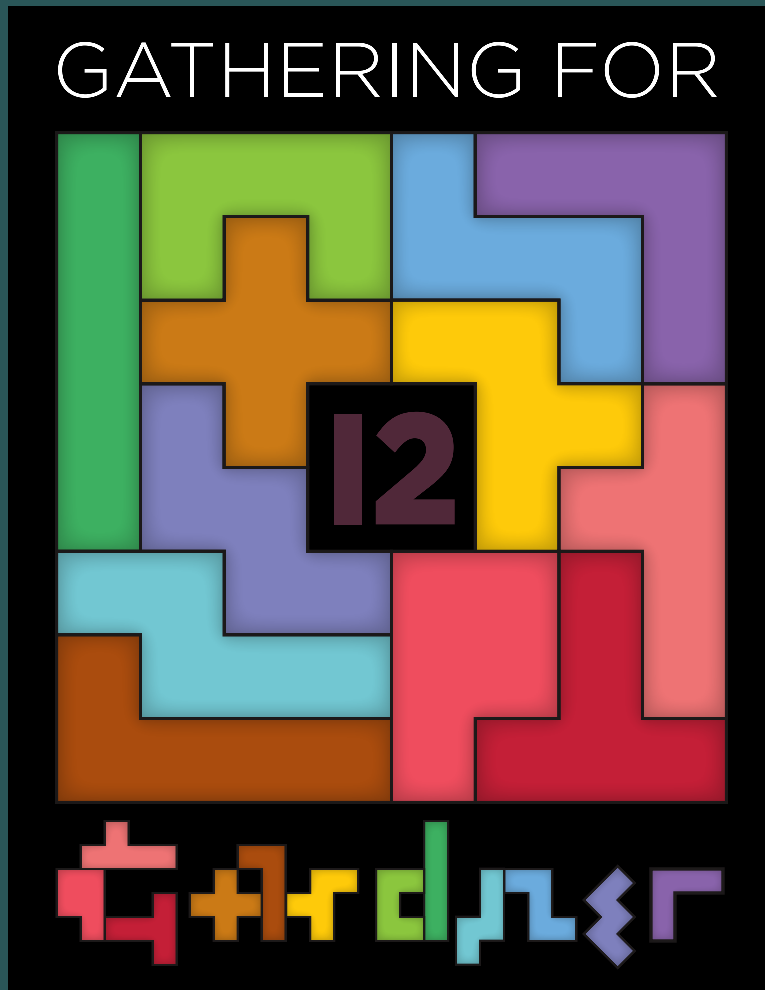


G4G12 Exchange Book

VOLUME 2

Legacy, Puzzles, & Science

Atlanta, Georgia

MARCH 30 - APRIL 3, 2016

G4G12 Exchange Book

VOLUME 2

The Gift Exchange is an integral part of the Gathering 4 Gardner biennial conferences. Gathering participants exchange gifts, papers, puzzles and other interesting artifacts. This book contains gift exchange papers from the conference held in Atlanta, Georgia from Wednesday, March 30th through Sunday, April 3rd, 2016. It combines all of the papers offered as exchange gifts in two volumes.

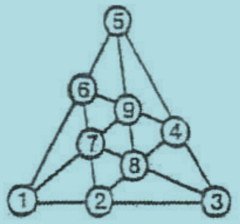
Table of Contents | VOLUME 2

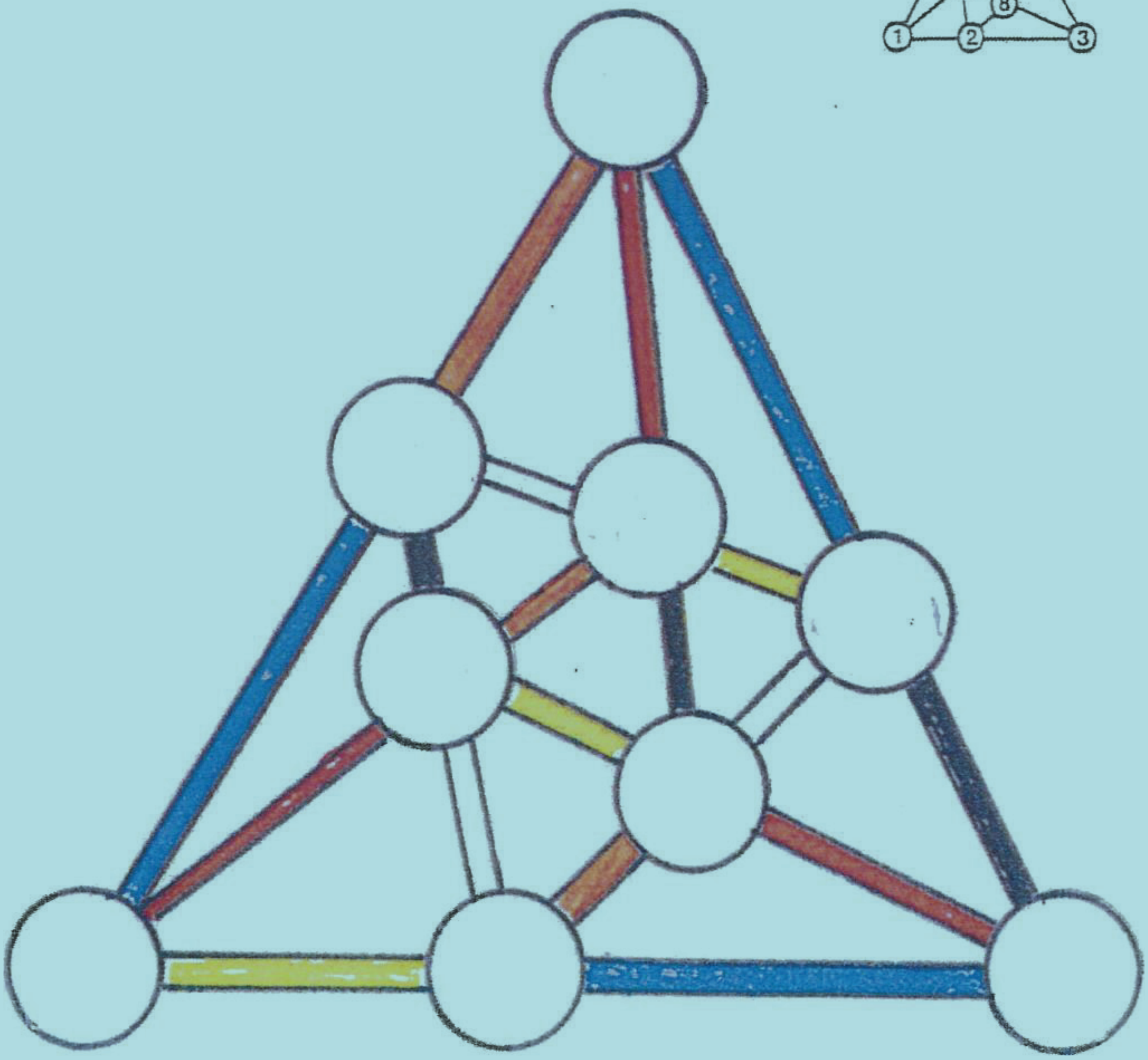
LEGACY

PUZZLES

Table of Contents (cont.)

KATe JONES





LEGACY

Kate Jones – A Tribute | Karen Farrell & Jeremiah Farrell | Page 11

Juvenilia

Dana Richards

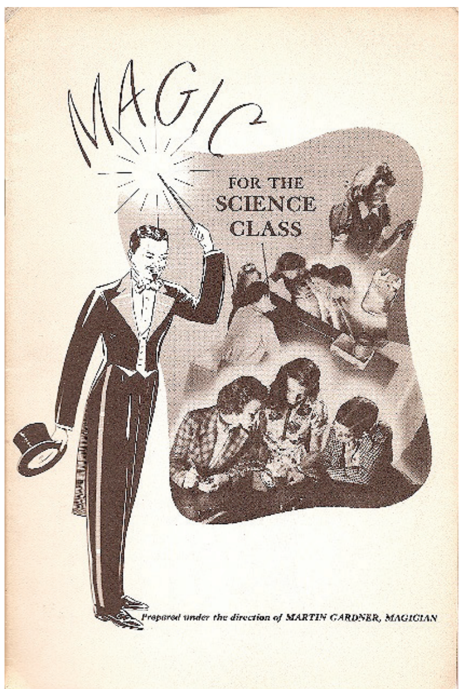
1 : compositions produced in the author's youth
2 : literary compositions suited to the young
Webster's Collegiate Dictionary

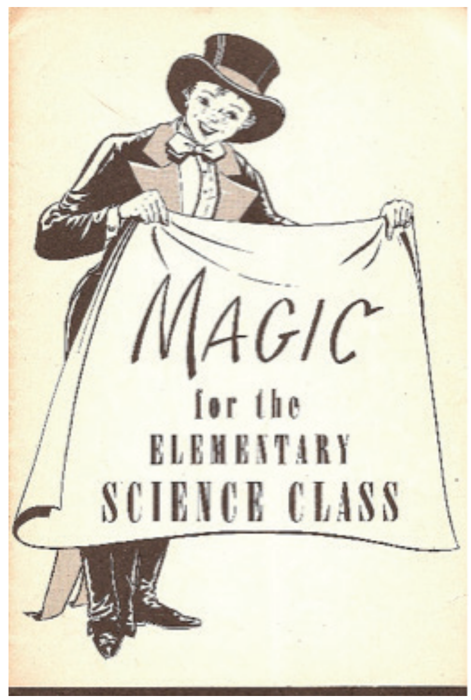
This is based on the talk given at G4G12 on the various efforts Martin Gardner made while young and those made for the young reader. We begin with a brief mention of his earliest influences.

Two periodicals he read avidly as a child were *John Martin's Magazine* and *Science and Invention*. His interest was so abiding that 70 years later he acquired complete runs of them, in hopes of passing on the appreciation to a new generation. The first was suited for younger readers and contained the puzzles of George Carlson. The second was published by Hugo Gernsback and was full of science, recreations, and debunking of pseudo-science. He was also a young fan of Sam Loyd.

Gardner wrote a lot of poetry at Central High School in Tulsa. Science-fiction was an early influence, hence the poem "An Ethraldrian Gazes at the Earth" and the like. In February 1930 Ripley's "Believe It or Not" published a submission of his. In April 1930 *Science and Invention* published a question Gardner sent to "The Oracle". And in May 1930 *The Sphinx*, a prominent magic journal, published a trick of his. The editors encouraged him and by June his name had moved to the title "The Best Pocket Tricks of Martin Gardner." He continued to contribute often. In May 1932 he had letter published in *The Cryptogram* encouraging more difficulty and in-depth articles. In September 1934 his article "A Puzzling Collection" appeared in *Hobbies*, which detailed his extensive collection of mechanical puzzles. At this point his public writing became associated with his studies at the Univ. of Chicago.

His first publication for children was an outgrowth of the pamphlets he was writing for magicians. In the 1930s he had begun in earnest to accumulate and sort his growing knowledge on several subjects including magic. Drawing on his files he wrote two pamphlets for a publisher of science textbooks in 1941: *Magic for the Elementary Science Class*, and *Magic for the Science Class*. His profession was listed as "magician."





After the war he contributed to *Uncle Ray's Magazine*. His column contained stunts, science tricks and math puzzles.

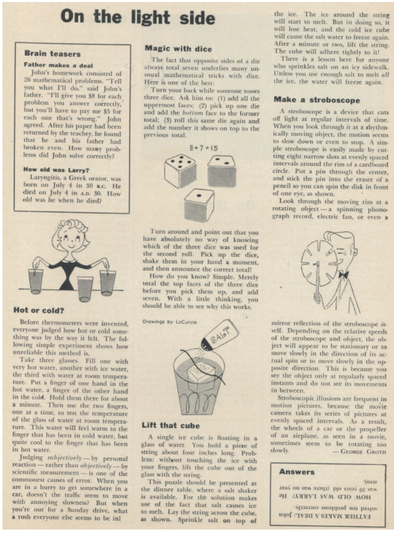
After moving from Chicago in 1948 to New York he came to the notice for publishers. In particular, he began contributing regularly to Parents Institute, which was keen to expand their magazine line.

- *Children's Digest* (Sept. 1951)
- *Parents Magazine* ("Family Fun," Jan. 1952)
- *Humpty Dumpty* (Oct. 1952)
- *Polly Pigtails* (Spring 1953)
- *Piggly Wiggly / Piggity* (Winter 1953)
- *Children's Playcraft* (Jan. 1954)

He contributed to the titles above irregularly, except for *Humpty Dumpty*, which edited from the first issue. He told his mother he was the "contributing editor in charge of gimmicks." He provided filler material, games, stunts, puzzles and the occasional article. But for *Humpty Dumpty* he also contributed a short story about the titular character and he also contributed a cautionary verse about good behavior. (Some of the latter were collected in *Never Make Fun of a Turtle, My Son* which he joked could have been called "Poems of Sage Fatherly Advice to Undisciplined Children in this Democratic Age of Moral Rot".) The short stories have never been republished despite his concerted efforts.

He considered his years with Parents Institute to be a good and rewarding experience. He always spoke of it with pride. However he left by 1961, when they were just reprinting his contributions and in general treating him miserly.

As he was winding down at Parents Institute and ramping up at *Scientific American*, he took on the task of writing a column "On the Light Side" for *Science World*, a high school science magazine. This lasted 5 semesters mostly under the alias of "George Groth". The material was largely collected in the books *Mathematical Puzzles* and *Science Puzzlers* (since reprinted with different titles).



On the light side

Brain teasers

Father makes a deal

John's homework consisted of 26 mathematical problems. "Tell you what I'll do," said John's father. "I'll give you $8 for each problem you answer correctly, but you'll have to pay me $5 for each one that's wrong." John agreed. After his paper had been returned by the teacher, he found that he and his father had broken even. How many problems did John solve correctly?

How old was Larry?

Laryngitis, a Greek orator, was born on July 4 in 30 B.C. He died on July 4 in A.D. 30. How old was he when he died?

Hot or cold?

Before thermometers were invented, everyone judged how hot or cold something was by the way it felt. The following simple experiment shows how unreliable this method is.

Take three glasses. Fill one with very hot water, another with ice water, the third with water at room temperature. Put a finger of one hand in the hot water, a finger of the other hand in the cold. Hold them there for about a minute. Then use the two fingers, one at a time, to test the temperature of the glass of water at room temperature. This water will feel warm to the finger that has been in cold water, but quite cool to the finger that has been in hot water.

Judging *subjectively*—by personal reaction—rather than *objectively*—by scientific measurement—is one of the commonest causes of error. When you are in a hurry to get somewhere in a car, doesn't the traffic seem to move with annoying slowness? But when you're out for a Sunday drive, what a rush everyone else seems to be in!

Magic with dice

The fact that opposite sides of a die always total seven underlies many unusual mathematical tricks with dice. Here is one of the best:

Turn your back while someone tosses three dice. Ask him to: (1) add all the uppermost faces; (2) pick up one die and add the *bottom* face to the former total; (3) roll this same die again and add the number it shows on top to the previous total.

8 + 7 = 15

Turn around and point out that you have absolutely no way of knowing which of the three dice was used for the second roll. Pick up the dice, shake them in your hand a moment, and then announce the correct total!

How do you know? Simple. Merely total the top faces of the three dice before you pick them up, and add seven. With a little thinking, you should be able to see why this works.

Drawings by LoCurcio

SALT

Lift that cube

A single ice cube is floating in a glass of water. You hold a piece of string about four inches long. Problem: without touching the ice with your fingers, lift the cube out of the glass with the string.

This puzzle should be presented at the dinner table, where a salt shaker is available. For the solution makes use of the fact that salt causes ice to melt. Lay the string across the cube, as shown. Sprinkle salt on top of the ice. The ice around the string will start to melt. But in doing so, it will lose heat, and the cold ice cube will cause the salt water to freeze again. After a minute or two, lift the string. The cube will adhere tightly to it!

There is a lesson here for anyone who sprinkles salt on an icy sidewalk. Unless you use enough salt to melt *all* the ice, the water will freeze again.

Make a stroboscope

A stroboscope is a device that cuts off light at regular intervals of time. When you look through it at a rhythmically moving object, the motion seems to slow down or even to stop. A simple stroboscope is easily made by cutting eight narrow slots at evenly spaced intervals around the rim of a cardboard circle. Put a pin through the center, and stick the pin into the eraser of a pencil so you can spin the disk in front of one eye, as shown.

Look through the moving rim at a rotating object—a spinning phonograph record, electric fan, or even a mirror reflection of the stroboscope itself. Depending on the relative speeds of the stroboscope and object, the object will appear to be stationary or to move slowly in the direction of its actual spin or to move slowly in the opposite direction. This is because you see the object only at regularly spaced instants and do not see its movements in between.

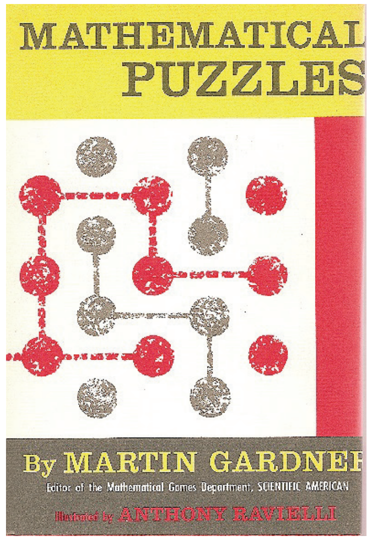
Stroboscopic illusions are frequent in motion pictures, because the movie camera takes its series of pictures at evenly spaced intervals. As a result, the wheels of a car or the propeller of an airplane, as seen in a movie, sometimes seem to be rotating too slowly.

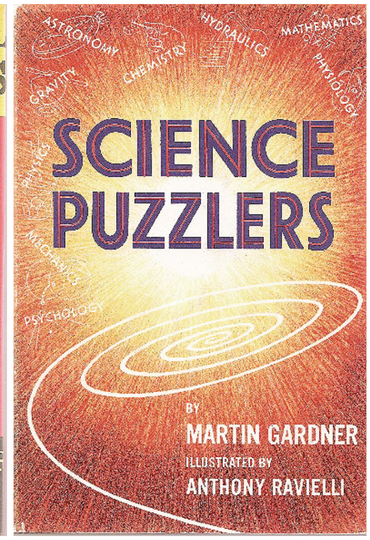
— George Groth

Answers

FATHER MAKES A DEAL: John solved ten problems correctly.

HOW OLD WAS LARRY? He was 59 years old (there was no year zero).





He became very busy and in demand after this. However he regularly found time to write for the youth market.

- *The Arrow Book of Brain Teasers*, 1959.
- *Archimedes, Mathematician and Inventor*, 1965.
- *Perplexing Puzzles and Tantalizing Teasers*, 1969.
- *Space Puzzles*, 1971.
- *Codes, Ciphers and Secret Writing*, 1972.
- *The Snark Puzzle Book*, 1973.
- *More Perplexing Puzzles and Tantalizing Teasers*, 1977.
- *Classic Brainteasers*, 1994.
- *Science Magic*, 1997. (aka *Science Tricks*, 1998.)
- *Mind-Boggling Word Puzzles*, 2001.
- *Smart Science Tricks*, 2004.
- *Optical Illusion Play Pack*, 2008.

In addition to these books he contributed the Oz literature in many ways. He edited reprints of his youthful hero George Carlson, in two "Peter Puzzlemaker" volumes. However his most recognized contribution to the education of young readers was the creation of two boxed sets of filmstrips—*The Aha Box* and *The Paradox Box*. These became the books *Aha*! and *Aha. Gotcha*!.

GIFT EXCHANGE
For
G4G12
March 2016

KATE JONES—A TRIBUTE

By Karen and Jeremiah Farrell

It would be a rare puzzlist who is not aware of Kate Jones, but perhaps her many accomplishments are not as well-known as they should be.

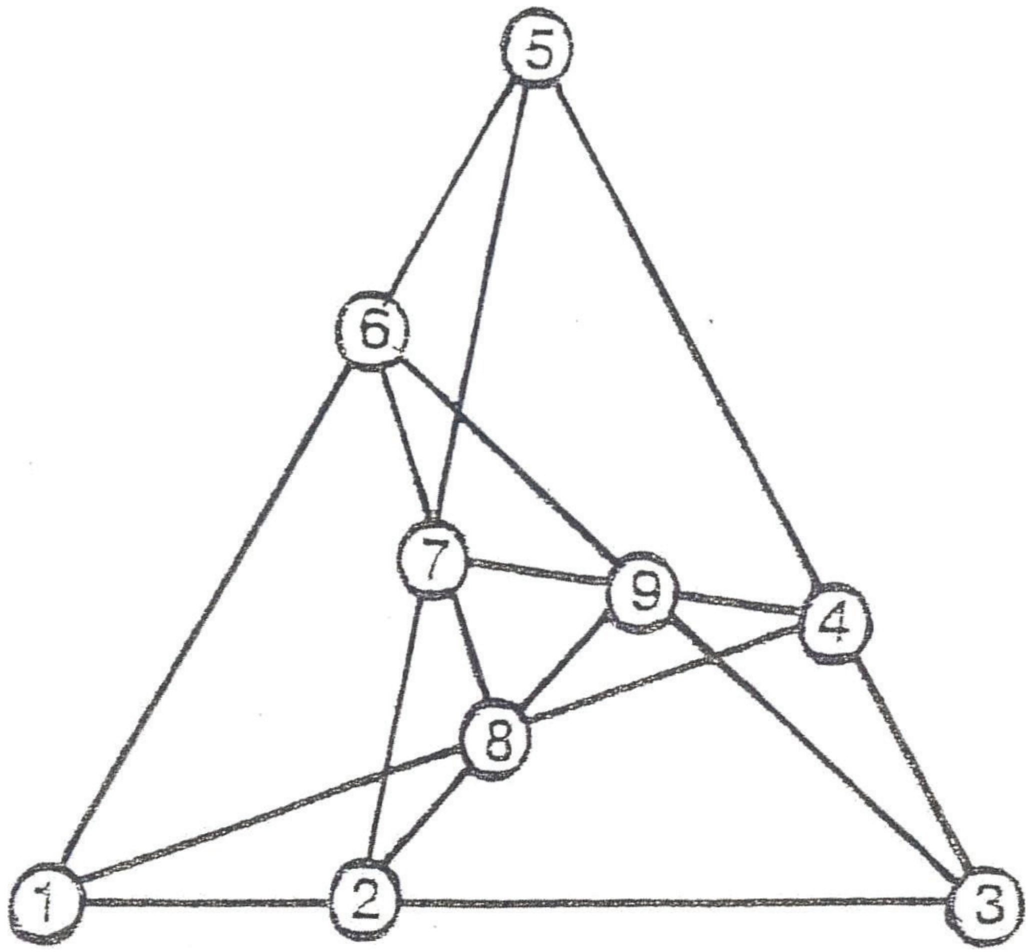
She has been active for over 35 years with puzzles; attending many, many International Puzzle Party events and Gatherings for Gardner. Representing her Kadon Enterprises' business, she has produced over 200 original puzzles and received 53 "Games 100" selections from *Games Magazine* (and lots of prize ribbons at art shows). Notable among her offerings at Kadon would have to be Martin Gardner's "The Game of Solomon" and Solomon Golomb's pentominoes puzzle. The late Tom Rodgers spoke highly of these two puzzle-games as well as all of Kate's other issues. Rodgers may well have had the largest private collection of her works.

Kate is also an accomplished recreational mathematician and poet. To try to match in a small way her creative ability, we offer three puzzle-games in her honor: O'BEIRNE's TRI-HEX, PAPPUS and "KATe JONES". These three are specific examples of (9,3) symmetric configurations. More generally an (n,r) configuration is a collection of n "points"and n "lines" subject to the following requirements:

R1: Any two points belong to at most one line.
R2: Each line has r points, and each point belongs to r lines.

