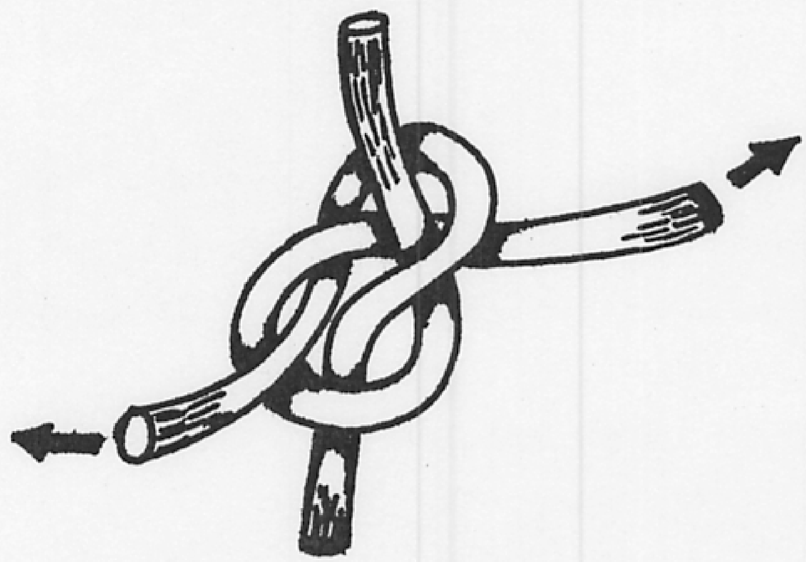
Construct this knot and watch how it grips itself.

The **granny** should probably be called the **devil**, it just won't hold under tension:

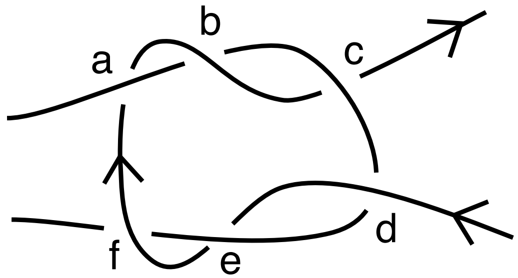


Granny Knot

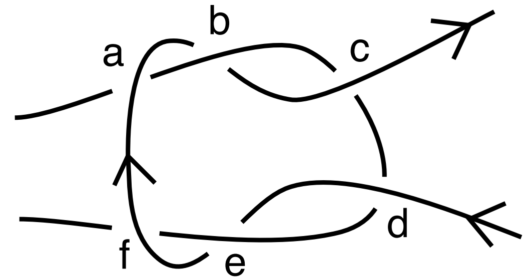
Try it! Ends A and B, are twisted perpendicular to ends A' and B' and the rope will feed through this tangle if you supply a sufficient amount of tension.



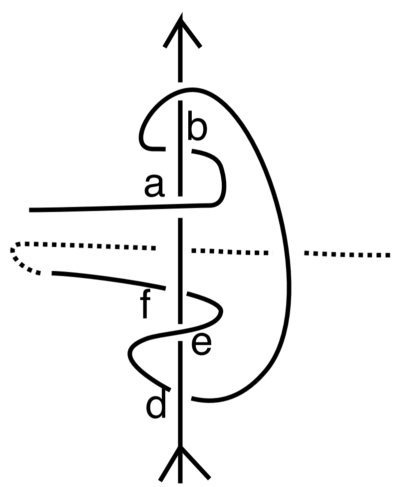
Interconversion of the Granny and the Square Knot to Corresponding Hitches.

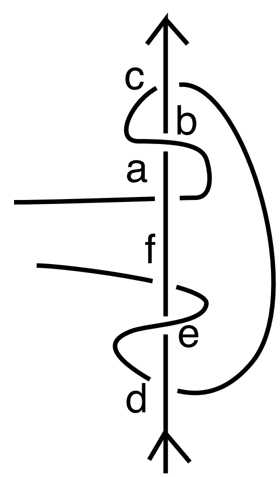


Granny Knot

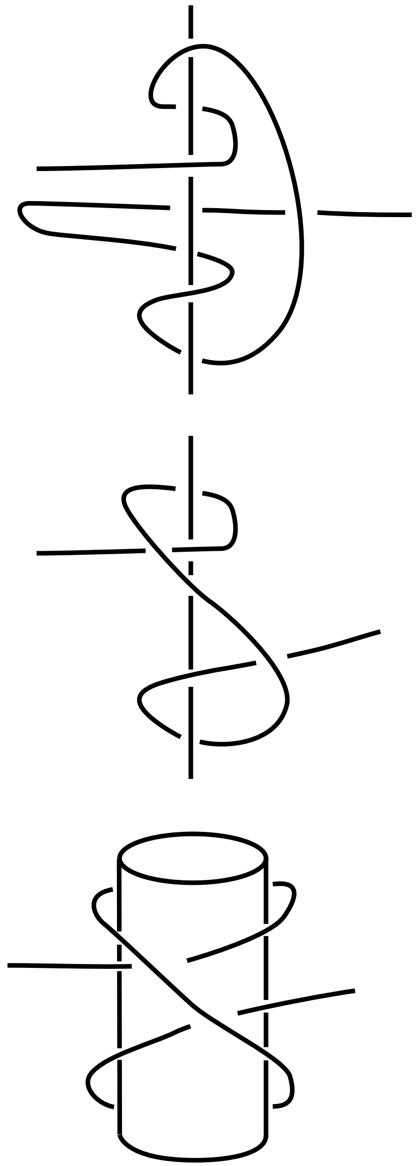


Square Knot

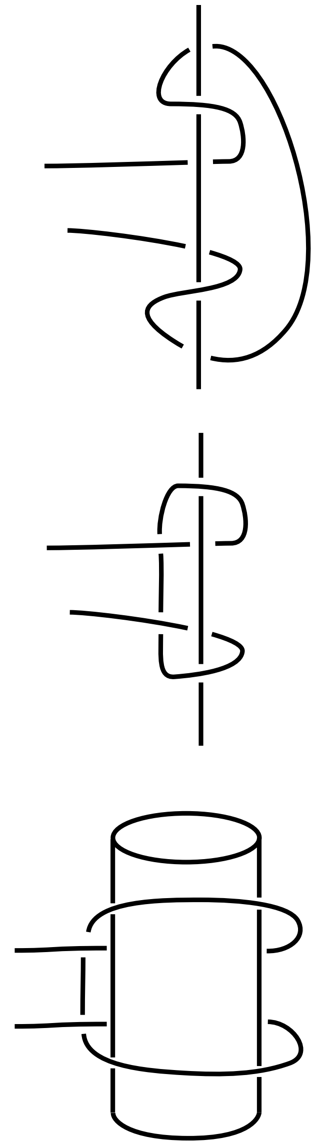




Straighten the Oriented Axis in order
to convert the knot (splice) to a hitch.