There are precisely three (9,3)s and our examples represent these. See (1) and (2) for more details.

(1) O'Beirne's Tri-Hex. T. H. O'Beirne (3) commercialized this game about 50 years ago and our puzzle version is played on the first diagram with the nine words listed. The words use the letters KATe JONES three times each (note that e and E are regarded as different). As a puzzle, place the nine words on the nodes so that each of the nine lines contains a common letter.

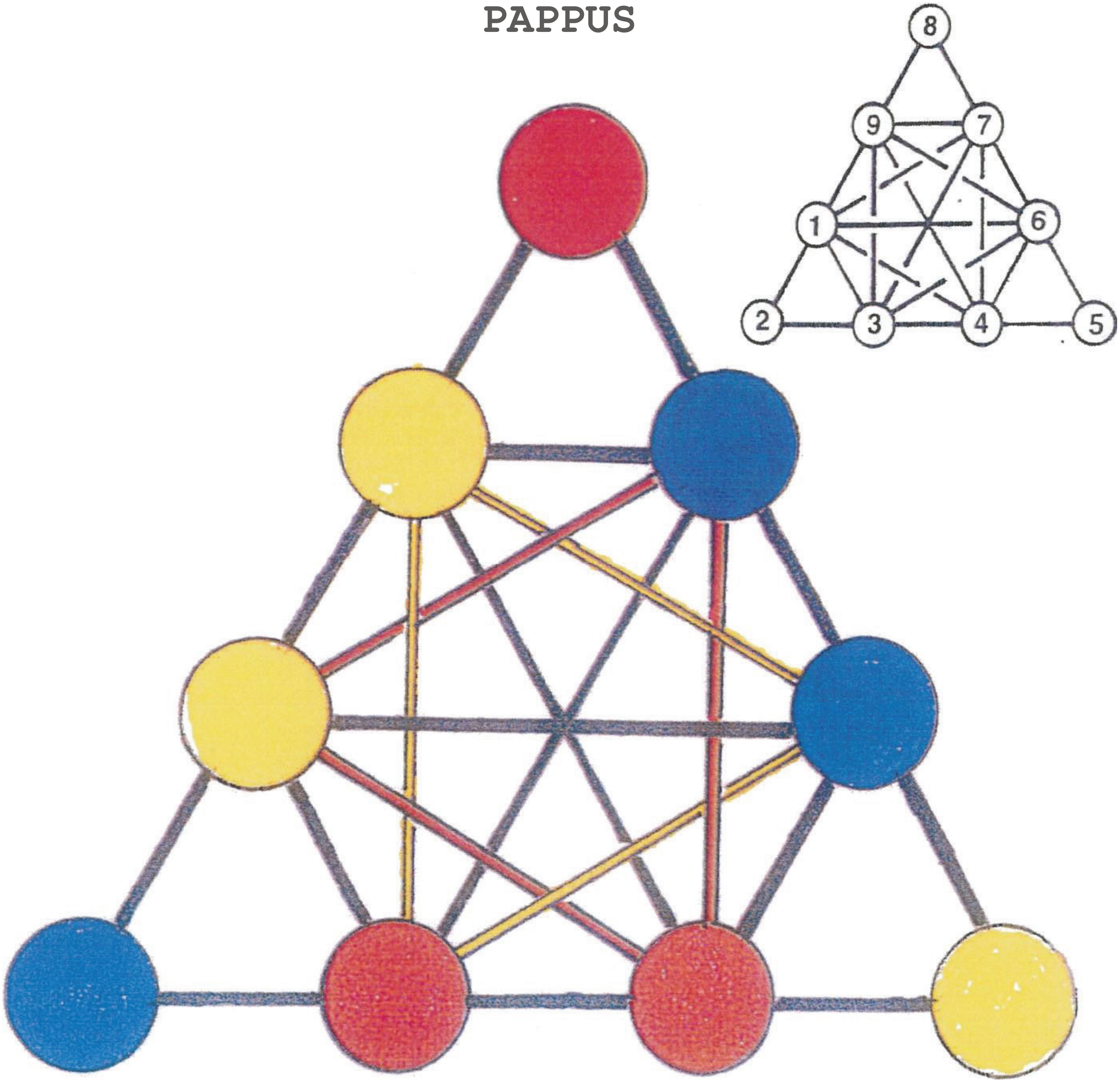


O'BEIRNE

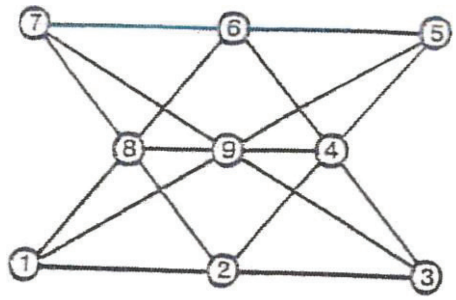
EKe	ENS	KAT
JAN	JET	JOe
OKS	SeA	TON

(2) PAPPUS. A new set of nine words from the letters of KATe JONES are to be placed on the second diagram so that the nine equilateral triangles contain a common letter at each of their nodes.

PAPPUS



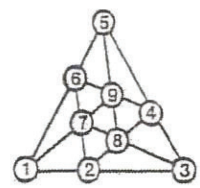
ATE	EKe	JAK
JeN	JOT	OKS
ONE	SAN	SeT

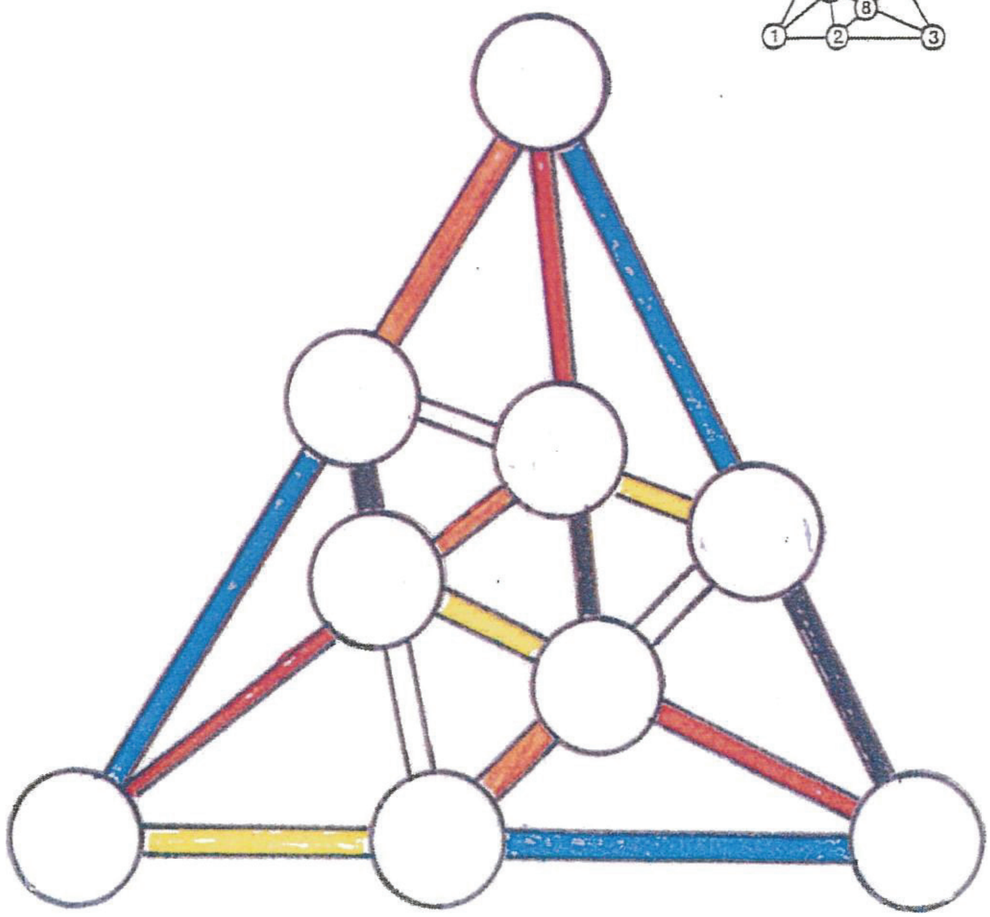


This puzzle is based on a theorem of Pappus (circa 300 A.D.) that describes a generalized hexagon on two lines that always results in three collinear points on its side intersections. Instead of triangles we could have used points and lines as per this diagram.

(3) KATe JONES. A final set of nine words are to be placed on the nodes of the third diagram so that each line contains a common letter.

KATe JONES





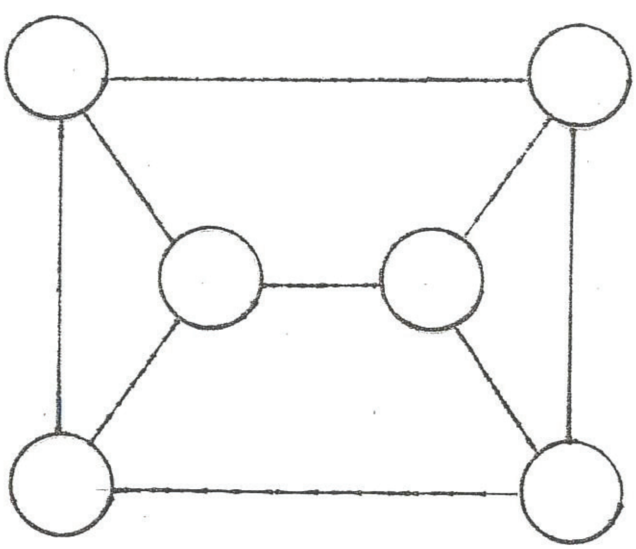
EKe	**JAK**	**JEN**
JOe	**OAT**	**OKS**
SAN	**SET**	**TeN**

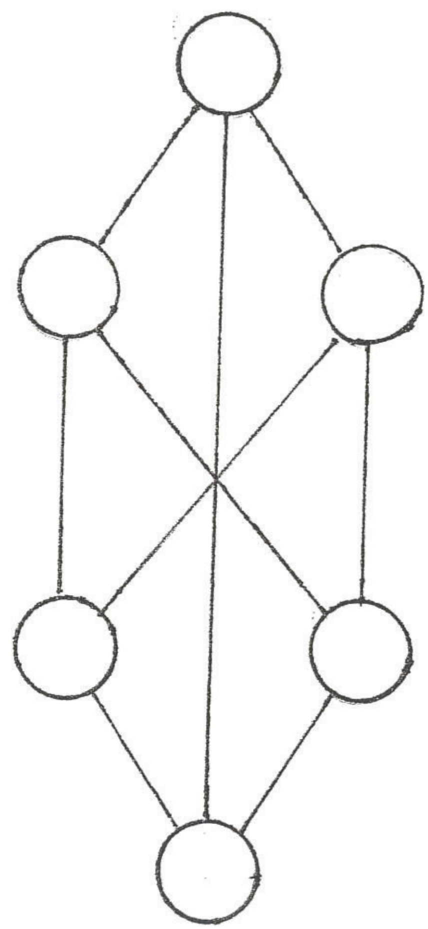
THE GAMES.

Each of three diagrams can be played as a "Tic-Tac-Toe" two-person game where each player has exactly four distinctive tokens. They alternately play a token on a node and the first to obtain a "line" wins. For fairness we rule that if First does not win in four moves then Second wins. There can be no ties.

Every (n,3) that we have studied save one is a first-person forced win. The only exception we have found is KATe JoNES where Second can instead force a win. Strategies will follow later. There are also three dual versions of the puzzles where the nine letters of KATe JONES can be written on tokens and placed on the nodes of a given puzzle so that each line anagrams into one of the possible nine words. Details are left to the reader.

Two additional puzzles. Place six three-letter words on the two following diagrams so that each of the letters in KATe JONES is used exactly two times.





ANSWERS.

Definitions of the more unusual words follow.

JAK: An Asian tree of the bread fruit genus

KAT: Ancient Egyptian unit of weight (or perhaps from "Krazy Kat")

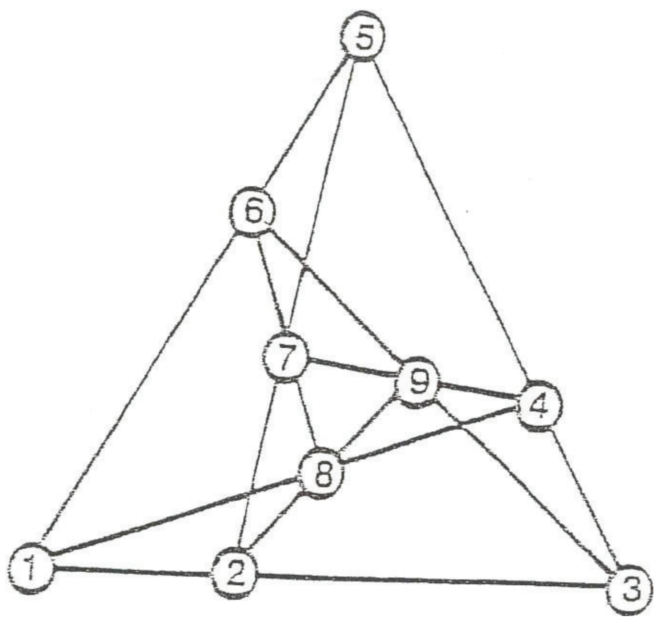
SAN: A member of a nomadic S. African tribe of huntsmen

ENS: Being or existence

JEN,JAN: Girls' names

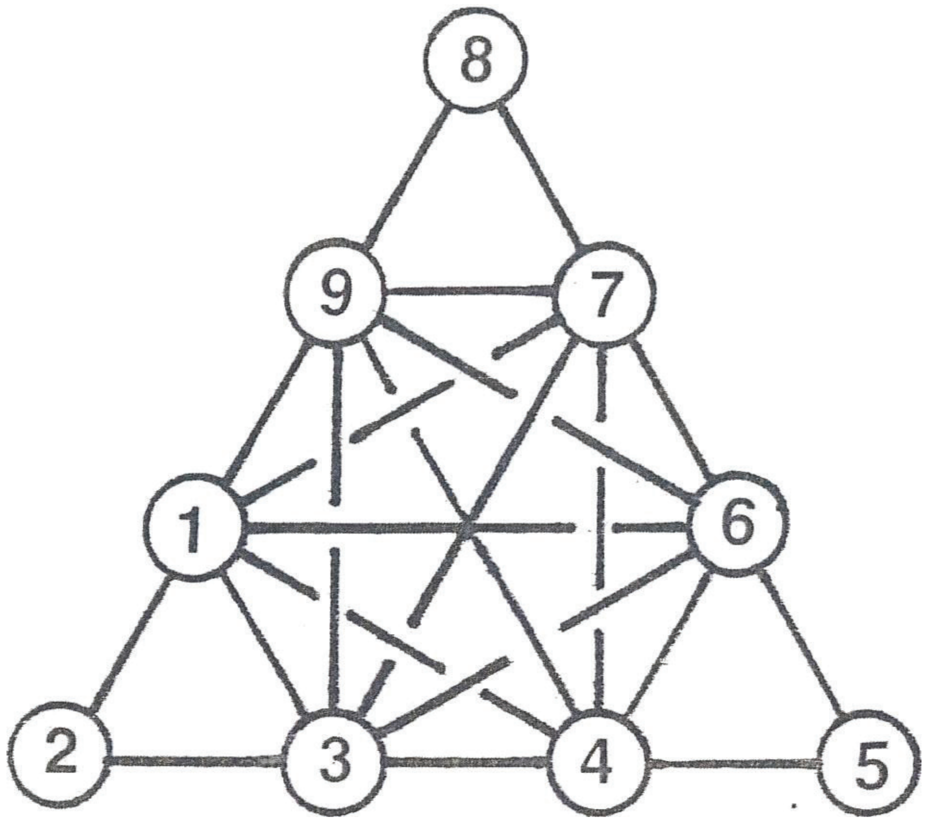
The answer sheet for KATe JONES supplies also an answer for the dual anagrammic puzzle. Also, if Second plays clockwise on the inner star-nonagon, Second can force First to lose in four moves. For example if First plays, say, OAT(7) then Second should play SAN(2) and thereafter play rationally.

For First to win at O'Beime's game, the player must start with oe of KAT, ENS, or JOe and play rationally afterwards. To win at the PAPPUS game, First can play on any color and if Second plays on that same color, First then plays on the third node of that color. If Second does not play on First's color then First plays to force Second to waste a move by playing a new color for which Second must block by playing a token on Second's original color.



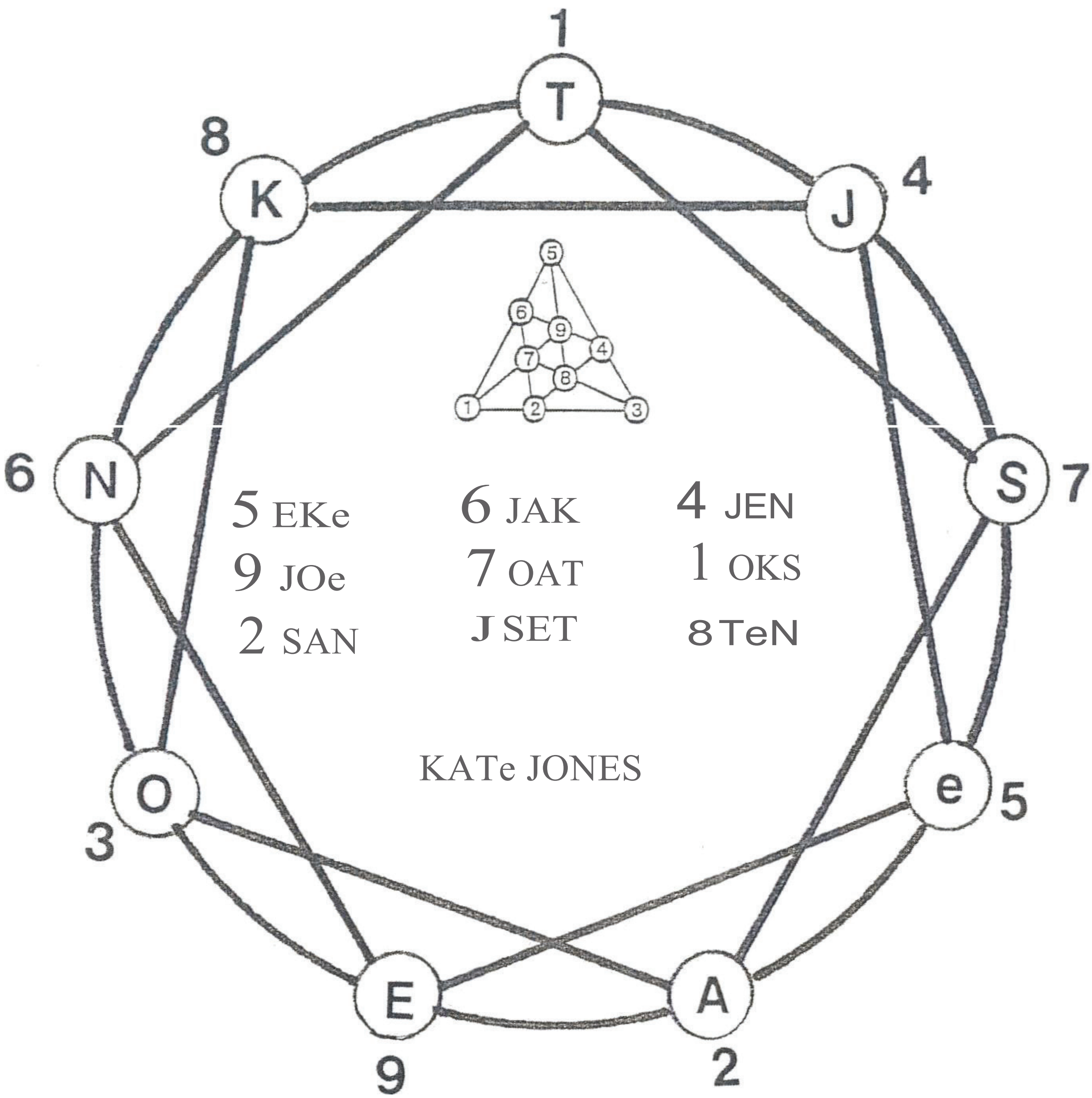
O'BEIR.i'l'E

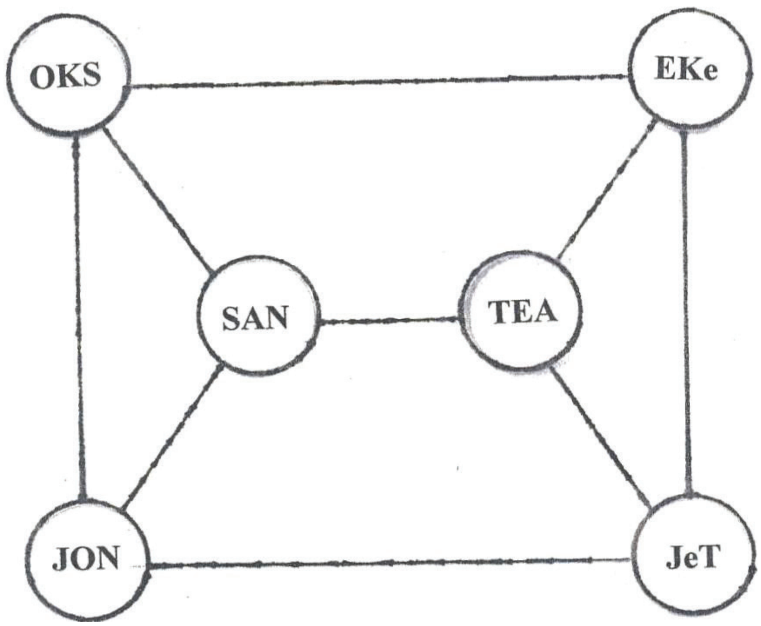
1 EKe	4 ENS	2 KAT
7 JAN	8 JET	6 JOe
3 OKS	5 SeA	9 TOT

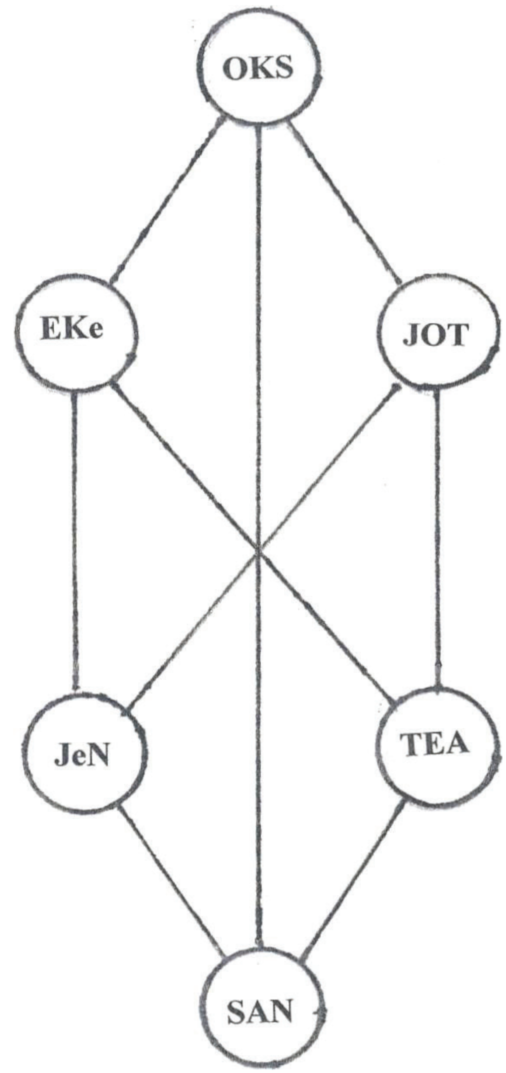


3 ATE	1 EKe	7 JAK
8 JeN	9 JOT	4 0KS
2 0NE	5 SAN	6 SeT

PAPPUS


1 T
8 K
4 J
6 N
7 S
3 O
5 e
9 E
2 A
5 EKe
9 JOe
2 SAN
6 JAK
7 OAT
J SET
4 JEN
1 OKS
8 TeN
KATe JONES


OKS
EKe
SAN
TEA
JON
JeT


OKS
EKe
JOT
JeN
TEA
SAN

BIBLIOGRAPHY

(1) Jeremiah Farrell. Games on Word Configurations, *Word Ways: The Journal of Recreational Linguistics,* 27(40: 195-205, November 1994.
(2) Jeremiah Farrell, Martin Gardner and Thomas Rodgers. Configuration Games, *Tribute to A Mathemagician.* Ed. B. Cipra, E. Demaine, M. Demaine, and T. Rodgers. Wellesley, MA: AK Peters, 2005, pp. 93-99.
(3) T. H. O'Beime. *Puzzles and Paradoxes*. New York: Dover, 1984, p. 109.

Martin Gardner and Scientific American:
The Magazine, Columns, and the Legacy

by Peter L. Renz

I worked with Martin Gardner as an editor and saw some of the action behind the scenes. How did those who worked with him see him? What resources did he draw upon? How did he do what he did? Here is a start at the answers to these questions.

At Scientific American. In 1974 I visited my colleagues at *Scientific American* before heading to Hastings-on-Hudson to meet Martin Gardner**.** Dennis Flanagan, editor of the magazine, told me that having columns like Martin's freed him for the trickier parts of his job. Reviewing Martin's *Colossal Book of Mathematics* in *American Scientist* in 2002, Dennis wrote that the column "was a big hit with the readers and contributed substantially to the magazine's success."

Gerard Piel, the magazine's publisher, wanted closer cooperation with it's subsidiary, W. H. Freeman and Company, where I was mathematics editor. I was sent scouting to see what Martin might suggest. Martin and I were in touch from then on, and I helped set up or sort out his publishing arrangements for him at Freeman, the MAA, and elsewhere.

In 1977 Morris Kline was putting together selections for the *Scientific American* Reader,*Mathematics: An Introduction to Its Spirit and Use* — a shorter and gentler version of his 1968 *Mathematics in the Modern World.* Morris wanted broader coverage and more elementary exposition. Martin's columns covered many basic topics, but when Morris put together the earlier reader Gerry Piel ruled them out, telling Morris that Martin controlled the rights. Freeman handled reprints and readers for the magazine then, and I knew Martin was liberal about permissions. Unlike Gerry, I was not Martin's boss, and I saw no harm in asking on Morris's behalf. Martin said, "Yes," and 14 of the 40 articles in Kline's 1978 reader were Martin's. Dennis and Gerry were protective of Martin and his material, while Martin was generous by nature. The words were his, but he saw the concepts part of a common heritage.

Your Choice: Skim or Peruse. Earlier, Morris Kline had called me about an error he spotted in a thought experiment Martin described in his April 1975 column,"Six Sensational Discoveries that Somehow or Another Have Escaped Public Attention." The experiment revealed an inconsistency in special relativity. Morris specialized in electricity and magnetism, so this got his attention. I suggested Morris look at the sixth discovery in the column, Dr. Robert Ripoff's psychic motor, popularized by Henrietta Birdbrain. We decided that Martin would handle all questions in the following month's column. (See Chapter 10 of Martin's *Time Travel and Other Mathematical Bewilderments* for the story.)

Martin's columns rewarded careful readers and skimmers. Morris looked carefully at the material that was down his alley. He skimmed the rest and it all looked looked fine to him. Thousands of readers did the same. One "discovery" was that $e^{\sqrt{136}}$ exactly equals $262,537,421,640,768,744$. The numbers match to one part in 10^{30} . Finding the discrepancy by calculation would have been difficult in 1975.

In 2007 I looked through all of Martin's columns finding the illustrators so they could be credited in new editions. This gave me a feeling for the columns: their variety and their ideal length and accessibility. I was reminded of how Martin's problems permeated the atmosphere in the column's heyday.

Editor, Artists, Management. Armand Schwab was Martin's editor at the magazine and he devised titles for the columns. Schwab and the art director lined up the artists, more than thirty over time. They are interesting. Bunji Tagawa, who did the first column, was a Sage fellow at Cornell in philosophy before turning to art. James D. Egleson was an early and frequent illustrator of Martin's columns and was famed for Hicks Mural Room at Swarthmore as well. Ed Bell, at *Scientific American* for more than 35 years and was its art director in 2010 when I was last in touch with him. He had fond memories of Martin's columns, as did Ilil Arbel, now a successful author, who illustrated many of the later columns.

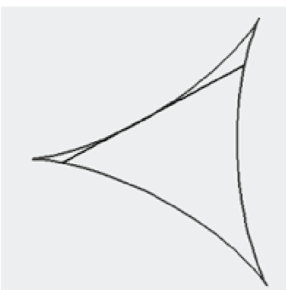
Scientific American was owned by technological optimists who were committed to reason and progress. Among them were the trio who engineered the rebirth of the magazine in 1947: Gerard Piel, publisher; Donald H. Miller, Jr., general manager; and Dennis Flanagan, editor. Backing them were Bayard Ewing, Leo Gotlieb, Nathan Levin, Frazer McCann, Julius and Lessing Rosenwald, and John Hay Whitney.

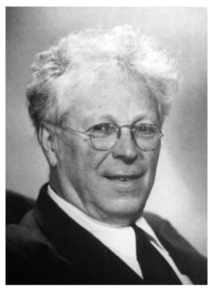
In the 1970s the magazine held its summer board meetings in San Francisco, and Freeman editors met with the directors. The directors were as keen about science and technology as they were about profits. They were tech-savvy and essential to the success of the enterprise.

How Did He Do It? Keys to Success? A restless and powerful mind, a superb memory (even into old age), skill as a writer, and wit, and great energy. *Scientific American's* audience devoured his columns and showered him with material. Many of you read, enjoyed, thought, and responded to his columns.

How did Martin work? Partly as a reporter, getting his stories from the sources: some examples are, John Conway's Game of Life, Mandelbrot's fractals, public-key cryptography, etc. Sometimes he drew a column from a book, for example, his April 1961 on H. S. M. Coxeter's *Invitation to Geometry*. Some columns he drew from many sources; for example, his February 1963 column, "Curves of Constant Width," cites the Watts drill for square holes, *The Enjoyment of Mathematics* by Hans Rademacher and Otto Toeplitz, and papers on "rotors" by Michael Goldberg from the *American Mathematical Monthly*. The article cites Franz Reuleaux and his triangle but not his book, *Kinematics of Machinery*However John Grafton of Dover Publications and I believe that Martin had looked at this book and suggested the Dover reprint, which appeared in 1964.

This "curves" column ends with the Kakeya problem: What is the least area in which a needle of unit length can be rotated through 360°. Sōichi Kakeya conjectured that it was a hypocycloid of three cusps, as shown on the left below with the unit needle inside it.





On the right above is A. S. Besicovitch, who showed that a needle of unit length could be turned through 360° in as small an area as you wish. This column is Chapter 18 in *The Unexpected Hanging* and in its new edition, *Knots and Borromean Rings, Rep-Tiles, and Eight Queens* (2014). The new edition gives the construction that solves the Kakeya problem and many surprising new results connected with it. This quote gives the sense of the story:

Despite its recreational flavor, the Euclidean Kakeya problem is a central open problem in geometric measure theory with deep connections to harmonic analysis (e.g., Fefferman's result on the convergence of Fourier series in higher dimensions) and other important problems in analysis. Proving the Euclidean Kakeya conjecture (which is widely believed) seems notoriously difficult, and most progress on it is via combinatorial "approximations." — from "Kakeya Sets: New Mergers and Old Extractions" by Zeev Dvir and Avi Wigderson in *The 49th IEEE Symposium on the Foundations of Computer Science* (2008).

Lasting Impact, Long Tail. Recreational problems often tie into deeper mathematics, as the Kakeya example shows**.** Looking at Martin's columns, I am struck by their lasting interest. Flexagons, the Game of Googol or Secretary Problem, and the Unexpected Hanging launched small industries after they appeared in the column. We will be chewing on new forms of puzzles Martin popularized for decades. Martin's trapdoor cipher column altered the cryptographic landscape. His columns on Conway's Game of Life fired interest in cellular automata. His columns on *Godel, Escher, Bach* and *The Planiverse* helped popularize the work of Douglas Hofstadter and A. K. Dewdney, who went on to became *Scientific American* columnists.

Sources, People. Martin mined gold from the New York Public Library and gleaned treasures from his correspondents. Material from more than 1500 of them can be found in the Martin Gardner Papers at Stanford's library. Stan Isaacs went through the archive and identified each item for the Guide to Martin Gardner Papers, which is now available online. The Papers take up 60 feet of shelves. Look at the PDF to get a sense of who contributed to the column and how Martin organized the material.

There is treasure in this collection. Don Knuth spent two weeks combing these files when he visited Martin in Hendersonville, North Carolina, and later arranged for them to come to Stanford.

Bear in mind that the Stanford archive has only those files Martin kept relating to his column. He probably discarded more than he kept and his column was but a part of his complete lifework. In 1979 he wrote Don Knuth outlining a typical month when he was doing his column. He allotted two weeks to write his column and reserved two weeks for other projects — ones like *The Annotated Alice.*

I looked through Stan Isaac's Guide to get a feel for the material. John Conway, H. S. M. Coxeter, and Solomon Golomb have the most citations. Other groups sprang to my eye. Artists and writers, among whom there were: Isaac Asimov, L. Sprague de Camp, M. C. Escher, Piet Hein, Scott Kim, Gershon Legman, Frederick Pohl, Constance Reid, and Carl Sagan. Other *Scientific American* columnists included were: A. K. Dewdney, Douglas Hofstadter, James R. Newman, Ian Stewart, and Jearl Walker. Some giants I noticed were: P. A. M. Dirac, Oskar Morgenstern, John Nash, Linus Pauling, Roger Penrose, Claude Shannon, John Tukey, Stanislaw Ulam, Marilyn vos Savant, Scott Morris, Will Shortz, and Mel Stover. These are some names that jumped out at me; if you look, you will see others, some expected and some quite surprising.

Legacy: Continuing Contributions . Thinking and writing were Martin's joys. He could not rest from them. After his wife died in 2000 he was depressed and told me he probably wouldn't write any more books. What does the record show? From 2001 on he published 22 books and 78 articles, reviews, or magic tricks.

Martin gathered his Mathematical Games columns into 15 books and found on the MAA CD, *Martin Gardner's*

Mathematical Games. In 2006 he made arrangements for second editions. This is a joint project of the the Mathematical Association of America and Cambridge University Press. After Martin's death in 2010 his son James made arrangements with *Scientific American* allowing the project to be completed by using Martin's files and contributions from others. David Tranah, editorial director of Cambridge University Press, is spearheading this effort.

The Gatherings 4 Gardner and Celebration of Mind carry on in Martin's tradition. Martin's support of other authors shows in his blurbs and reviews. He defended reason and rooted out folly of every sort. He was my first source for news of political folly or hypocrisy. He crusaded against injustice based on intellectual fraud. See, for example, "False Memory Wars" in *The Skeptical Inquirer*, reprinted in *The Jinn from Hyperspace*.

Martin was a Platonist, and he critiqued humanist or relativist views of mathematics. See his review of Philip Davis and Ruben Hersh's *The Mathematical Experience*, in *The New York Review of Books.* He also critiqued reform mathematics textbooks in the same publication. Search under "The New New Math." We disagreed about Platonism, and other things, but his barbs were aimed at my ideas, not my person. So far as I knew, Martin harbored no animus against those whose ideas he attacked.

The delight he took in intellectual play, his regard for reason, his interest in and sympathy with human foibles, and his skill and productivity as a writer enriched us all these sixty years, and they will continue to do so for decades to come. It was a pleasure to have known him.

The books. Martin Gardner collected his Mathematical Games columns into fifteen books. The brief titles of these are listed below in the order of the Mathematical Association of America's CD *Martin Gardner's Mathematical Games.* Brief titles are used and 1/e indicates the first edition title and 2/e the second.

1. **1/e** *The Scientific American Book of Mathematical Puzzles* . . . (1959); **2/e** *Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi*. . . (2008).

2. **1/e** *The 2nd Scientific American Book of Mathematical Puzzles* . . . (1961) **2/e** *Origami. Eleusis, and the Soma Cube* . . . (2008).

3. **1/e** *New Mathematical Diversions from Scientific American, etc.* (1966); **2/e** *Sphere Packing, Lewis Carroll, and Reversi*. . . (2009).

4. **1/e** *The Unexpected Hanging and Other Mathematical Diversions* (1969); **2/e** *Knots, Borromean Rings, and Eight Queens* (2014).

► Suggestions for the books from this point forward are in order as of April 2016. These should be sent to David Tranah at Cambridge University Press or to me, Peter Renz.

5. **1/e** *Martin Gardner's Sixth Book of Mathematical Games*. . . (1971); **2/e** *Klein Bottles, Op-Art, and Sliding-Block Puzzles*. . .. **Note.** There was a short collection of Dr. Matrix columns, *The Numerology of Dr. Matrix*, before the *Sixth Book*, but these columns were combined with others in *The incredible Dr. Matrix*, which is Book 9 listed below.

6. **1/e** *Mathematical Carnival* (1975); **2/e** *Sprouts, Hypercubes, and Super Ellipses*. . ..

7. **1/e** *Mathematical Magic Show* (1977); **2/e** *Nothing and Everything, Polyominoes, and Game Theory*. . ..

8. **1/e** *Mathematical Circus* (1979); **2/e** *Random Walks, Hyperspheres, and Palindromes*. . ..

9. **1/e** *The Incredible Dr. Matrix* (1978); **2/e** *Words, Numbers, and Combinatorics: Martin Gardner on the Trail of Dr. Matrix.*

10. **1/e** *Wheels, Life and Other Mathematical Amusements*, (1983); **2/e** *Wheels, Life, and Knotted Molecules*. . .

11. **1/e** *Knotted Doughnuts and Other Mathematical Entertainments*, (1986). **2/e** *Knotted Doughnuts, Napier's Bones, and Gray Codes*. . ..

12. **1/e** *Time Travel and Other Mathematical Bewilderments* (1988); **2/e** *Tangrams, Tilings, and Time Travel*. . ..

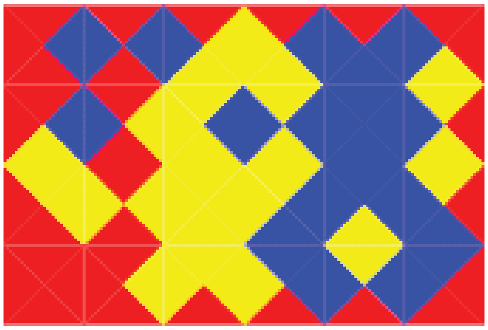
13. **1/e** *Penrose Tiles to Trapdoor Ciphers* (1989); **2/e** *Penrose Tiles, Trapdoor Ciphers, and the Oulipo*. . ..

14. **1/e** *Fractal Music, Hypercards, and More* . . . (1992); **2/e** *Fractal Music, Hypercards, and Chaitin's Omega*. . ..

15. **1/e** *The Last Recreations, Hydras, Eggs, and Other Mathematical Mystifications* (1997) **2/e** *Hydras, Eggs, and Other Mathematical Mystifications*. . ..

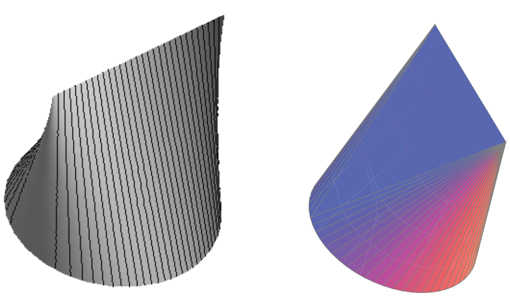
What the new editions showed us. Martin Gardner kept files on his columns noting new results. When the columns were collected into books he regrouped his files by book. Not all of these new developments can be handled at the level of Martin's books, but much can be illustrated or pointed to. The treatment of the Kakeya problem in Book 4 is an example of this. Martin and I went to the Editorial Board listed in the new editions, especially to John Conway, Richard Guy, and Don Knuth.

Computers and the Web changed everything. Curves that artists drew can be explored on screen. Variants of the Soma cube can be 3D printed. Here is an example of computing related to Chapter 16 of Book 3. MacMahon's squares quartered along their diagonals, and the quarters are coloreds red, yellow, or blue so that every possible distinct coloring appears once. There are 24 of them, The puzzle is to make 4 x 6 rectangle so that edges that meet are the same color and the outside of the rectangle is of one color. Here is one solution:



The question is "How many solutions are there?" In Martin's column he said there was just one, mistaking what Mac Mahon had said. Readers found other solutions and sent them in. Federico Fink, working by hand in Buenos Aires, estimated that there were 12,224 solutions. This was in 1963. In 1964 Fink got Gary Feldman at Stanford to coount the solutions using a mainframe. Feldman found 12,261. This is not the end. In 1977 Hilario Fernandez Long, in Buenos Aires, did a computer count giving 13,328 patterns. This number was number later confirmed by John Harris in Santa Barbara. As Ronald Reagan suggested, "Trust, but verify."

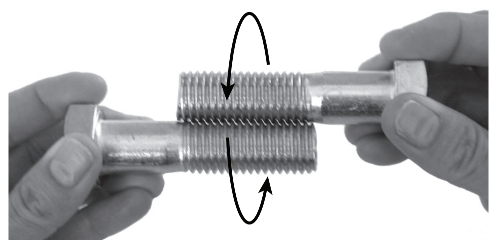
The Cork Plug, Book 5, Chapter 5, shows what computer graphics can tell us. This plug has a horizontal circular base. Above a diameter of the base raise a vertical square. The figure is filled out by taking the cross sections perpendicular to both the circular base and the vertical square to be isosceles triangles whose bases are chords of the circular base and whose apexes lie on the top edge of the square. The Mathematica™ image below, left, is the Cork Plug. The image at the right is its convex hull, slightly rotated.



Martin asked for the volume of the Cork Plug. There is a nice *Aha!* answer. He remarks that this is plug the least convex volume having three orthogonal projections that are respectively a circle, a square, and an isosceles triangle. Using Mathematica ™ to check the illustrations, I noticed that Martin's plug was not convex. Indeed, the illustration in the column showed that. So what is the least volume of convex body having these three orthogonal projections? This question is also has an easy answer. Returning to the illustration in the old edition, the figure suggests that the triangular projection is equilateral, but Mathematica ™ or thought will show this can not be.

The artists were not credited on the pages of the columns nor on those of the earlier editions of the books. Martin and I set out to identify them, credit them, and secure permission for reuse. This is a difficult task. For some images replacement was the better option. In the case of the Cork Plug, computer graphics gave a more accurate image, and the software allows readers to explore related shapes. These new editions gave us a better appreciation for how good the old illustrations were and how some can be improved.

Another example is the Twiddled Bolts, Book 2, Chapter 5. The photograph below shows the arrangement. The left and right bolts are interchangeable and they can be twiddled in either direction. The question is: When twiddled in the direction indicated will the bolts move together, apart, or keep the same distance? Note the top bolt is moving toward you while the bottom bolt is moving away. Experiment will answer the question. For *Aha!* that proves what must be true see the last line of this article.



Using photographs or computer graphics puts things in a context that invites the reader engage with the material. As these new editions come along we should expect more readers to suggest such additions. And I hope for more material on the Web. The Gatherings for Gardner and the Celebrations of Mind are the perfect places to begin the search for such contributions.

The list of books is found above. They are all available on the CD, *Martin Gardner's Mathematical Games.* We are moving ahead at Cambridge University Press, at The Mathematical Association of America, and at the Martin Gardner Littrary Trust. We look forward to your contributions and suggestions. ► **As for the bolts**, turn the page upside down. This interchanges the left and right bolt but changes nothing else. Which way will the bolts be twiddling and moving then? *Aha!*

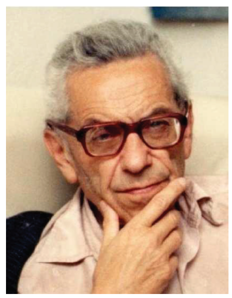
Peter Renz MoxonsMaster@gmail.com May 2016

Martin Gardner's little known collaborations with Paul Erdős & Nicolas Bourbaki

"Card Colm" Mulcahy, Spelman College

1 Apr 2016

Everyone knows of Martin Gardner's stimulating collaborations with artist Salvador Dalí, fractalist Benoit Mandelbrot, and numerologist Dr. Matrix. Less well known are his interactions with legendary Paul Erdős and the fabulous Nicolas Bourbaki.



Scientifically American

There is an online article in *Scientific American*, a reputable publication, called **John Conway Reminiscences about Dr. Matrix and Bourbaki** (see `https://blogs.scientificamerican.com/guest-blog/john-conway-reminiscences-about-dr-matrix-and-bourbaki`). In it, there is a link to a video clip from 1 April 2015 in which John H. Conway discusses a meeting at Martin's house that he'd attended decades earlier where he'd met both Bourbaki and Mandelbrot (see `https://www.youtube.com/watch?v=mM6bc6Ogrjs`).

In a second clip, Conway discusses a paper Martin wrote with Bourbaki, dedicated to Erdős (see https://www.youtube.com/watch?v=zDPD7qtKFw0). A third clip reveals that Martin also wrote a paper with Erdős, which was both deep and profound, and apppeared in the *Kurdish Journal of Mathematics* (see https://www.youtube.com/watch?v=IJHumNVgbxU).

Aperiodically Humourous

Just today, *Aperiodical* reported on a previously unknown Erdős-Bacon collaboration. The article reveals that: "The brief footage of the collaboration was found in unused material filmed for the 1993 documentary *N Is a Number: A Portrait of Paul Erdős*" (see http://aperiodical.com/2016/04/erdos-bacon-previously-unknown-collaboration.)

Aperiodical is a good mathematical website so obviously the story is true.

China warns against April Fools' Day jokes

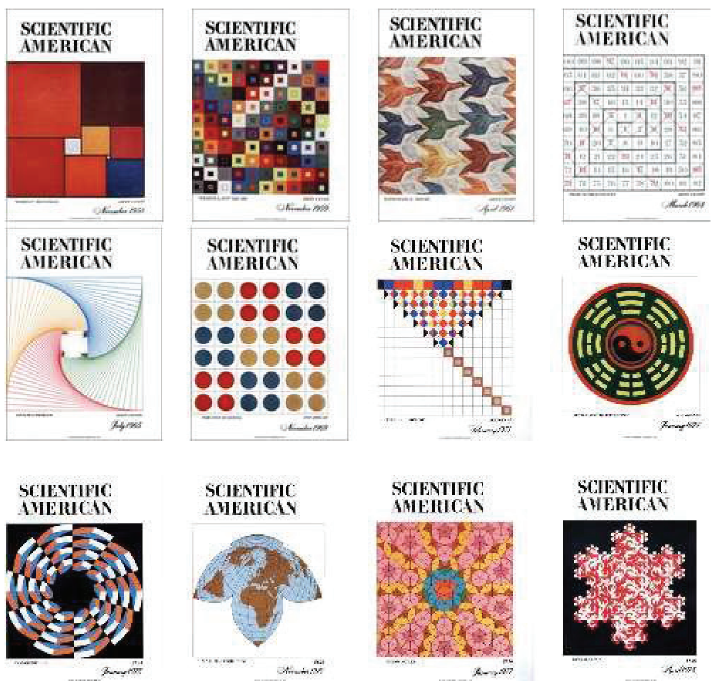
A recent news item proclaimed,"An official of the Chinese Communist Party has slammed April Fools' Day as an ideologically unsound Western conspiracy ... We hope that everyone does not trust, make, or transfer rumours." In 2013, the government there introduced a rule whereby anyone who tweeted something suspicious or seditious that was retweeted more than 500 times faced prosecution.