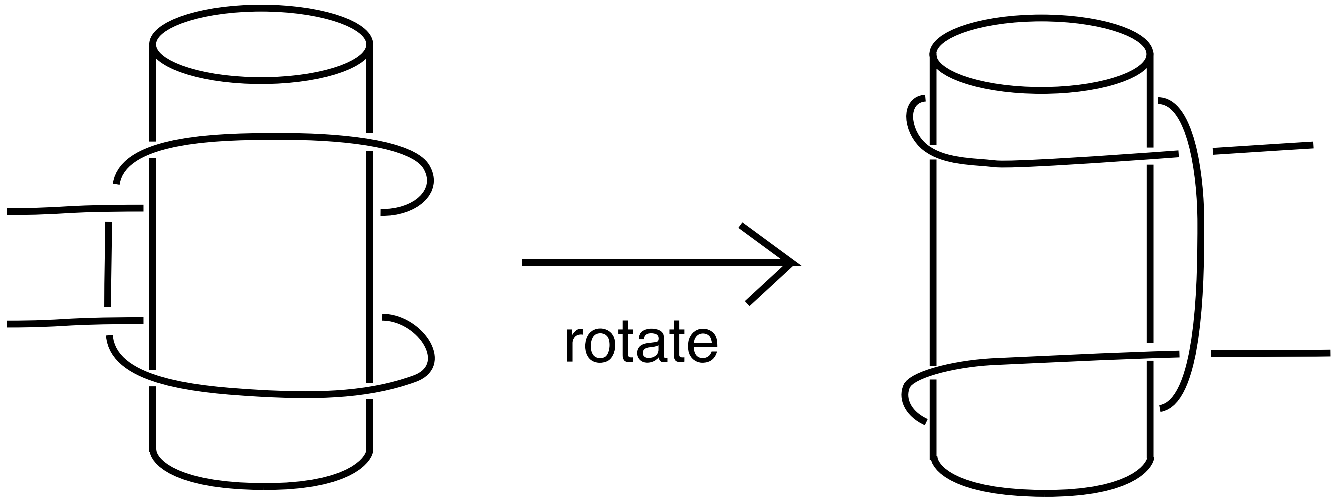


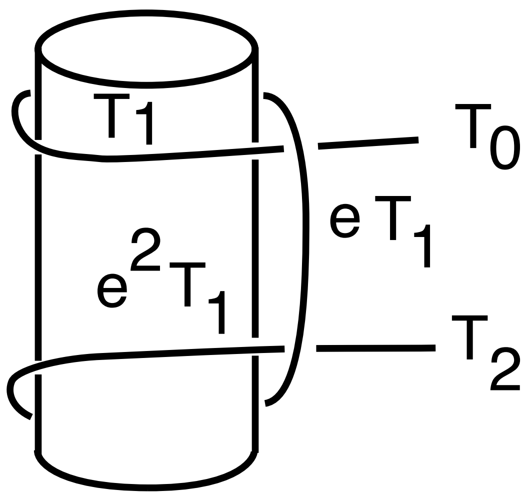
Granny Hitch
same as the
Clove Hitch



SquareKnot Hitch

SquareKnot Hitch





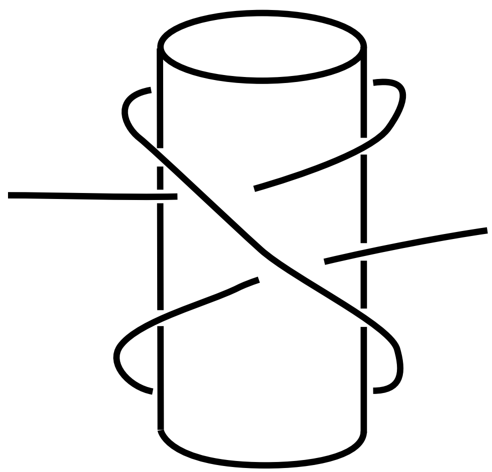
$$T_1 < T_0 + ueT_1$$

$$T_2 < e^2T_1 + ue\,T_1$$

Need ue >1 and so hitch will hold for sufficiently rough rope.

In our Bayman-type analysis of the square knot hitch, we found that if the parameter $u > 1/e$ where e is post's friction parameter, then the hitch will hold. This has the same appearance as the end-result of our analysis of the clove hitch in the middle section of the paper, but if you make these hitches you will see that there is a world of difference between the square hitch and the granny hitch (aka clove hitch). The granny hitch makes good contact with the post, while the square hitch does not make good contact. The weaving points in the square hitch tend to be pulled away from the post. This means that much more depends upon the friction of the rope against itself than in the granny hitch.

The moral of our story is that there is much physical lore to be extracted from individual weaving patterns. A full mathematical analysis of knots and their behaviour under forces and friction is a subject for the future.



Granny Hitch
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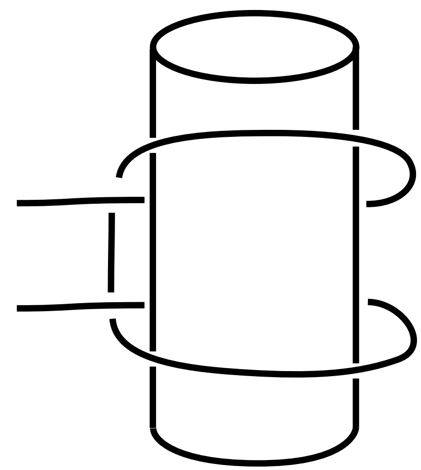

SquareKnot Hitch