Spaghetti-Harvest in Ticino

One wonders if Martin knew about the famous hoax BBC *Paronama* newsreel from 1 April 1957, narrated by the very trustworthy Richard Dimbleby. In it, an exceptional heavy spaghetti harvest in Switzerland is featured (see http://hoaxes.org/archive/permalink/the_swiss_spaghetti_harvest).

Of course, one of Martin's more famous *Scientific American* columns was a brilliant April Fool's one from 1975; it's worth tracking down.

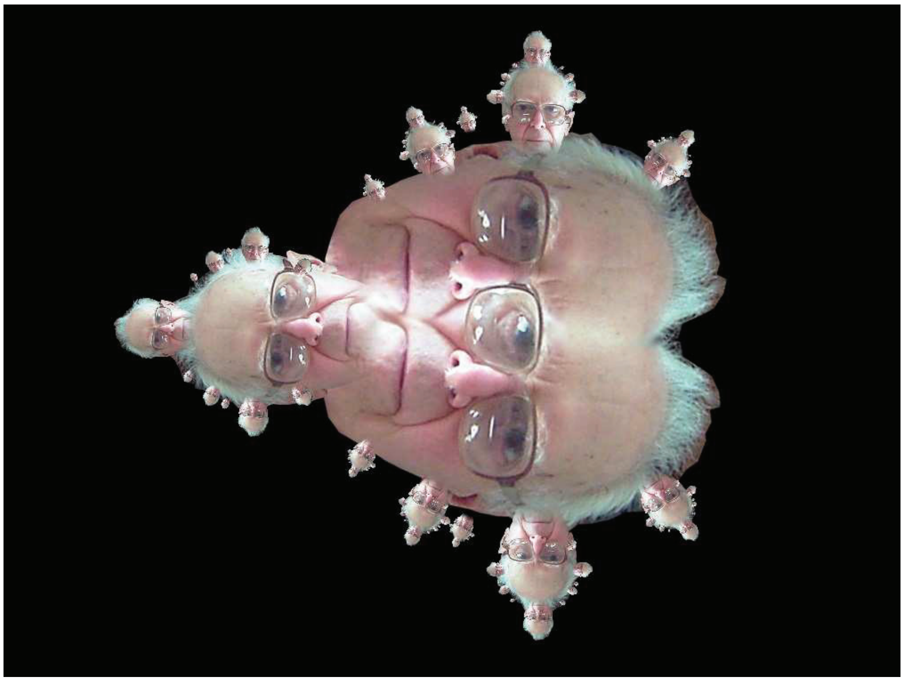
Martin's contribution to G4G12



As it happens, 12 is the number of *Scientific American* covers spawned by Martin's roughly 300 articles for the magazine, shown here in the order in which they appeared (see `http://www.martin-gardner.org/SciAm12.html`).

Mandelbrot's contribution to G4G12

"Given any Mandelbrot, B, there is a Euclidean neighbourhood of Martin, i.e., a neighbourhood N of Euclid Avenue, which contains B." (Mandelbrot lived near Martin back in the 1970s.)



The above image was created by Henry Segerman & Craig Kaplan in the hour or two before the presentation `https://www.youtube.com/watch?v=mqB2r35RvAk` was given.

Twitter's contribution to G4G12

We recommend following both @WWMGT ("What would Martin Gardner Tweet?") and @MGardner100th on Twitter; 2844 and 1605 people (respectfully) do. Just don't assume that $2844+1605 = 4449$: Twitter may be addictive, but it's sub-additive, rathen than additive.

AN EXCHANGE FOR
G4G12
Atlanta, March 2016

PAUL SWINFORD, A TRIBUTE

By Jeremiah Farrell

Paul Swinford (1/29/1931-4/26/2000), a Cincinnati magician, entertained at many G4Gs.

PAUL SWINFORD
(*photo by* ROLING)

To my friend Jerry, with every good wish, Paul Swinford 3-15-00

A protégé of Stuart Judah he was best known for his two books *Faro Fantasies (1968)* and *More Faro Fantasies* 1971. He also was the Parade editor for the *Linking Ring* (1974-78) and was a major contributor to the *Pallbearers' Review.* His invention of the Cyber Deck in 1983 included an 8-page introduction on the binary system by his friend Martin Gardner. He claimed he had read and absorbed everything available of Gardner and in 1999 published a lecture on numberplay and wordplay.

THE WONDROUS WORLD OF

NUMBERPLAY & WORDPLAY

A Lecture by

Paul Swinford

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$
$$12345 \times 9 + 6 = 111111$$
$$123456 \times 9 + 7 = 1111111$$
$$1234567 \times 9 + 8 = 11111111$$
$$12345678 \times 9 + 9 = 111111111$$

To Jerry, with every good wish! Paul Swinford 4-9-00

In the wordplay part we quote Swinford:

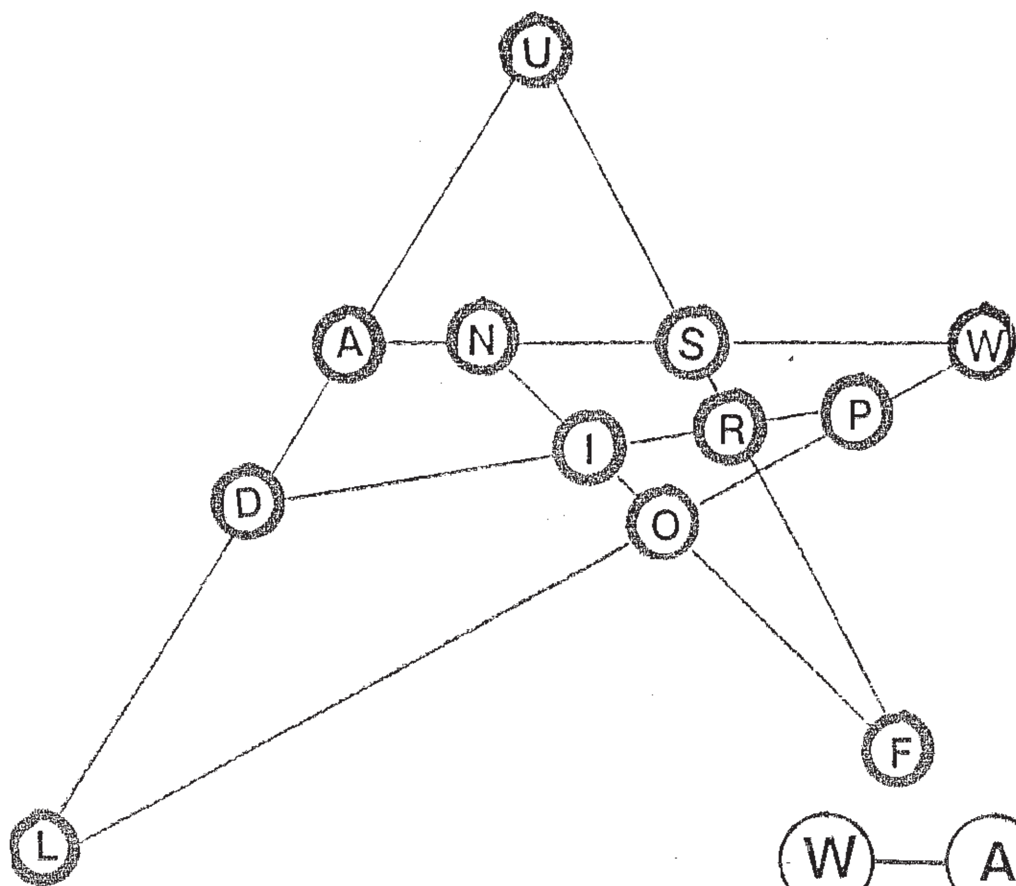
ANAGRAMS - Webster defines an anagram as "a word or phrase made by transposing the letters of another word or phrase." The word derives from the Greek word "anagrammatismos" which translates as "to transpose letters." Anagrams have been around for a while. They were invented in 260 B.C. by the Greek poet Lycophron.

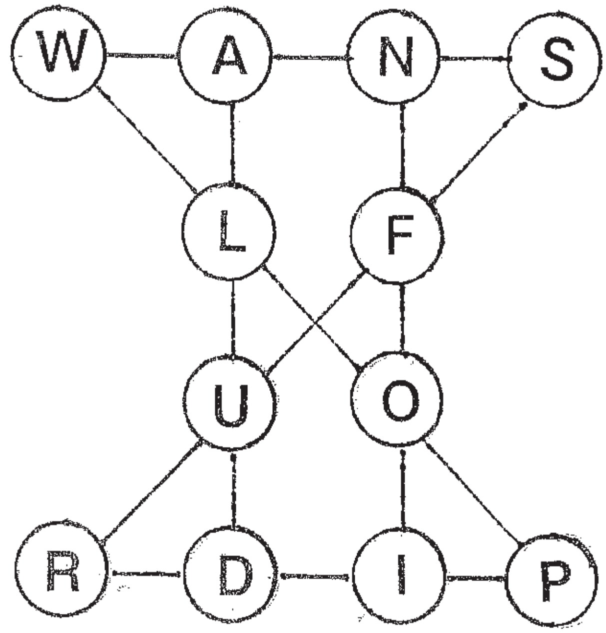
A book entitled *Anagrammasia* containing about 5,000 anagrams was published in 1925. Expert opinion holds that only two or three copies are known to have survived. It was compiled by a dedicated anagrammatist who signed on as "Amaranth." Little is known about him except that he was a Pittsburgh attorney with the surname of Lovejoy.

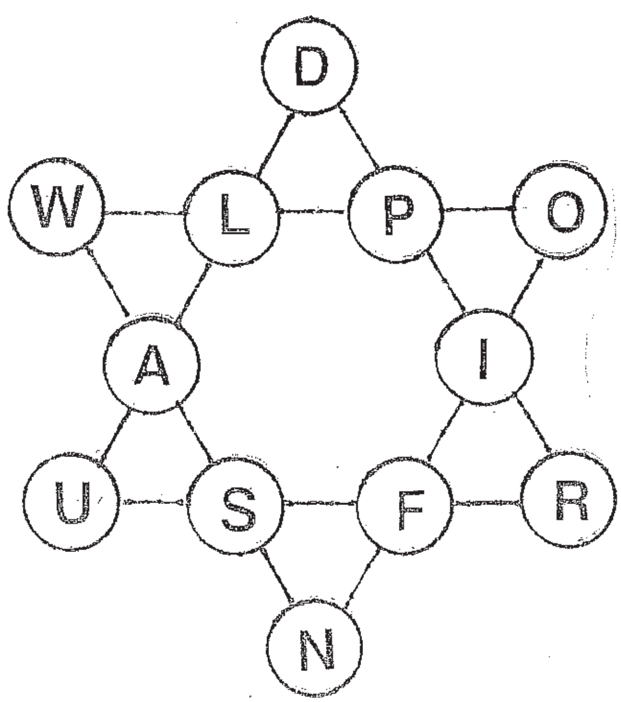
Coming up with anagrams derived from personal names is very much in vogue. The famous mathematician Agustus de Morgan was once presented with a list of 800 anagrams on his name as compiled by one of his admirers. The Internet now offers a program through which one can have a list of computor generated anagrams on any word, phrase or name submitted.

If the personal reference will be excused, I will tell you of the outcome when I submitted my name to "Anagram Genius Server." I was particularly interested to explore this since my name has the rare characteristic of having no repeating letters. I submitted my request and very shortly thereafter down-loaded a list of 200 anagrams based on the 12 letters of my name. Unfortunately, most of them were just nonsensical phrases of little value such as: "Fail drowns up" and "Spur lad if now" and so forth. Things improved a little with "Is darn fowl up?" (Is that lazy pet rooster of mine awake yet?) and "No awful drips?" (Did the plumber do a good job?) With a bit of a stretch, we can justify "Fluids or pawn?" by saying that I was attempting to decide if I wanted to drink two or three beers or go hock my watch.

We now exploit the 12 letters of his name for a variety of puzzles. We first note that using each letter exactly twice one can form the six words DRIP, DUAL, INFO, PLOW, SURF and SWAN. Not so remarkable until one notices that each of the six fit on at least three non-isomorphic graphs.







For some numberplay assign the integers 1 to 12 to the letters of PAUL SWINFORD so that every row of four sums to the same constant.

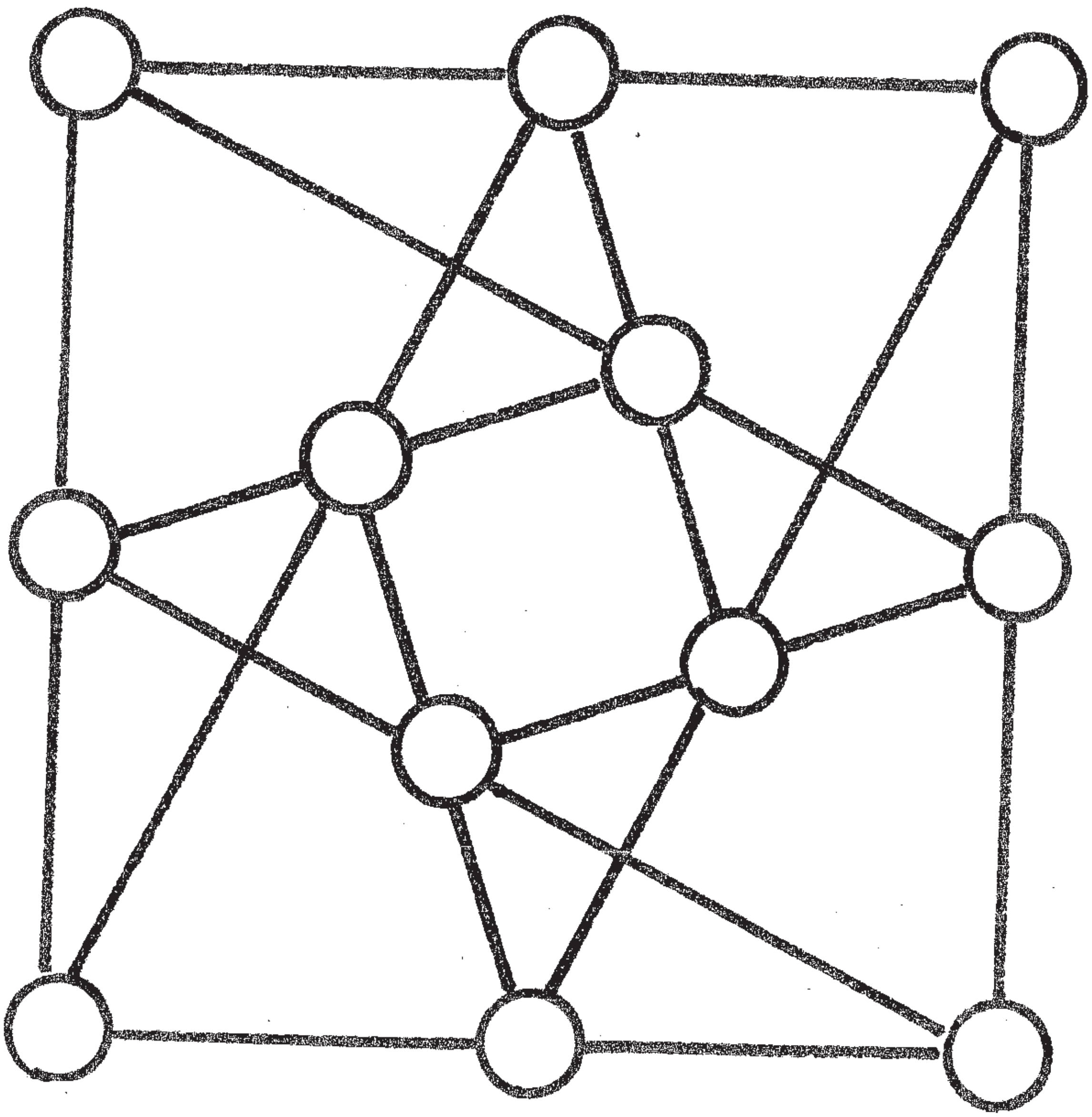
This is an illustration of the concept of isoagonic graphs. That is non-isomorphic graphs that have the same puzzle "struggle".

Another puzzle. The following 16 words use the letters of PAUL SWINFORD four times each. Arrange the 16 in the square so that every row and column anagram into our honoree's name.

> AND, FAW (a gypsy), FID (a hardwood pin), FLU, FOR, LIP, LON (Chaney), POW, PUD (a fist), RAP, RUN, SAL, SIR, SOD, WIN, WUS (a S. Wales companion).

This square is sometimes called semimagic (the diagonals don't "sum" to the constant) but can be considered as a generalization of the old problem of arranging the 16 court cards so that each row and each column contains one of each suit and one of each value. This is a problem of Jacques Ozanam (1640-1717). In this case the two main diagonals can conform as well. More generally these are instances of Euler orthogonal squares.

Use this symmetric diagram to address several puzzles that use the letters of Paul Swinford.



(1) Place these 12 words on the nodes so that every abutting node has a letter in common.
DIP, DOW, FAD, FLU, IFS, LAR (household god), LOP, NOR, PUN,
SUR (above, Fr.), WAS, WIN

(2) Place the 12 letters on the nodes so that each line of three letters anagrams into one of the words AND, FIR, FLU, OFS, OLD, PAL, PUN, RUN, ROW, SAW, SIP,
URD (a bean), WIN

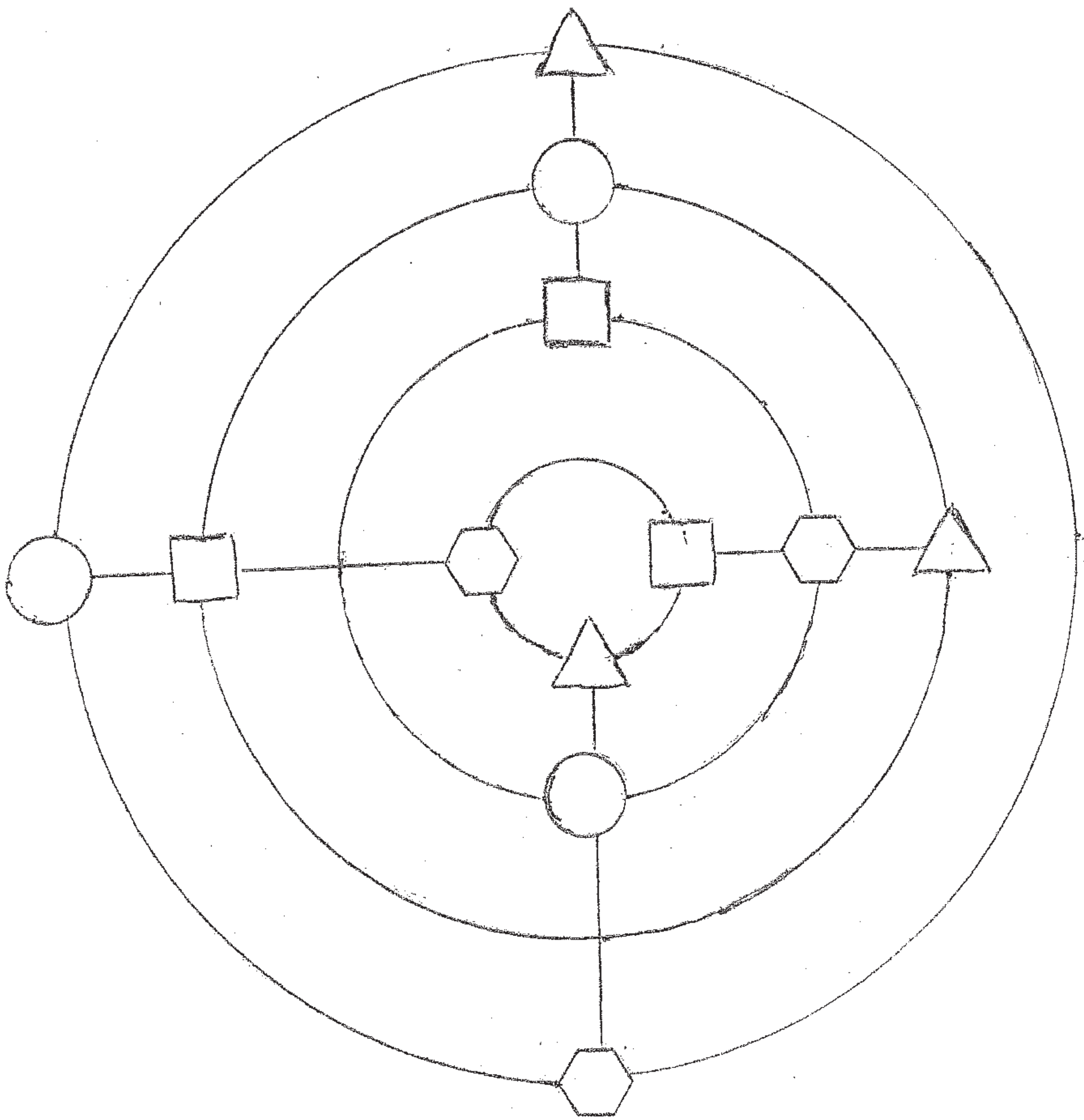
Each of these puzzles is a word example of a (12,3) symmetric configuration. See "Configuration Games", p. 93-99 by J. Farrell, M. Gardner, and T. Rodgers in *2005 Tribute to a Mathemagician*, AK Peters.

Our final puzzle-game is another (12,3) configuration (there are 229 different such configurations).

The Puzzle. Place the following 12 words on the diagram so that every line of three on the diagram, every diamond of three and every shape of three have a common letter.

DIS (a name for Pluto), DUN, FAN, FLU, FOR, LAW, LOP, RAD (radiation unit), RIP, SOW, SUP, WIN

The Game. Two players alternately place a token on a node and first to obtain three in a row, diamond or shape, wins. We know that such symmetric configurations exist for (9,3), (12,3) and (15,3) similar diagrams. We conjecture that there are infinitely such types.



SOLUTIONS.

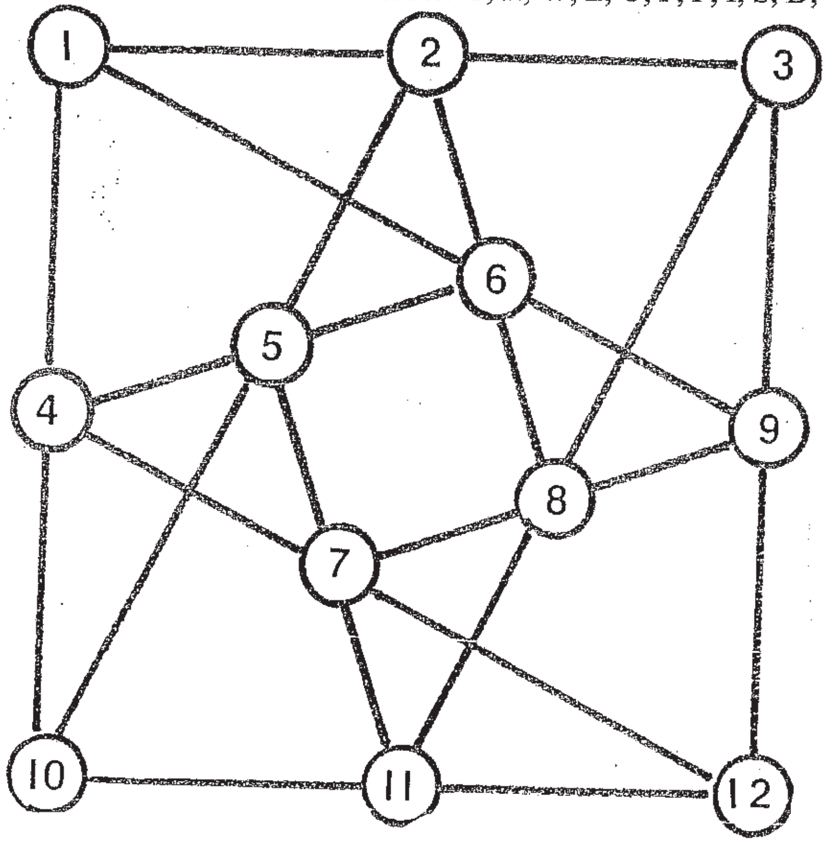
The magic diagrams will sum to 26 on each line of four if the following are given the numbers 1 to 12. O, W, D, R, A, U, F, I, S, N, P, L. Note that the six points of the star also will sum to 26.

One solution to the semimagic square is:

LIP	SOD	FAW	RUN
AND	FLU	SIR	POW
WUS	RAP	LON	FID
FOR	WIN	PUD	SAL

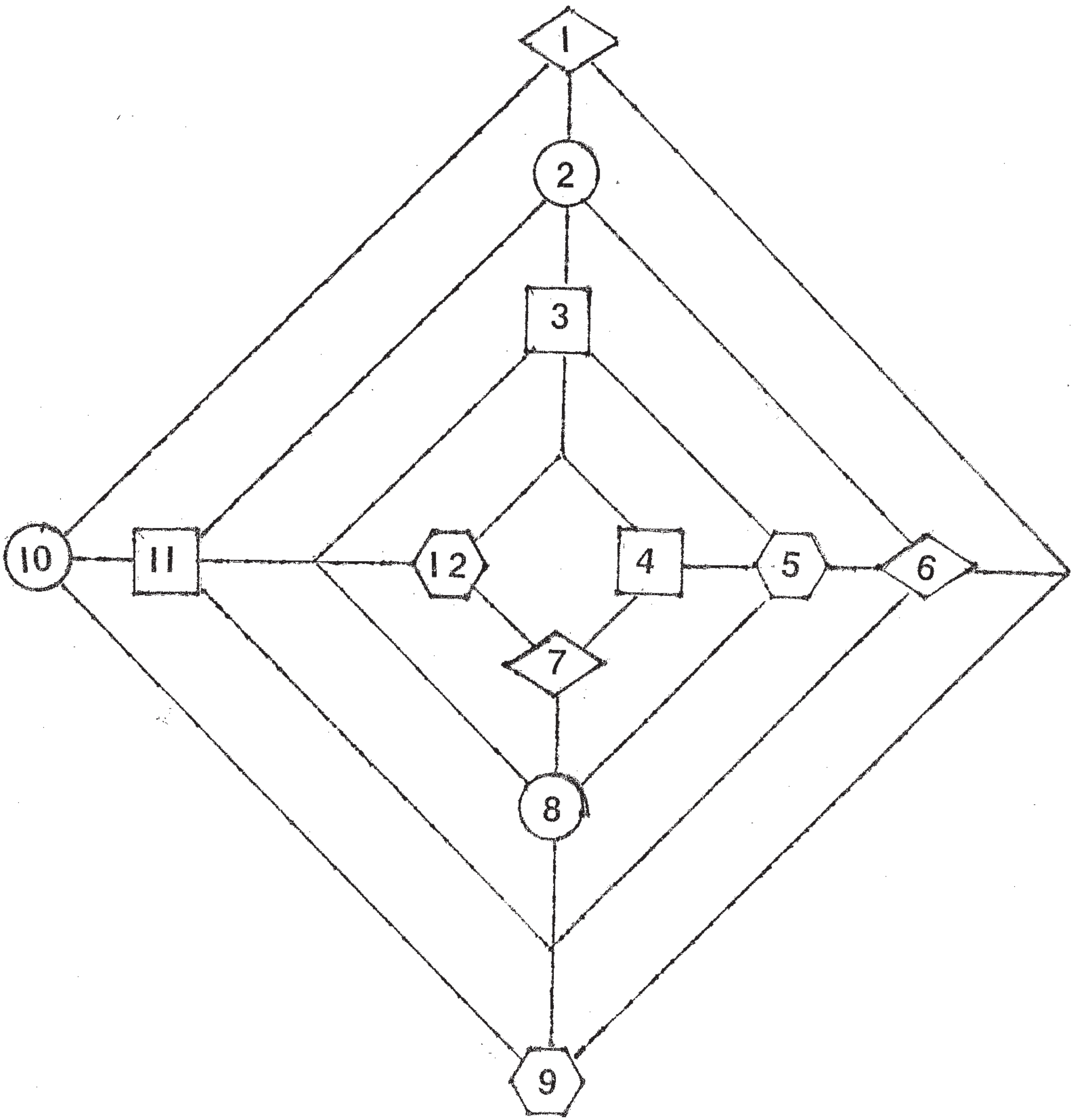
First (12,3). Place in order 1 to 12 the words:
LOP, PUN, DIP, LAR, SUR, NOR, WAS, WIN, DOW, FLU, IFS, FAD

Second (12,3). Place these letters in 1 to 12 order: O, R, W, L, U, F, P, I, S, D, N, A.



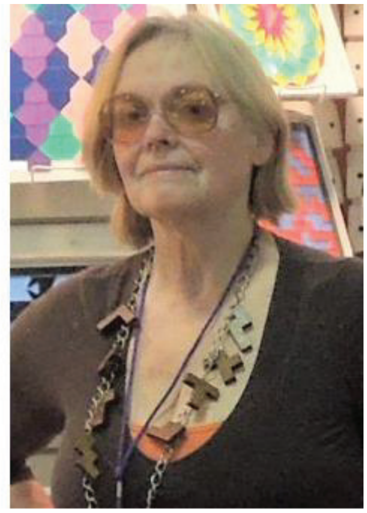
Third (12,3). In 1 to 12 order place these words:
FOR, LOP, SOW, DIS, WIN, RIP, RAD, LAW, FAN, FLU, SUP, DUN

On all of the three symmetric (12,3)s a tic-tac-toe type game may be played and First player can always win from any starting position. The key is to carefully choose a move for which Second must block and waste a turn.



Artist Information Statement

The playable art of Kate Jones



What are "gamepuzzles"?

Gamepuzzles are Kate's own original creations that can be played both as competitive games for two or more people, and as puzzles for one player alone. Kate has been designing them since 1979.

How do they work?

The sets usually consist of geometrically shaped tiles or chess-like counters. The players figure out how to arrange the tiles to form patterns and designs, with many different objectives and levels of difficulty. The numbers of ways you can fit them together may be in the thousands. It's the unique nature of the way Kate designs them that it's a new adventure every time you play with them. The figures and game rules are illustrated in each set's handbook. The games require strategy for how you place or move the pieces around on the board to achieve a winning position. They're easy to learn and you can keep improving your skill each time you play.

How are they made?

They're made in our own craft workshop, from unique ideas that Kate develops and designs. The pieces are cut from acrylic sheets with our laser, using a computer program to guide the table, then handfinished. The most intricate patterns can thus be programmed for cutting. Wood components are handcrafted with traditional tools and the laser. Kate also writes and illustrates the rule books.

What is your philosophy, your artistic vision, in designing these gamepuzzles?

It sums up as a celebration of mind—the joy of thinking—playable art—truth and beauty. Here's Kate's philosophy in brief:

> Kadon's goal is to make and sell good and true and beautiful things at decent prices. The products should bring pleasure to the widest range of individuals, from child to adult, from beginner to expert. The style and concept of the products should be universal and timeless, each idea presented in its purest form. Each product must be worthy of the customer's time and money.
>
> The human mind is such a marvelous instrument, with its unique capacity to perceive, observe, learn, understand, discover, create, invent, solve and reason. I want to give it the undiluted essence of each idea, captured in a form that you can hold in your hand and eye and mind, and move about at will. Each gamepuzzle set contains a bit of cosmic truth, nourishing the mind's need to recognize system and structure. In symmetry and order the mind finds its fullest sense of its own efficacy and of beauty.

One of the themes implicit in Kate's gamepuzzles is that harmony can be achieved from diversity—that, with some ingenuity and patience, you can fit many all-different pieces into one beautiful composition, where the uniqueness of each individual piece assures and enhances the balance of the whole. There's a great sense of freedom in knowing there's more than one right answer and in having many options. Minds that exercise regularly with puzzles stay sharp longer, and play helps reduce the stresses of life. Play on!

Email: **kadon@gamepuzzles.com**. Phone/fax, 410-437-2163. Web: **www.gamepuzzles.com**.
Snailmail: Kadon Enterprises, Inc., 1227 Lorene Drive, Suite 16, Pasadena, MD 21122 –U.S.A.

Self-Description

Louis H. Kauffman, Math UIC, Chicago, Illinois 60607-7045, <kauffman@uic.edu>

Naming objects occurs, and when it does, a pointer is notated in memory from A (the name) to B (the object) as

$$A \longrightarrow B.$$

When there is a naming

$$A \longrightarrow B,$$

it is shifted to

$$\sharp A \longrightarrow BA.$$

That is, the name is appended to the object and sets up a new or meta-name $\sharp A$ for this composite object made of thing and name.

This shifting process is noticed and given a name M. Thus

$$M \longrightarrow \sharp.$$

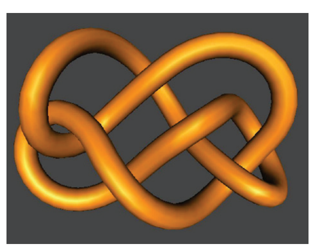
M is the name of the meta-naming process. This naming is shifted to form

$$\sharp M \longrightarrow \sharp M$$

and abbreviated to

$$I = \sharp M.$$

I am the meta-name of my meta-naming process.



A SPECIAL TRIBUTE
To
MARTIN GARDNER
At
G4G12
Atlanta, Georgia 2016

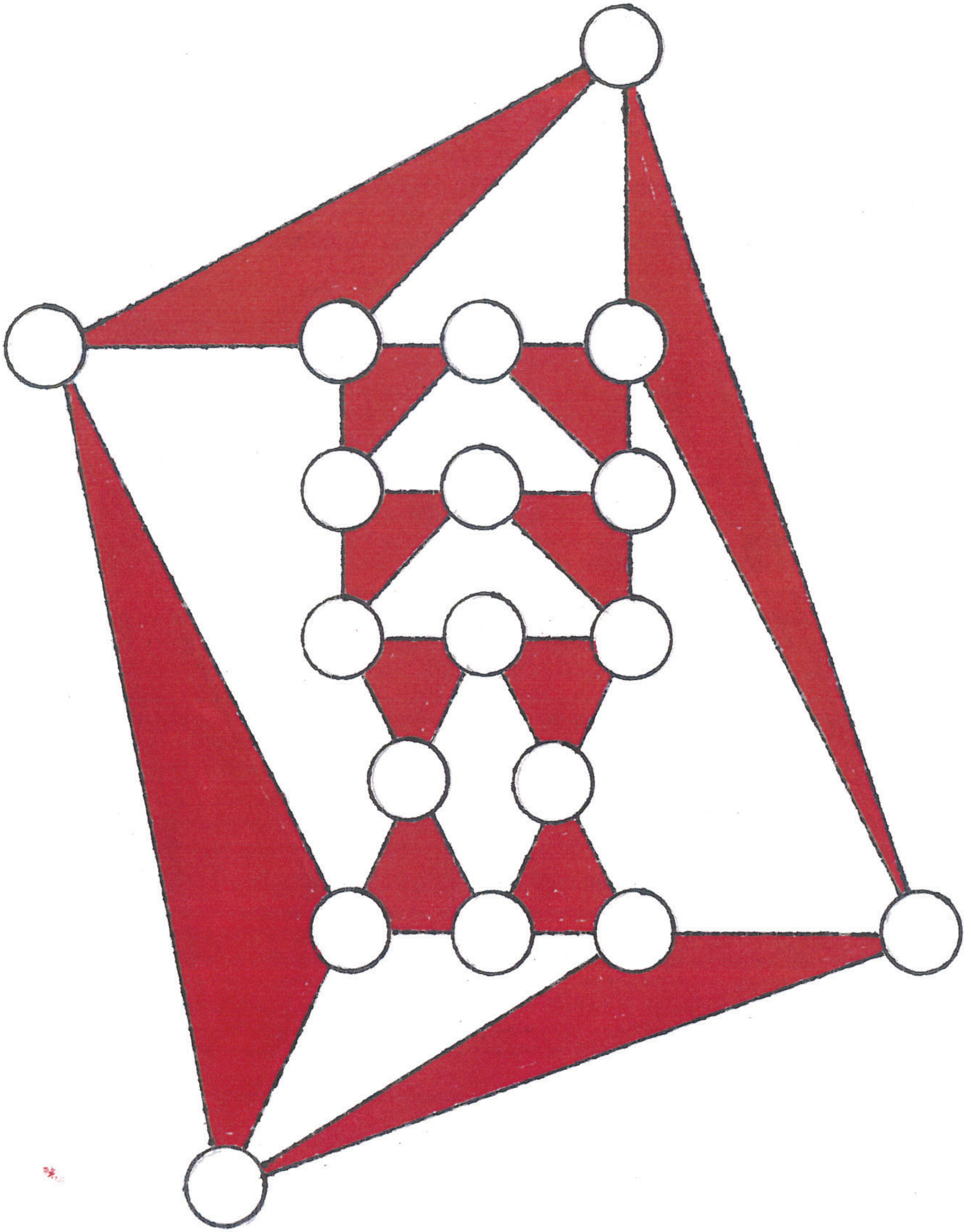
by

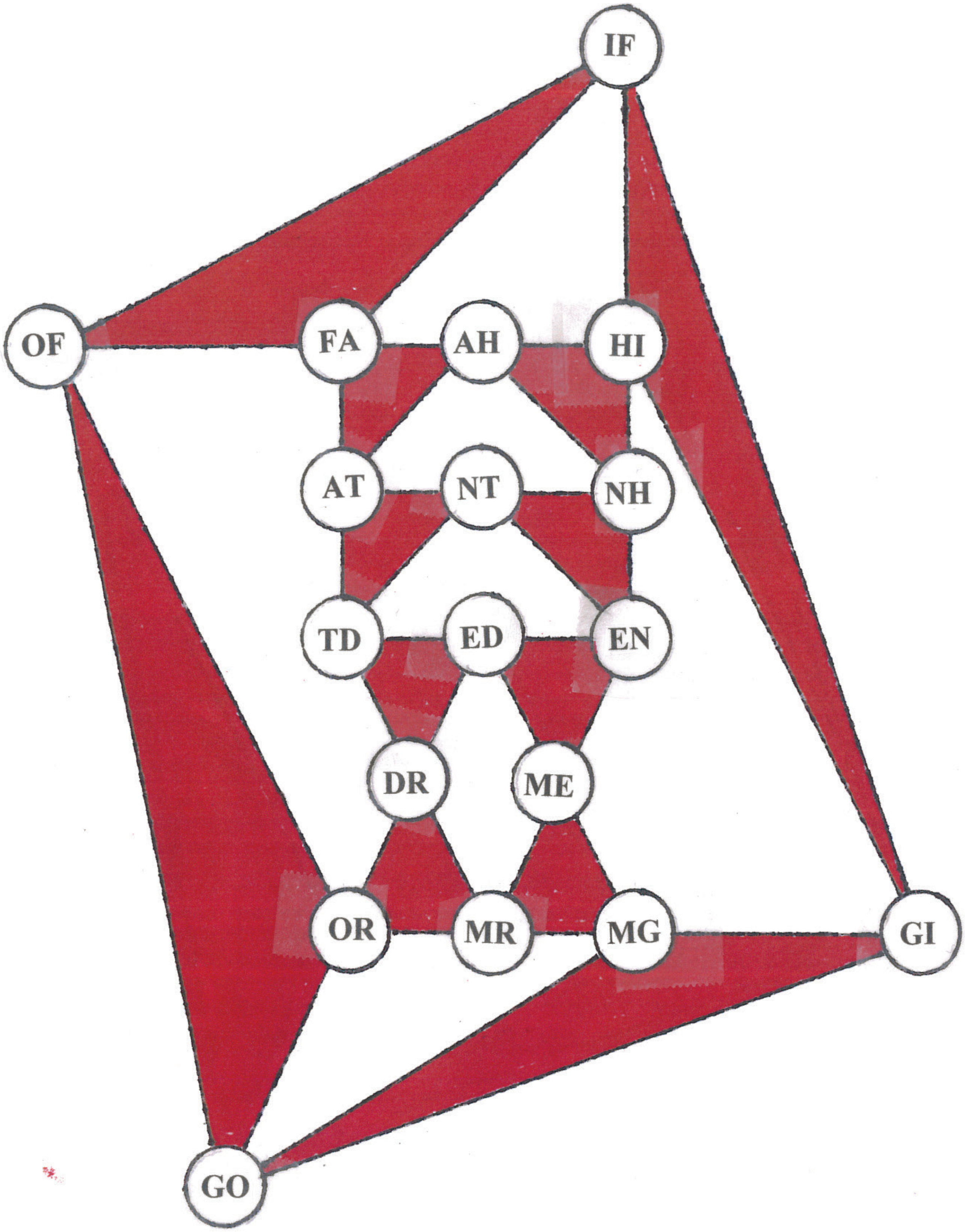
Jeremiah Farrell

There are exactly 12 different letters in the phrase GATHERING FOR MARTIN GARDNER. We use each of the 12 letters three times each in 18 different two-letter words that are to be placed on the nodes of the graph so adjoining nodes have a letter in common.

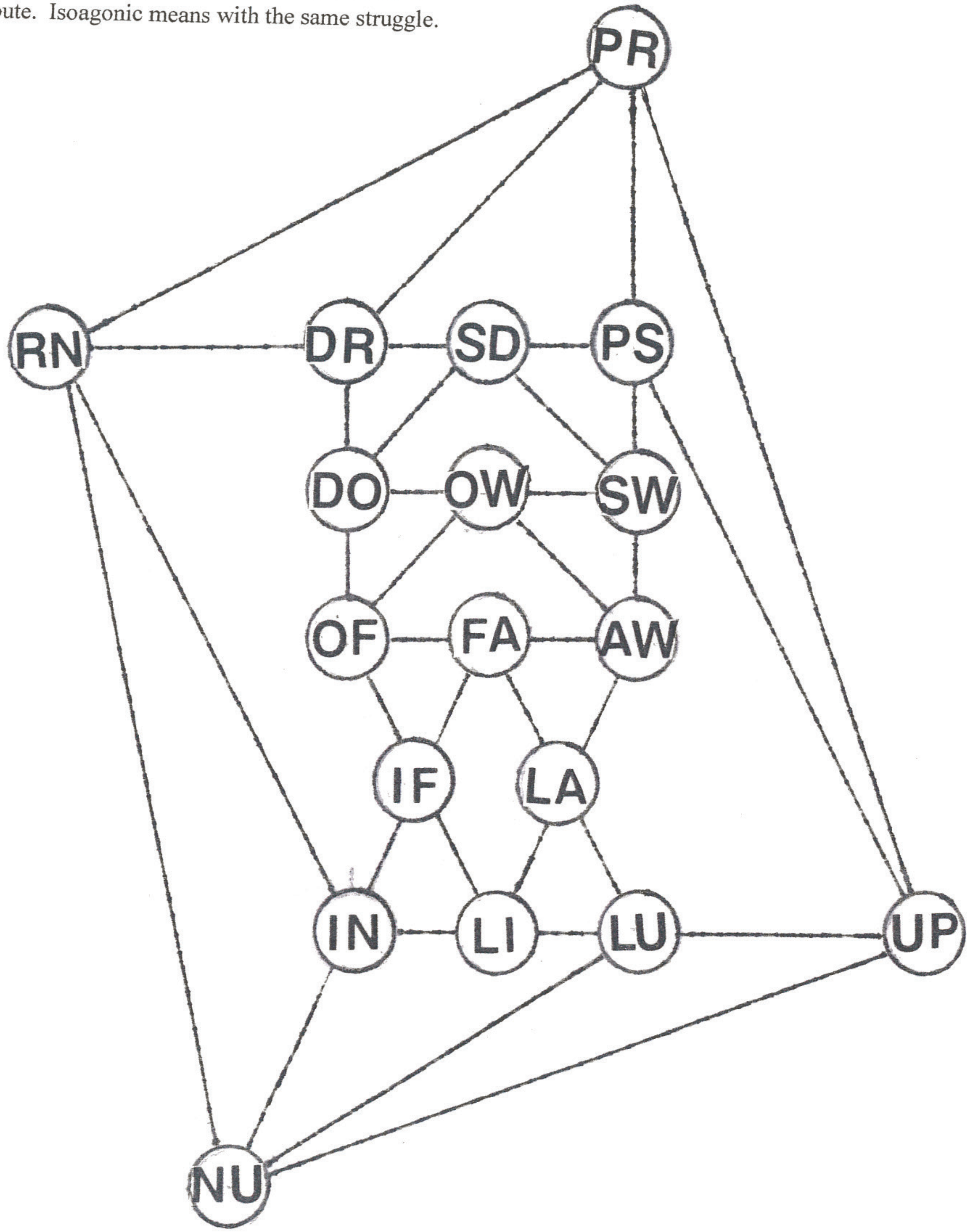
Our words are: AH, AT, DR, ED, EN, FA, GI, GO, HI, IF, ME, MG (Martin Gardner), MR, NT (New Testament), NH (New Hampshire), OF, OR, TD (Touch Down)

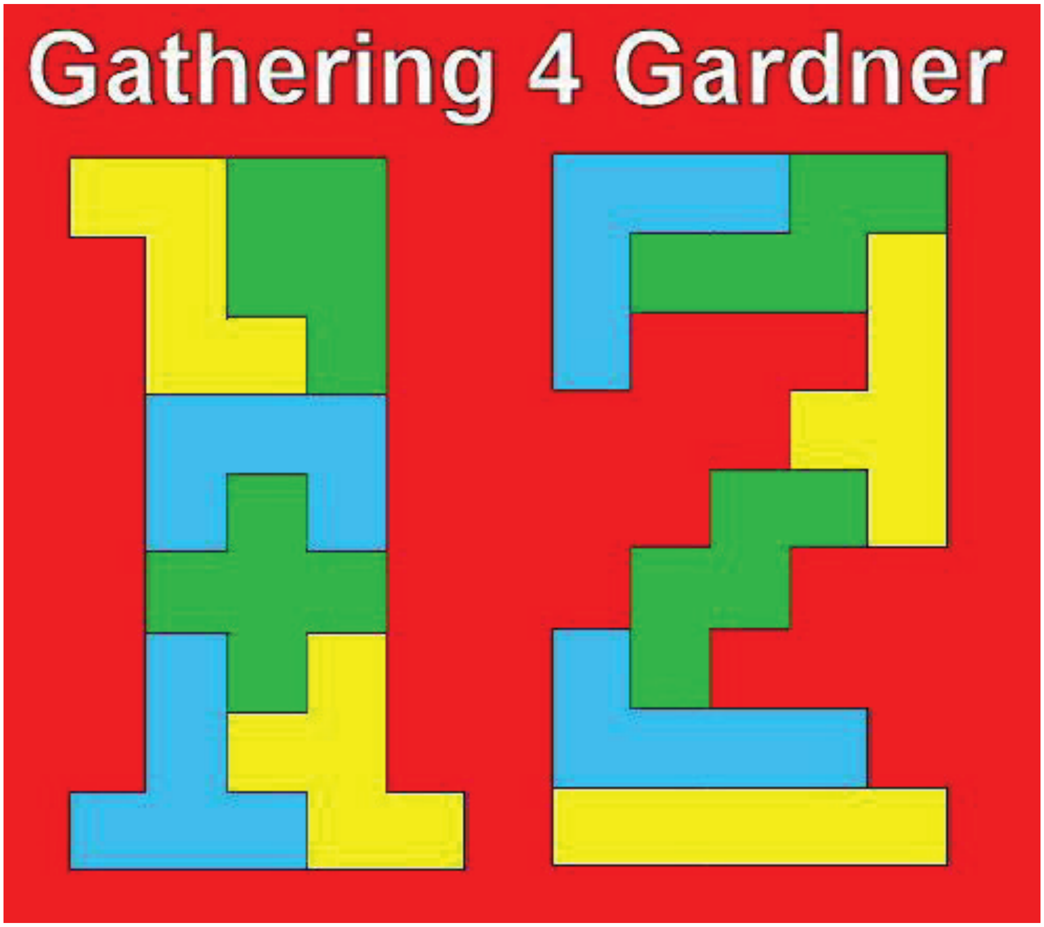
It is of some interest to note that this grid and the grid on Todd Estroff's exchange are certainly not graph isomorphic but each of the 18 two-letter word sets can fit on either diagram. This not so obvious result could perhaps be called "isoagonic", that is with the same puzzle struggle.




IF
OF
FA
AH
HI
AT
NT
NH
TD
ED
EN
DR
ME
OR
MR
MG
GI
GO

This proves the 12-puzzle on 18 two-letter words using PAUL SWINFORD (Todd Estroff's exchange) is isoagonic with the Martin Gardner tribute. Isoagonic means with the same struggle.





Twelveness

A Fibonacci verse celebrating the 12 pentominoes

1 1 2 3 5 8 13 21 34 55 89 144

Presentation by Kate Jones
at Gathering for Gardner 12
Atlanta, GA – 2016

 www.gamepuzzles.com

1 Martin

1 Gardner

2 Long ago

3 Wrote about pentominoes,

5 Brainchild of young Solomon Golomb,

8 The coolest recmath set in all the world.

13
Soon everybody played them, Gabriel made them,
Even Arthur Clarke became their addict.

21
Through a feat of fate along came Kate
And started a business, because she could,
Founded on 12 pieces of wood.

34
And this one set begat lots more—
Combinatorial puzzles by the score—
As awards rolled in and ribbons flew
And a beautiful mathematical product line grew,
Lovingly crafted… sold only in our traveling store.

55

As decades flowed by, the pents we had named Quintillions
Stood ever in first place, and their fans grew by the millions.
Their shapes showed up in a whole parade
Of other creations that we made.
And dear Martin Gardner, friend and mentor,
Let us produce the two games of which *he* was the inventor.

89

Polyominoes are everywhere, just take a look around —
On floors and walls, on every web page as pixels they are found.

From the Singularity to infinity, particles join in ever more fanciful arrays
Like elements in galaxies, where energy with space-time plays;

Then living beings happened along, from single cells to the giant whale,
And played with variations, inventiveness at every scale,

And somewhere in the middle are these humans on a planet blue—
They have minds that play with puzzles, math and the magic they can do.

144

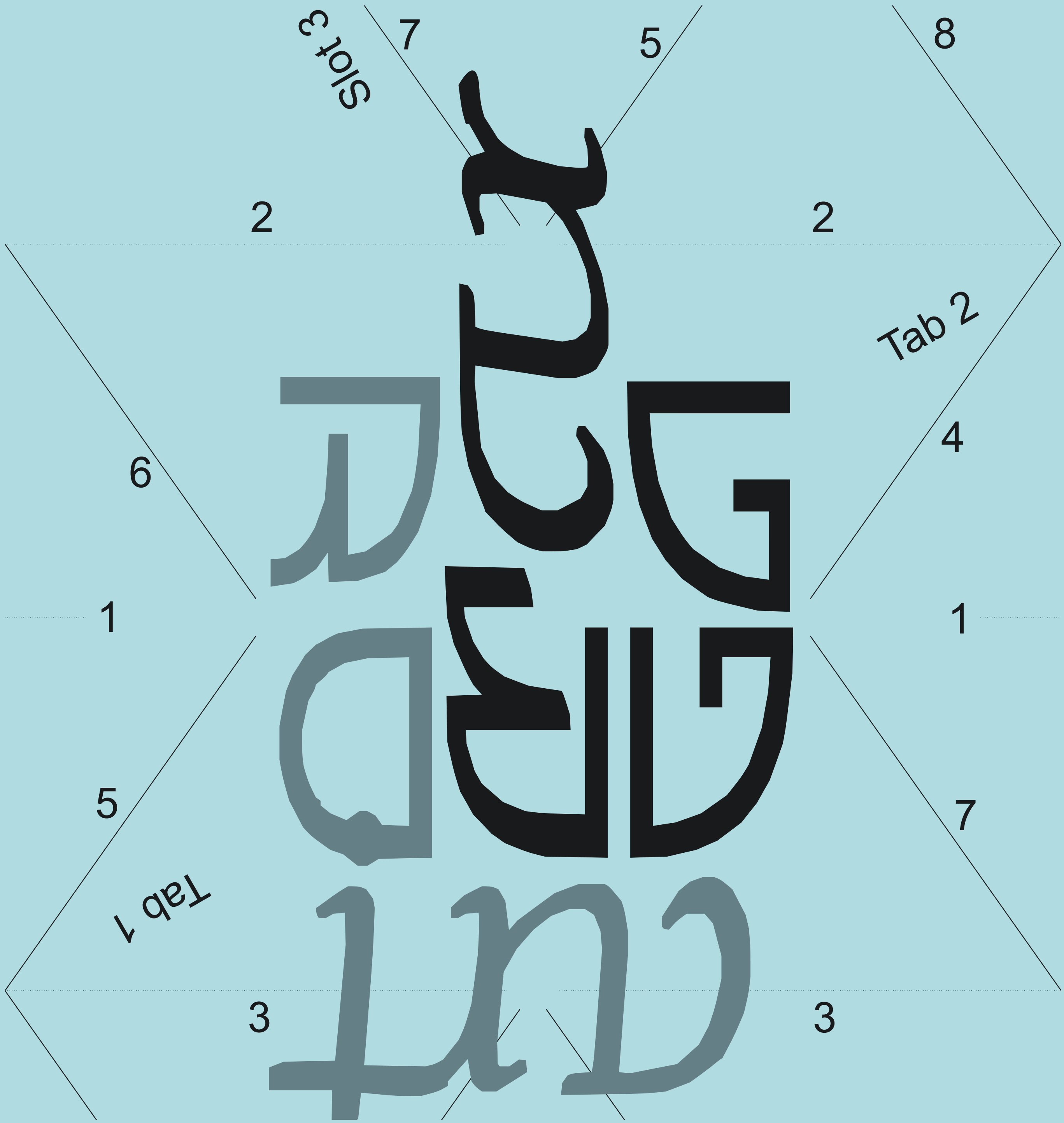
**Wrapping up the last line, with a 12-times-12 word string--I counted them with care—
Here is the list of all our games where you'll find pentominoes demand their share:**

Archimedes' Square, Boats, Brace, ChooChooLoops, Color Up, Colormaze, Combinatorix, Dezign-8, Diamond Rainbow, Doris, Dual Quintachex, Fill-Agree, Fractured Fives, Gallop, Grand Multimatch, Heptominoes, Hexacube, Leap, Lemma, L-Sixteen, Mini-lamond Ring, MiniMatch I, Multimatch I, Multimatch II, Multitouch I, Multitouch II, Octominoes, Pentomino necklace, Perplexing Pyramid, Pocket Pentominoes, Pocket Vees, Poly-5, Quantum, Quintachex, Quintapaths, Quintillions, Rhom-Antics, Rhombiominoes, RhombStar-7, Rombix Jr., Sextillions, Six by Six, Six Disks, Snowflake Square, Snowflake Super Square, Super Quintillions, Ten-Yen, Throw a Fit, Tiny Tans, Transpose, Triangoes Jr., Triangoes, Triangule-8, Trifolia, Trio in a Tray, Vee-21, Void and Warp-30.

**So thank you, Sol, for what you started,
And thank you, Martin, for what you imparted,
And thank you, World, for what you hearted.**

See the full illustrated PowerPoint presentation here: www.gamepuzzles.com/g4g12-2016.ppt

PUZZLES



Martin Gardner Modular Origami | Peter Knoppers | Page 111

3D SUDOKU and 4D SUDOKU

Hideki Tsuiki, Kyoto University

On a 8 x 8 x 8 grid of cubes, we can consider two SUDOKU-like puzzles.

Puzzle A_0: Assign numbers from1 to 64 to an 8 × 8 × 8 grid of cubes so that each 8 × 8-plane (3 × 8 exist) and each 4 × 4 × 4-block (8 exist) contains all 64 numbers.

Puzzle B_0: Assign digits from 1 to 8 to an 8 × 8 × 8 grid of cubes so that each 8-sequence (3 × 64 exist) and each 2 × 2 × 2-block (64 exist) contains all 8 digits.

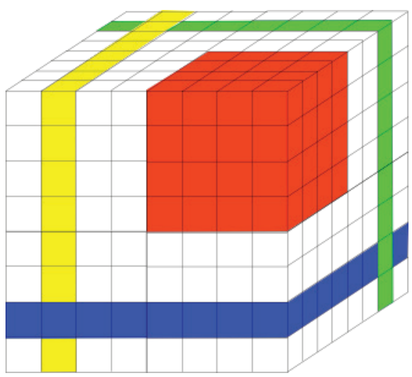


Figure 1: The constraints of Puzzle A_0.

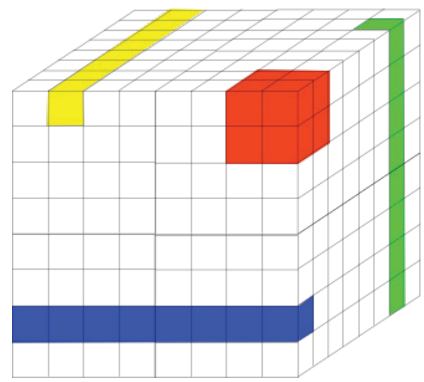


Figure 2: The constraints of Puzzle B_0.

Moreover, one can consider more complicated puzzles based on them. A 16-cell, which is a 4-dim object, has cubic projections in 8 different ways, and the third level approximation of the 16-cell fractal, which is composed of 512 16-cell pieces, is projected to 8 × 8 × 8 grids of cubes in 8 different ways. Therefore, one can consider two puzzles Puzzle A (resp. Puzzle B) to assign 64 (resp. 8) numbers to the 512 pieces of this object so that all the 8 projections form solutions of Puzzle A (resp. Puzzle B). In addition, the 8 cubic projections are divided into two sets of 4 orthogonal projections. Therefore, one can also consider simpler puzzles which only consider a set of 4 orthogonal projections to define Puzzle A_S and Puzzle B_S. In [1], it is shown that puzzle B does not have a solution but all the other five puzzles have solutions, and constructed solutions through algebraic methods. It also calculated the number of solutions for Puzzle B_S to be 1148928.

The purpose of this note is to present that it is possible to enjoy Puzzle B_0 and Puzzle B_S as pencil puzzles. Puzzle A, A_S, and A_0 require 64 different digits whereas Puzzle B_0 and B_S, use only eight digits. It is almost impossible to draw a 8 × 8 × 8 grid of cubes on a paper, but one can draw eight 8 × 8 grid of squares instead. The following is an easy to medium level of such a puzzle to fill the blanks to complete a solution of Puzzle B_0.

```
8123456_   2______5   4_6_2_3_   _5_7_1_8   __3_1_7_   3_5_7_1_   _6__8__3   __7_5__2
4__52__1   __4_8_2_   ____4_6_   2____7_5   7_1_3_5_   1______2   __8_1__4   82____1_
___6_2_3   _____3__   _8___4__   _2__7_1_   _74_2_3_   4__73__8   8_2_6_4_   __8_4_6_
____1___   8__3_6__   7_1_3_5_   _______2   3_____4_   _8_1__6_   6__5_3__   4__7__2_
__5__7_8   _4__1__7   ____5__4   _7__2___   __6_____   _1___5__   3__8____   7__5___1
1_3____4   _71__4__   2__4__1_   ______8_   __7____2   _____2__   4__1___5   __4__5__
________   ________   ________   4__2__7_   1__7___6   8__3__5_   __5____8   5____8_7
________   ________   _5__6___   ___5___1   _4___713   56___3_7   ___7_42_   __1__6_4
```

It is not easy to understand the constraint of Puzzle B_S. However, it can be restated on a 8 × 8 × 8 grid of cubes that, on all the 24 8 × 8-planes, the squares with the same color in Figure 3 have different digits.

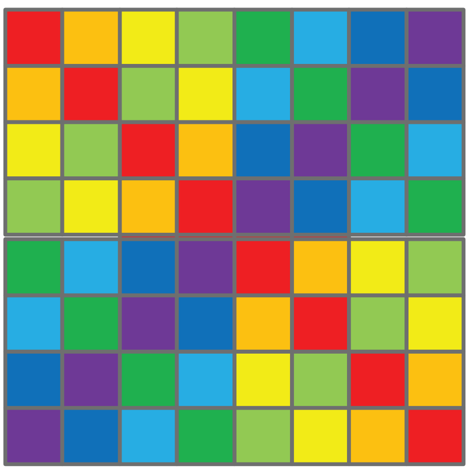


Figure 3: Constraints of Puzzle A_S.

Figure 43: A Physical solution of Puzzle A_0.

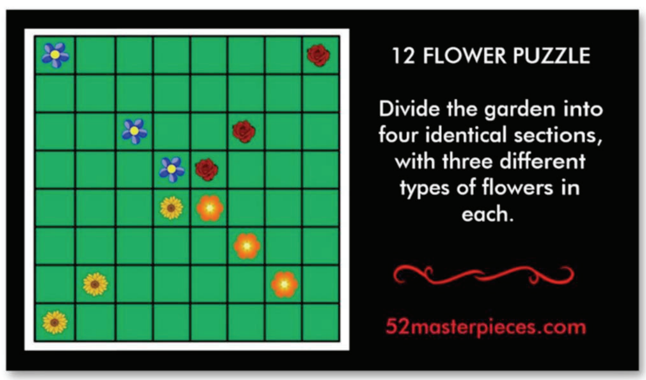
With this constraint, the following puzzle has a unique solution, which is the same as the solution of the B_0 puzzle above.

```
8123456_   ________   ________   ________   ________   3_______   ________   ________
4_______   ________   ________   ________   ________   ________   ________   ________
________   ________   ________   ________   ________   ________   ________   ________
________   ________   ________   ________   3_______   ________   ________   ________
________   ________   ________   ________   ________   ________   ________   ________
________   _7______   ________   ________   ________   ________   ________   ________
________   ________   ________   ________   ________   ________   ________   ________
________   ________   ________   ________   ______1_   5_______   ________   ________
```

[1] Hideki Tsuiki and Yasuyuki Tsukamoto: Sudoku Colorings of a 16-cell Pre-Fractal, in Discrete and Computational Geometry and Graphs, Proceedings of JCDCGG 2015, to appear in LNCS, Springer, 2017.

12 Flower Puzzle

by David Cohen



A 12-Loop 4 G4G12

George I. Bell
7! Ingersoll Pl, Boulder CO, 80303, USA
`gibell@comcast.net`
`http://www.gibell.net`

Equipment: Together with the 39-hole board on the next page, you will need a set of 38 counters (the pegs). Pennies are commonly used, but dice or sugar cubes are easier to pick up.

Puzzle #1 "The Central Game"
Place a peg on every circle except the center, d5. Now jump any peg over an adjacent peg into an empty circle, removing the jumped peg. Continue in this fashion, your goal is to clear the board and finish with a peg in the center, d5. Note that jumps are only allowed along columns or rows (there are four possible starting jumps: d3-d5, b5-d5, f5-d5 or d7-d5).

Puzzle #2 "The G4G12-Loop"
Fill the board except for g4, play to reach the **loop position** denoted by the shaded circles on the board. The peg at g4 can then perform 12 jumps in a big loop (g4-g6-e6-e8-c8-c6-a6-a4-c4-c2-e2-e4-g4), removing all remaining pegs and finishing at g4[1].

Puzzle #3 "The Unique Solution"
Fill the board except for d1, play to finish at d1. In this paper [1] we prove that this puzzle has a unique solution (up to symmetry and jump order).

Note: The three puzzles are in approximate order of increasing difficulty. For solutions, email me or see the web site at the top of this page.

[1] George I. Bell and John D. Beasley, "New Problems on Old Solitaire Boards", Board Game Studies #8, presented in Oxford, England (2005), published online in 2014 at `http://bgsj.ludus-opuscula.org` or see `http://arxiv.org/abs/math/0611091`

[1]Hint: Try working backwards from the loop position. Equivalently, play forward from the complement of the loop position.

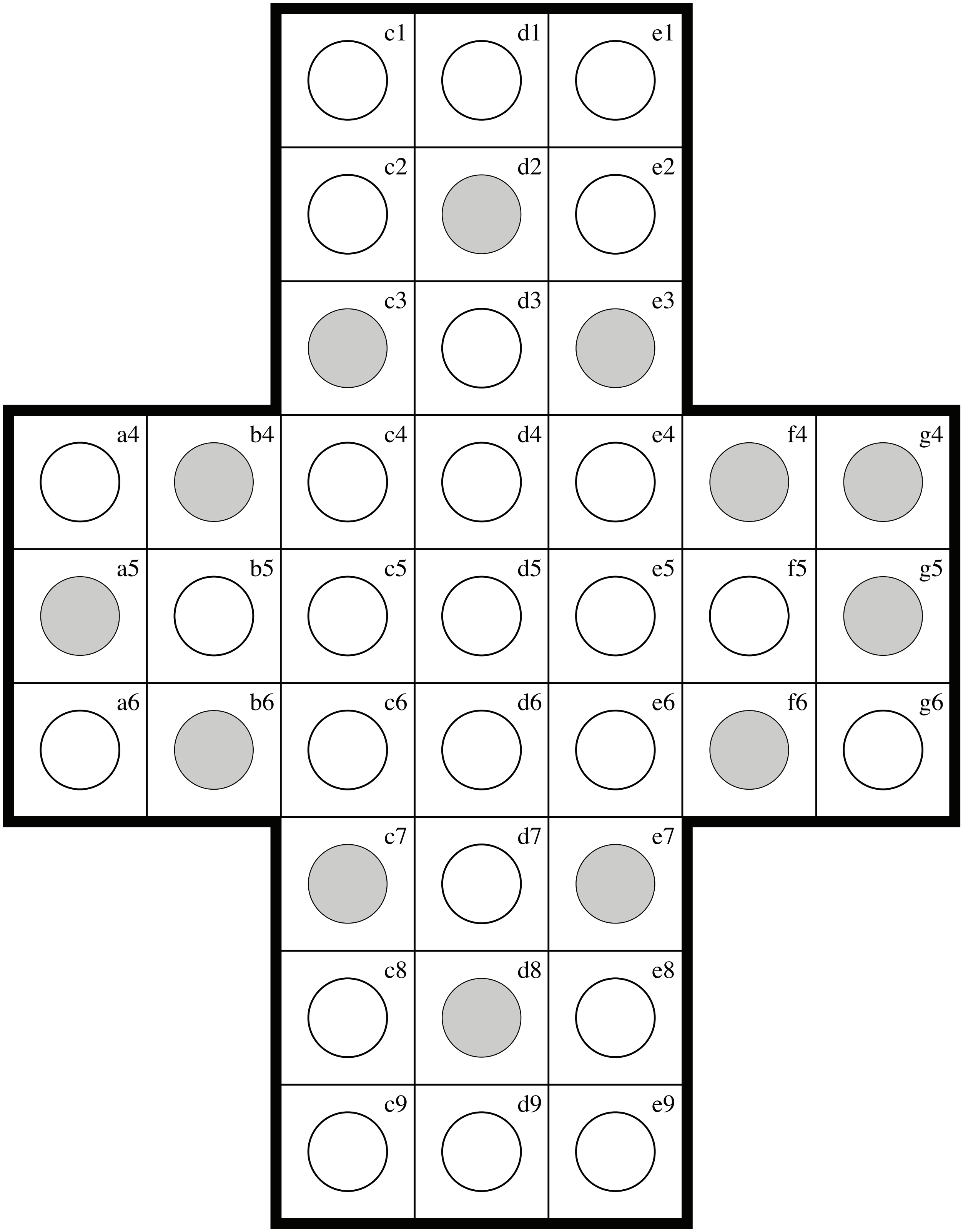


Figure 1: The 39-hole board.

The 100 Prisoners Puzzle Revisited

Yossi Elran

The Davidson Institute of Science Education, The Weizmann Institute of Science

Introduction

The 100 prisoners problem was first posed by Peter Bro Mitersen in 2003 and a few versions of the puzzle were subsequently published [1-3]. Being a probabilistic puzzle, it is not easy to derive the general solution for n prisoners. Even though the solution itself is a simple, straightforward expression that converges to ln 2, a horrible intermediate expression is used:

$$1-\frac{1}{(2n)!}\sum_{l=n+1}^{2n}\binom{2n}{l}(l-1)!(100-l)! = 1-\sum_{l=n+1}^{2n}\frac{1}{l}$$

Eq. 1 General solution to the 100 prisoners problem

In this paper, we present the problem as a soccer team puzzle and show a direct way to derive and explain the solution without using the intermediate formula.

The soccer team problem:

A new coach has been appointed to your favorite world cup soccer team, or football team if you happen to be British. The coach has decided that the 18 player team has good control of the ball, but unfortunately constantly loses games by making stupid decisions on the field, so he decides to exercise their mental skills. He rounds up the players, who are wearing t-shirts numbered 1 through 18, and shows them 18 identical boxes neatly lined up in a row.

“In each of these 18 boxes there is a slip of paper marked with a number between 1 and 18. In a moment, each of you will be sent to an isolated room. I will then call each and every one of you one by one and ask you to open any <u>nine</u> boxes, read the slips of paper inside each box, return them, close the boxes and return to your room. You can open any nine that you please, but you are not allowed to communicate with each other in any manner once you’ve entered your isolated rooms, nor are you allowed to mark the boxes in any way. If, after you have all visited the boxes, each of you has found within one of the nine boxes the same number that is written on his shirt, you will all get a $100,000 bonus. If, however, even one of you fails to reveal his own t-shirt number you will all be fined $100,000 and sent to play soccer for one month on an isolated island where they use coconuts as the ball.

One of the brighter players objected, “ Wait a minute! If I randomly open nine out of eighteen boxes, then I have a fifty percent chance of finding my t-shirt number. So does John here, and so does Sam. The probability that all of us succeed is the multiplication of these probabilities, which is 50% times 50% times 50% - 18 times - in other words 1/2 to the power of 18, which comes out less than four ten-thousandths of a percent?! There is no chance we’ll beat those odds?! You might as well send us off right now!”

“It’s true” retorted the coach “that if each player chooses nine boxes randomly, then the chances are negligible, however, I’m giving you now 5 minutes to come up with a strategy that will greatly improve the chances of success to over 30%!”
The players huddled together and after 5 minutes were sent to isolation. One by one, the coach took the players from their rooms and each opened nine boxes. Amazingly enough, they all found their numbers! What was their strategy?

Solution to the soccer team problem:

When huddled together, the players decided to mentally number the outside of the boxes with numbers from 1 to 18: The first in the row is mentally labeled 1, the second 2, and so on, until 18 for the last box in the row. The boxes had now a mental number on the outside and a number on the slip inside the box. Each player was to first open the box whose number on the outside was the same as the number on his own t-shirt. Then, he should read the number inside the box and go to the box whose number on the outside is the same as that number. He opens that box, reads the number on the inside and then opens the box that is numbered with that number, and so on. If within 9 boxes he finds the number on his t-shirt, then he is done because he has found his number.

For example: suppose the 18 boxes labeled on the outside are
1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18, and the slips of paper on the inside labeled: 13-8-5-18-3-15-11-7-9-12-2-10-4-14-6-16-17-1. The player with the number 7 on his shirt first opens the box labeled 7 on the outside. He then reads the note inside the box. On it is written the number 11, so he then goes to box number 11, opens it and reads the note inside which is numbered 2. He then goes to box number 2, opens it, reads the note which is numbered 8, goes to box 8, opens it and reads the note which is numbered 7. Here the player can stop because he found his number - the number on his shirt. Surprisingly, if every player uses this strategy, the odds are about 33%!

Explanation of the solution

To understand this strategy, we first note that the players have created a set of cyclic chains of numbers. Each chain depicts the route of a few players. Using the example above, there are the chains: 7-11-2-8-7, 1-13-4-18-1, 3-5, 6-15, 9-9, 10-12, 14-14, 16-16, 17-17.
For the players to succeed, there must be no chain larger than 9. A chain larger than 9 means that more than nine boxes have to be opened for each player whose number is included in this chain to find his number. This cannot happen since only nine boxes are opened. Moreover, if there is a chain larger than 9, then there is only one such chain. It may be of length 10 or 11 or 15 or whatever - but it is the only one with a length greater than 9.

We need to calculate the probability that within a random permutation of 18 numbers, we won’t find a cycle or ‘chain’ of length 10, 11, 12 etc. up to 18

Let’s side step a bit and look at a simpler calculation. Suppose there are four players and four boxes and each player is allowed to open only two boxes. In this case we need to calculate the probability that there are no chains of length 4 or 3.

First, we calculate that there are 4! (4 factorial = 24) possible arrangements of 4 numbers:

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

From these, six are *distinct* chains of length 4: 1234, 1342, 1423, 1243, 1324, 1432. The rest are just the same chains starting from a different number. A nice way to show this is to write the four numbers 1-4 at the corners of a square and draw all possible lines connecting them. We can trace six different cyclic paths along the lines, where each path visits each vertex once. These correspond to the six chains: 1234, 1342, 1423, 1243, 1324, 1432 - each direction (clockwise, anti-clockwise) is considered a separate path. Such cycles are called Hamiltonian cycles after the Irish mathematician William Rowan Hamilton who discovered them.

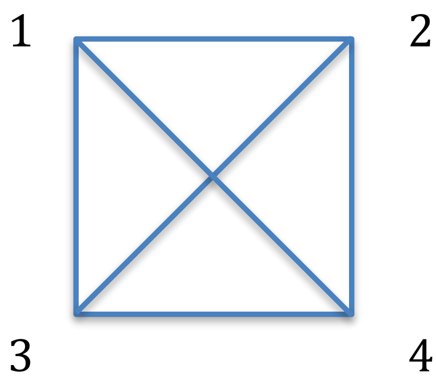


Fig. 1 Hamiltonian cycles on a square

Since 6 out of the 24 permutations are distinct length-4 chains, the possibility that a chain of length 4 is created is 6 in 24 - or - 1/4.

Now, we calculate the probability that a length-3 chain is created from the 4 numbers: 1,2,3 and 4. The number of possibilities of arranging the 4 numbers is still the same: 24. The number of Hamiltonian cycles on the triangle is 2. Since there are four possible triangles like this, depending on which number of the 4 you leave out - 1,2,3 or 4 - we multiply the number of Hamiltonian cycles - 2 - by 4 and get 8. So there is a chance of 8 in 24 or 1/3 to create a length-3 chain.

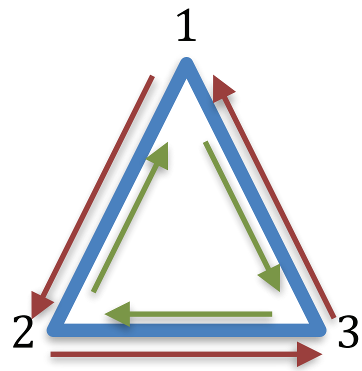


Fig. 2 Hamiltonian cycles on a triangle

This means that with 4 players and 4 boxes, the probability of creating a chain larger than 2 is one third plus one quarter, which equals 7/12 or about 58%. So, the complimentary probability of creating chains of length 2 or less is 100%-58%=42% - which is a fair chance for the players.

This argument can be generalised to *n* players and boxes to get the simple expression:

$$\lim_{n\to\infty}\left(1-\sum_{l=n+1}^{2n}\frac{1}{l}\right)=\ln(2)$$

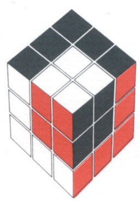
Eq. 2 General solution to the soccer team problem

It is well-suited to the occasion, to note that the connection between Hamiltonian cycles and chains was first brought to the attention of the wider public by Martin Gardner who wrote about it in an article on the binary gray code[4].

References

1. Flajolet Philippe and Sedgewick Robert, “*Analytic Combinatorics”*, Cambridge University Press, Cambridge, 2009.
2. Stanley Richard P., “*Algebraic Combinatorics: Walks, Trees, Tableaux, and More*”, Springer, New York Heidelberg Dordrecht London, 2013.
3. Winkler Peter, “*Mathematical Mind-Benders*”, A. K. Peters ltd., Wellesley, MA, 2007.
4. Gardner Martin, “*Knotted Doughnuts and Other Mathematical Entertainments*”, W. H. Freeman and Company, New York, 1986.

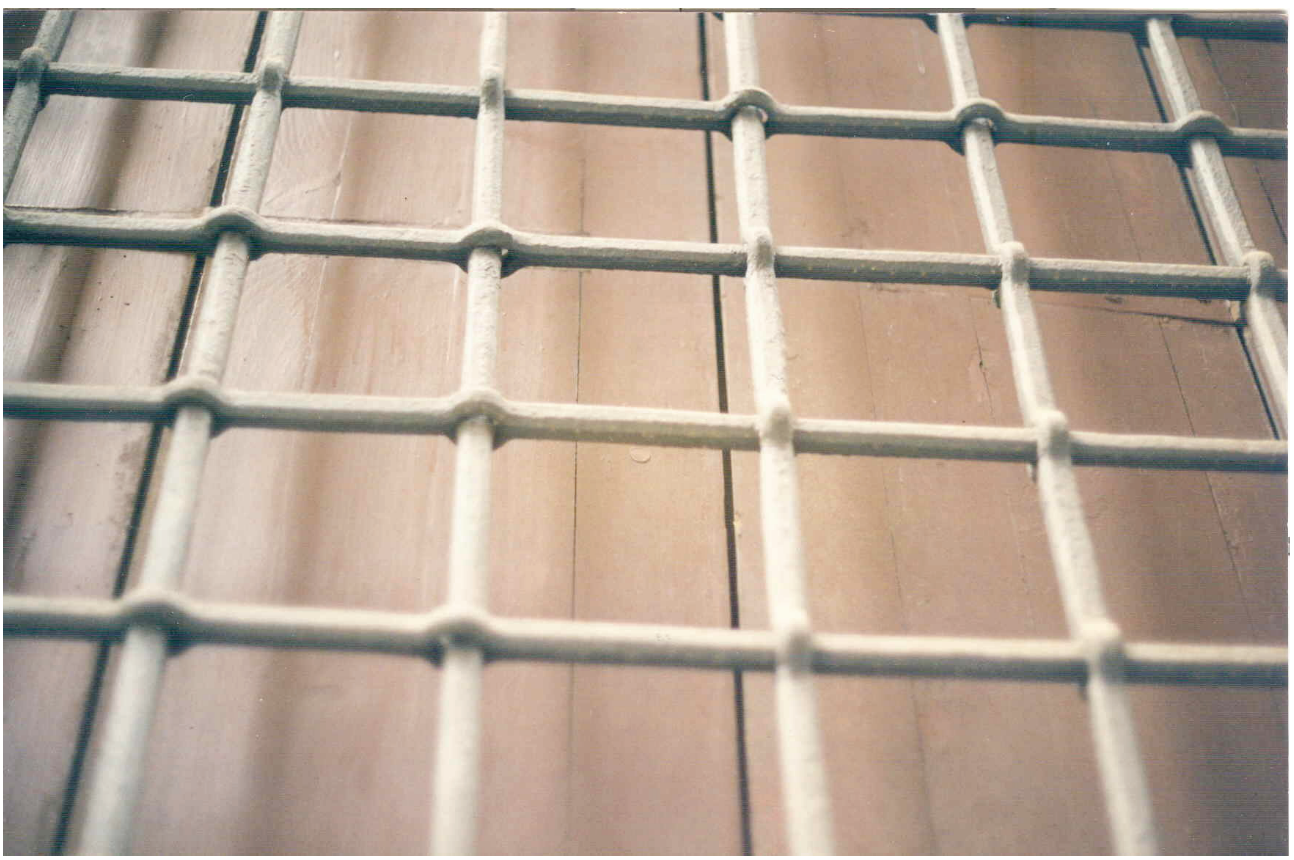
David Singmaster



Mathematician and Metagrobologist
87 Rodenhurst Road
London, SW4 8AF, England
Tel/fax: +44-(0)20-8674 3676
E-mail: ZINGMAST @ GMAIL.COM

ABOUT PUZZLE GRILLS

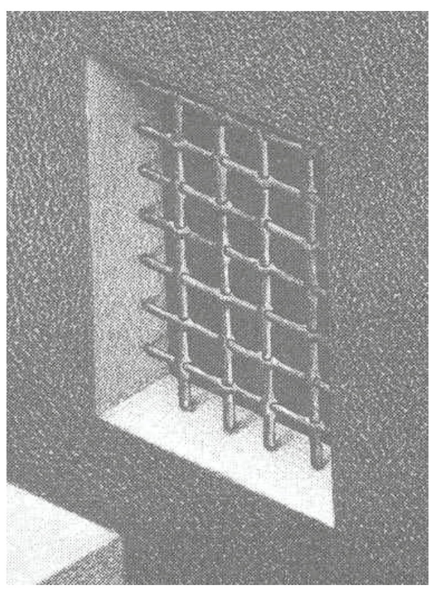
Travelling through medieval cities, one notices substantial window grills. Some years ago, I noticed that some of these had a puzzling central area where four rods formed a square with each rod passing through the next in sequence. After some contemplation, I realised that one could assemble such a square by a kind of uniform convergence. But the pattern continues outward and this prevents the uniform converging method. I had discussed this with Jean-Claude Constantine and he was going to make some pieces for me, but I never heard from him. I later discussed it with James Dalgety and he made an example from bended wire. Looking at his example, I saw how to assemble it, and I've thought it would make a nice puzzle.



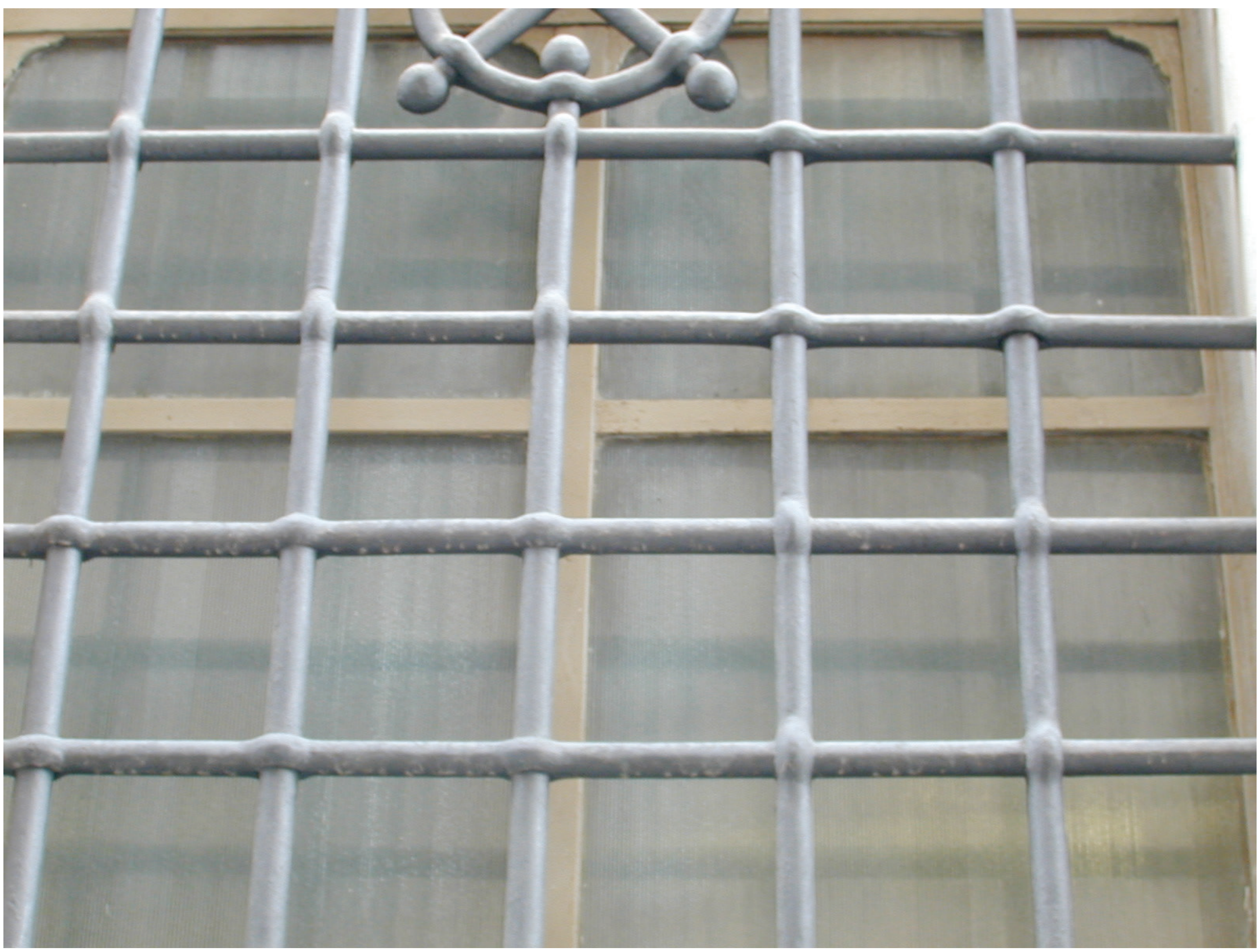
This is from the Palazzo Thieme in Vicenza, designed by Palladio.

Close up of the same grill, showing the impossible central square.

This has recently resurfaced in my thinking because Escher uses the idea in his Cycle (1933) and Belvedere (1968)

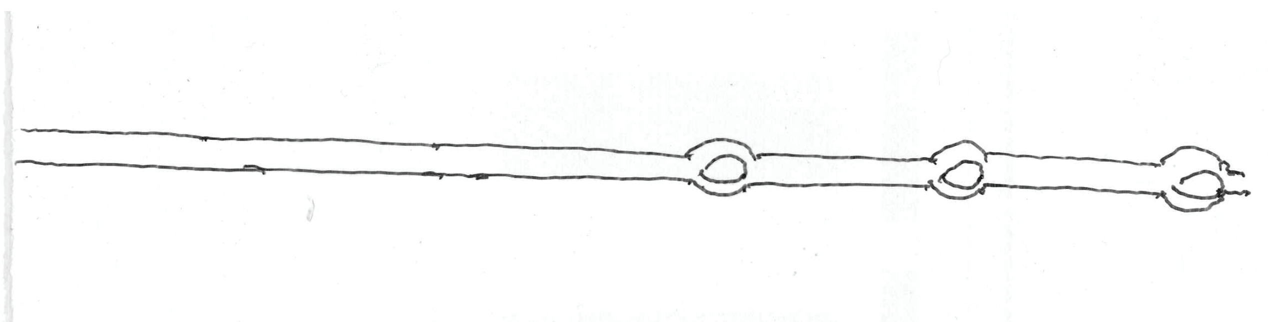


However, examination shows Escher has continued alternating horizontal and vertical holes and the result is genuinely impossible to assemble (I think).

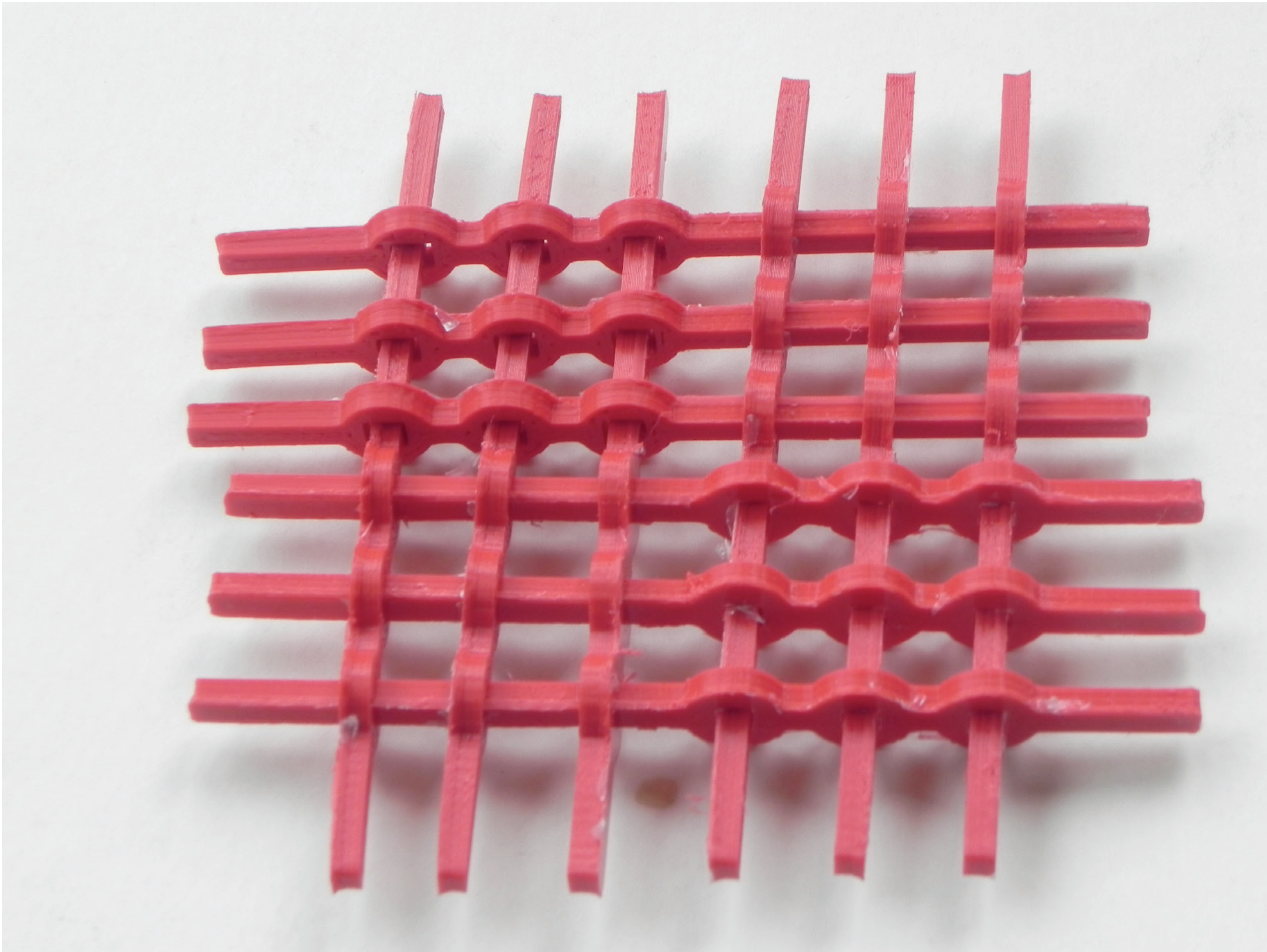


This is from Citta di Castello.

On Bank of Italy, Florence.



On 7 Nov. 2015, I gave a short talk on this topic at Maths Jam 2015. I asked how I could get an example made and showed the crude version I had made from bent wire. Someone suggested 3-D printing. Simon Bexfield was present and had brought two 3-D printers. By the time I asked him, he said he had already programmed in the pattern. That evening, the first example was ready. Unfortunately, the holes weren't big enough and Simon adjusted the program. In the morning a good set of pieces was ready and I assembled them and showed it around. Since then, Simon has made me a few more examples.



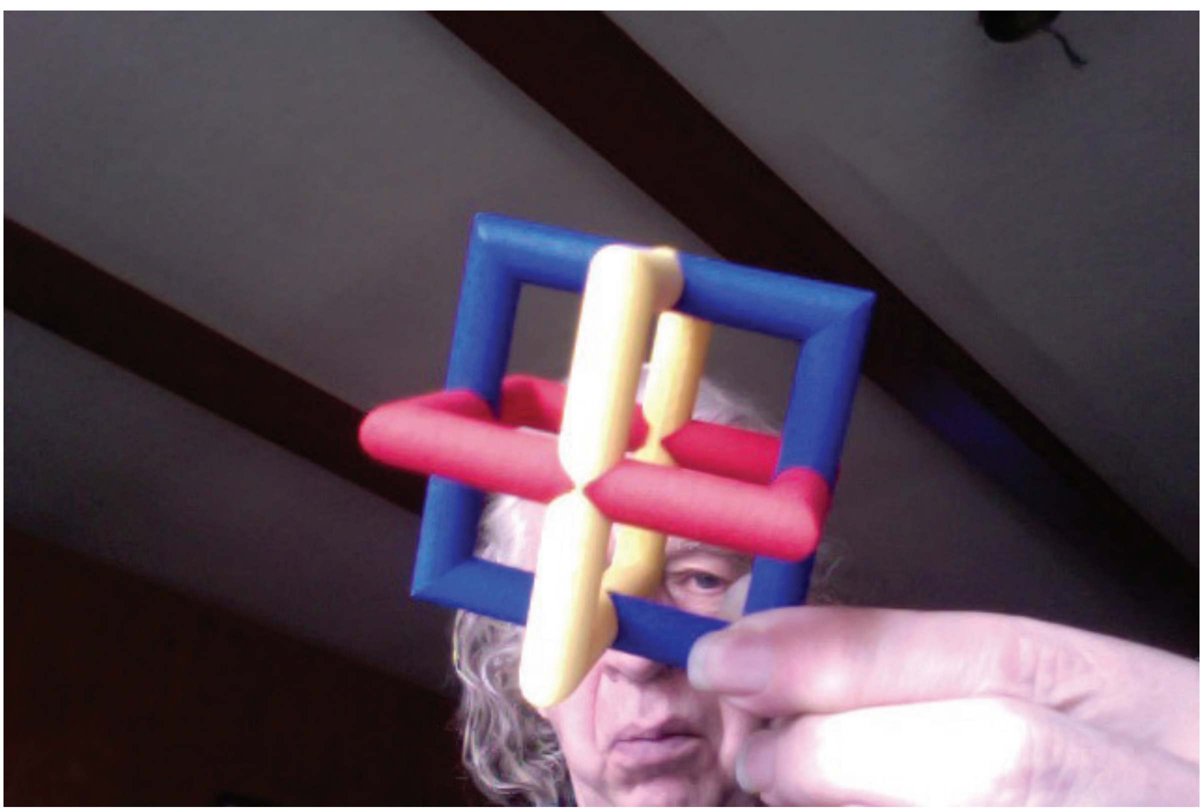
The first correct 3-D printed example, made by Simon Bexfield, 7-8 Nov 2015.

Bill & Neil's ill-fated Space-cube gift.

We sought to redesign the popular commercial version,

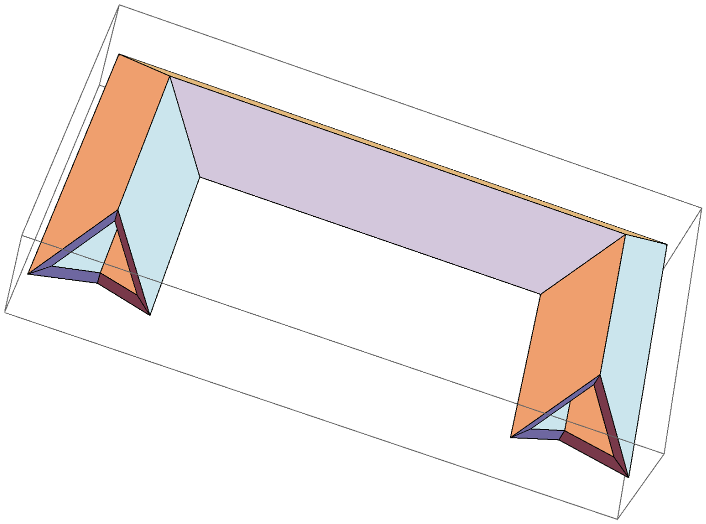
whose first step of assembly requires the delicate opposition of the small flat tips, which only exist because the structure is not really a cube. If it were, the flat tips would become pairs of sharp points which would be impossible to oppose. So we tried

Out[1171]=



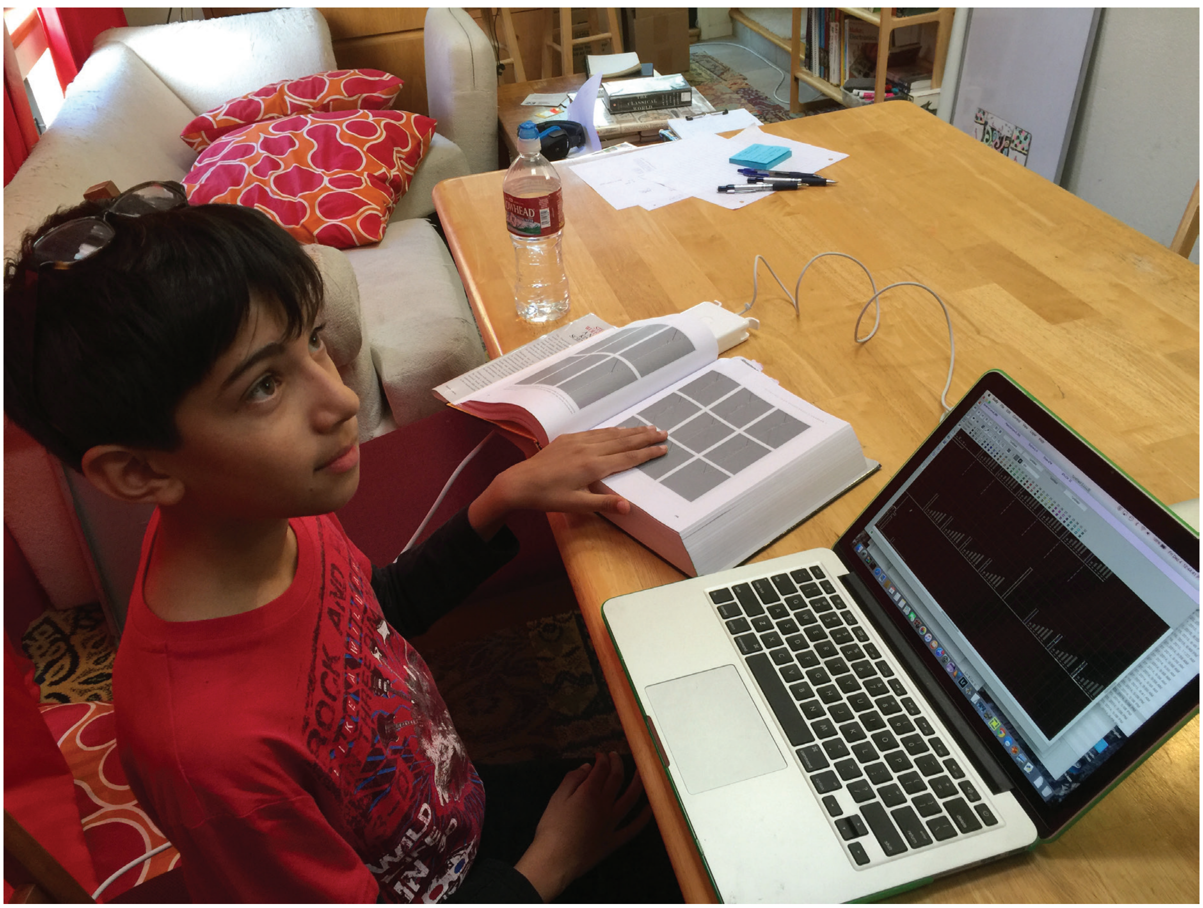
Problem 1: The sharp points were fragile. They bent and broke.
Problem 2: The puzzle is too easy!
Just squeeze the yellow pieces between the red pieces, twist them aside, drop in a blue piece, secure it by untwisting the yellows, flip it over, and repeat for the other blue piece.

But try twisting a square peg in a square hole!



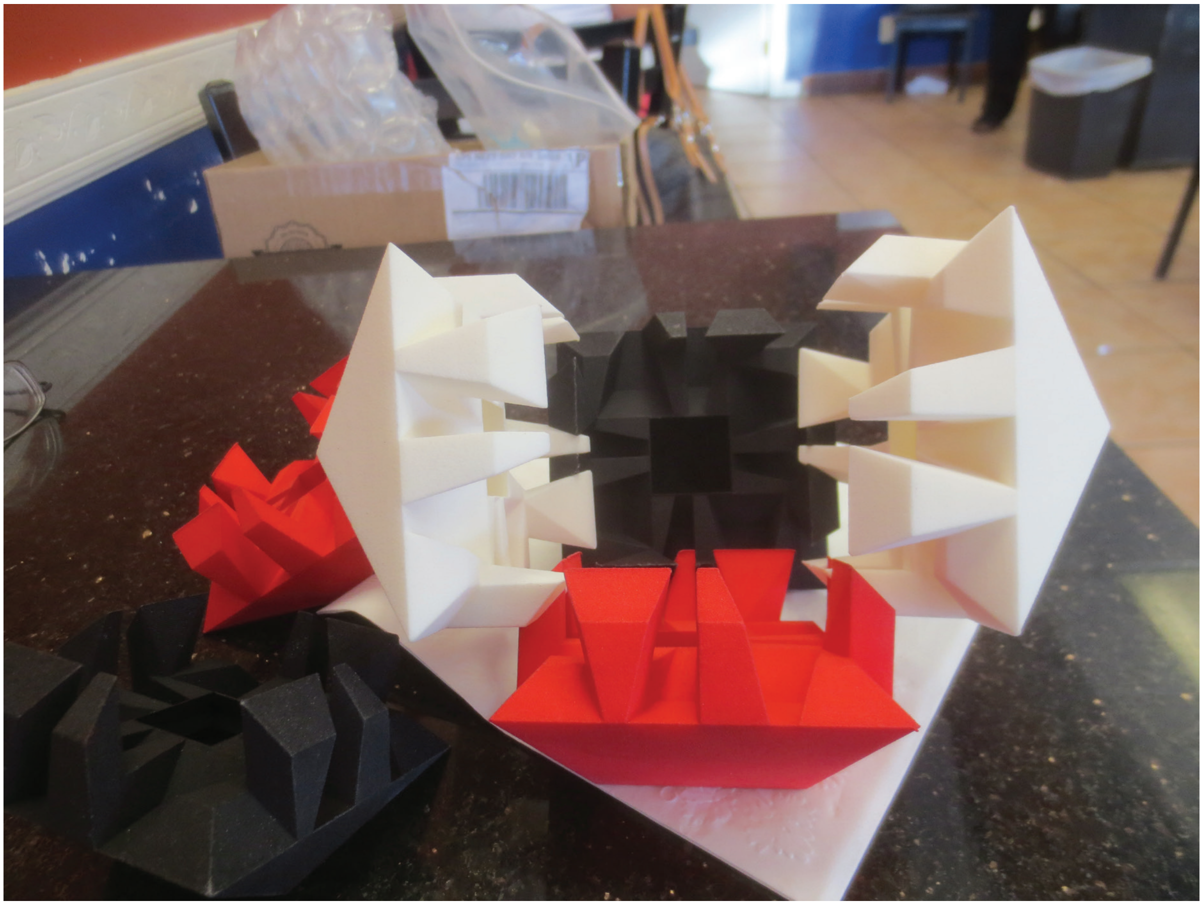
Is this even feasible? Rohan Ridenour

Out[1172]=



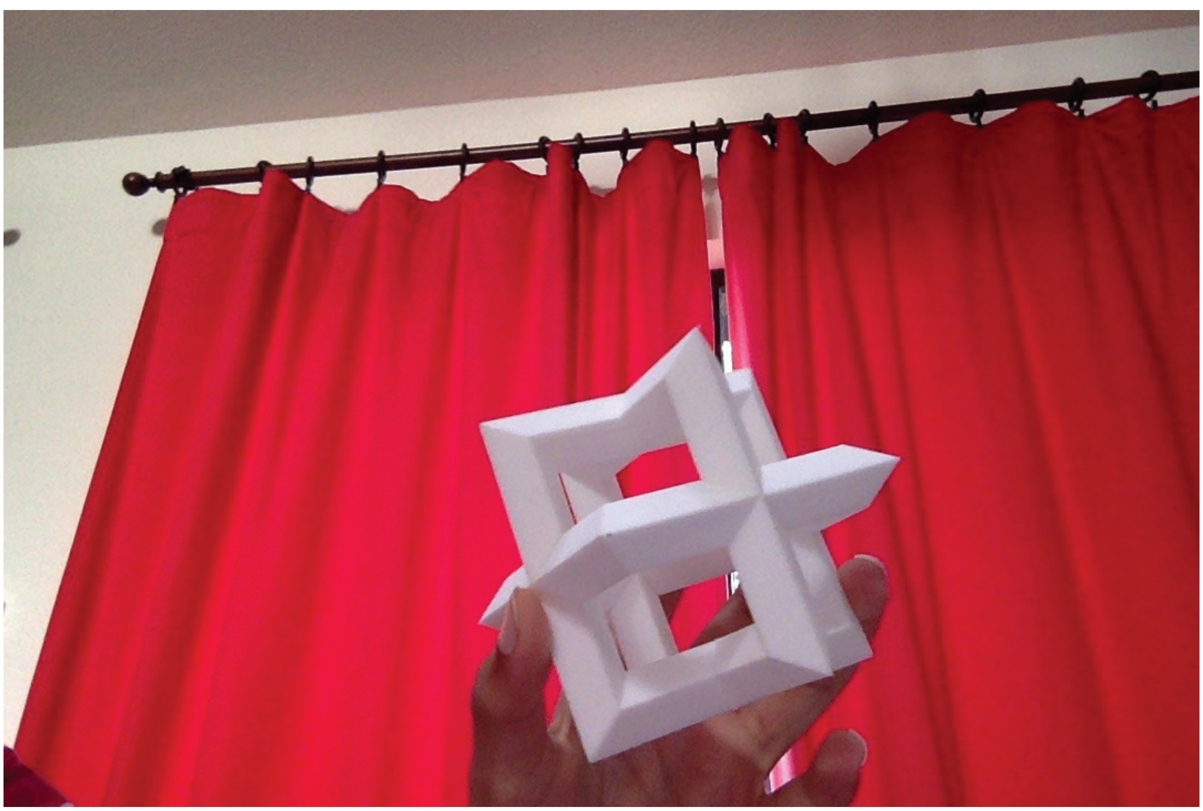
(age 10) said it obviously was, via the same coordinated motion required for our cube puzzle:

Out[1174]=



As proof, Rohan holds one up:

Out[1173]=



But sadly, we ran out of time and money. Commercial Spacecubes run about $6, but Shapeways wanted ten times that for our Tubecube, which still needs to have some detents embossed on it to compensate for wear. And poor Neil had five finals and missed G12 entirely!

Carl's Puzzles

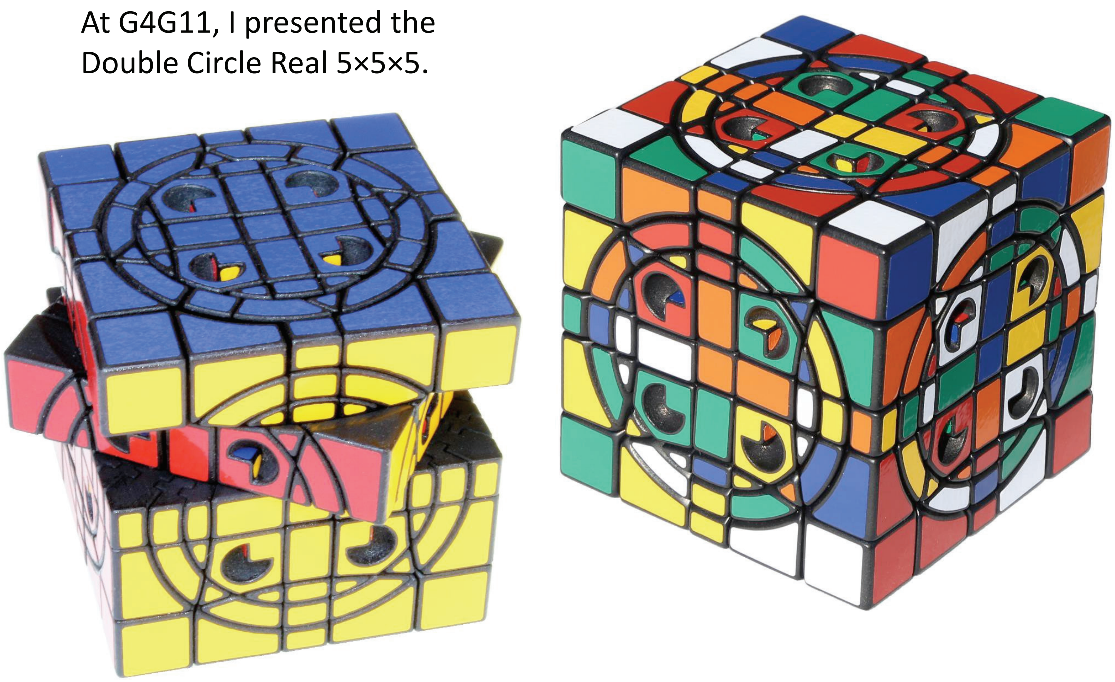
Update on Double Circle Real 5×5×5 and Introduction of the WOW5 Puzzle Ring

by: Carl Hoff

G4G12

Double Circle Real 5×5×5

At G4G11, I presented the Double Circle Real 5×5×5.

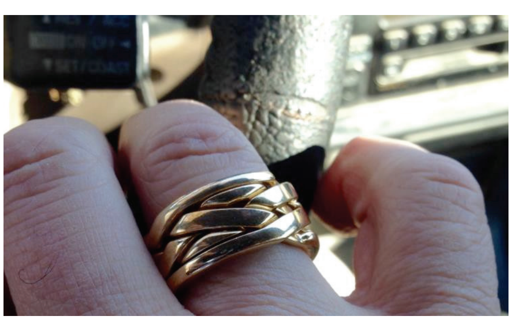


The Double Circle Real 5×5×5 is a 1×1×1 inside a 3×3×3 all inside of a 5×5×5. At G4G11, it was still a work in progress. It is now complete and functions as designed. I am so proud of it that I wrote an article about it for the *Game & Puzzle Design* journal. That article has now been published and it was so well received that the image chosen for the cover was a rendering of the Double Circle Real 5x5x5 mechanism.

Hoff, C., ‘The Double Circle Real 5×5×5’, Game & Puzzle Design, vol. 1, no. 2, 2015, pp. 5–14. c 2015

WOW5: Wrap O-round Weave Five

WOW5 is a 5 band puzzle ring where the weave pattern interlocking the 5 bands extends around the entire 360 degree circumference of the ring.



Why make the WOW5 puzzle ring?

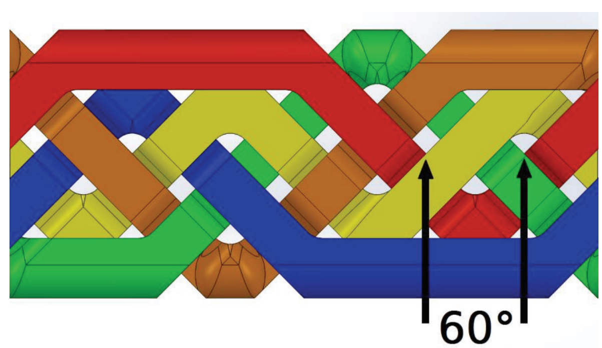
On traditional puzzle rings the weave only covers a fraction of the circumference. There are reasons for this:

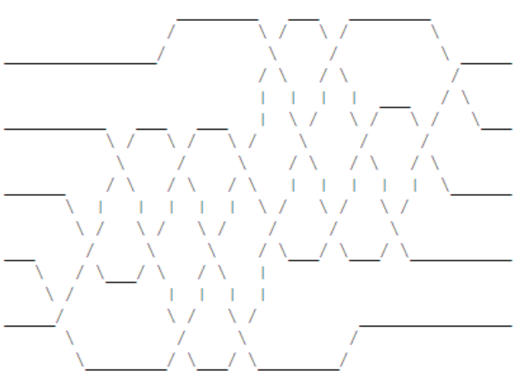
- Puzzle rings are cast from molds and puzzle ring producers want to make all the various sizes of rings from a single set of molds. The straight sections opposite the weave allow a region where material can be cut and removed without affecting the puzzle.
- Many weave patterns would simply result is a ring which was locked in the solved state if the weave were extended all the way around.

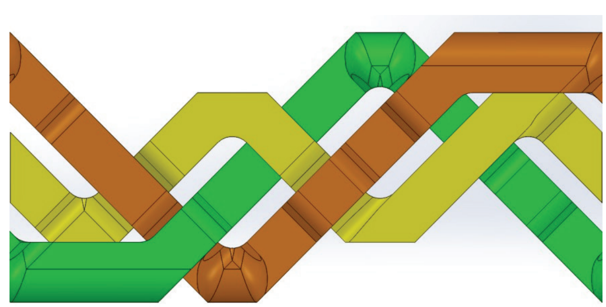
The weave tends to be the feature that is highlighted at the top of the ring when worn. But the ring can rotate on the finger and can look odd or be uncomfortable if its not centered at the top of the finger.

The Weave Pattern

Weave Five is a weave pattern published by Bram Cohen as ASCII art. I examined the weave and believed it would allow for disassembly as a wrap around design but wasn't certain until I had made one.







How was WOW5 made?

It was 3D printed. But the ring must be printed in the scrambled state.

- A 3D model is created via CAD and this model is trivial to resize on the computer.
- Shapeways has a pilot program called "interlocking metal" where they connect the interlocking bands with sprues.
- This is now printed in wax using a high resolution printer.
- The wax model is placed in a container. It then has liquid plaster poured over it.
- After the plaster is set the wax is melted and removed allowing the plaster to serve as a mold.
- Molten metal is poured into the mold.
- Once the metal hardens the plaster is broken away.
- The sprues are then cut away and the remaining puzzle bands can be cleaned and polished.

End result…one very hard puzzle ring…

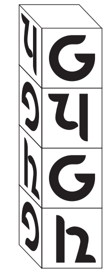
You will all get one printed in laser sintered nylon from i.Materialise as your exchange gift from me. Fortunately nylon allows it to be printed in the solved state. Enjoy…

I'm currently writing up another article for the *Game & Puzzle Design* journal about WOW5.

Where to next...

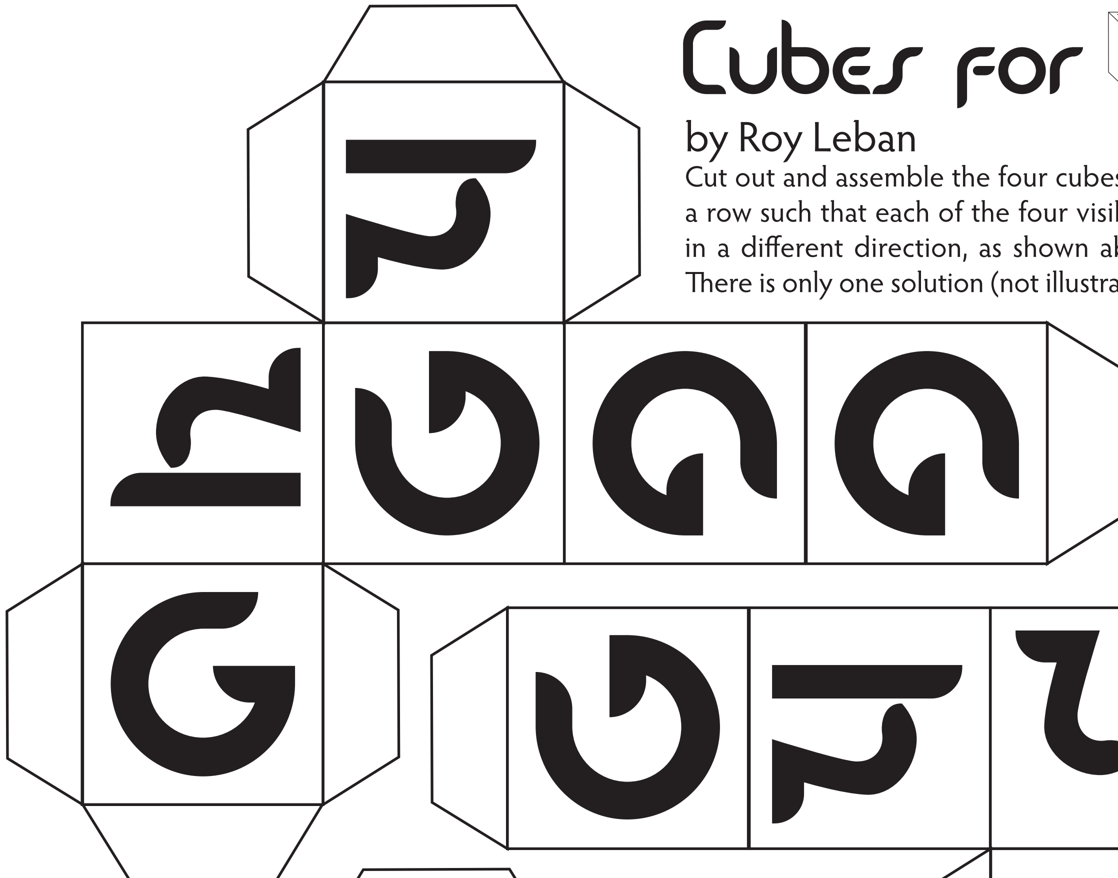
You may contact me at:
carl.n.hoff@gmail.com

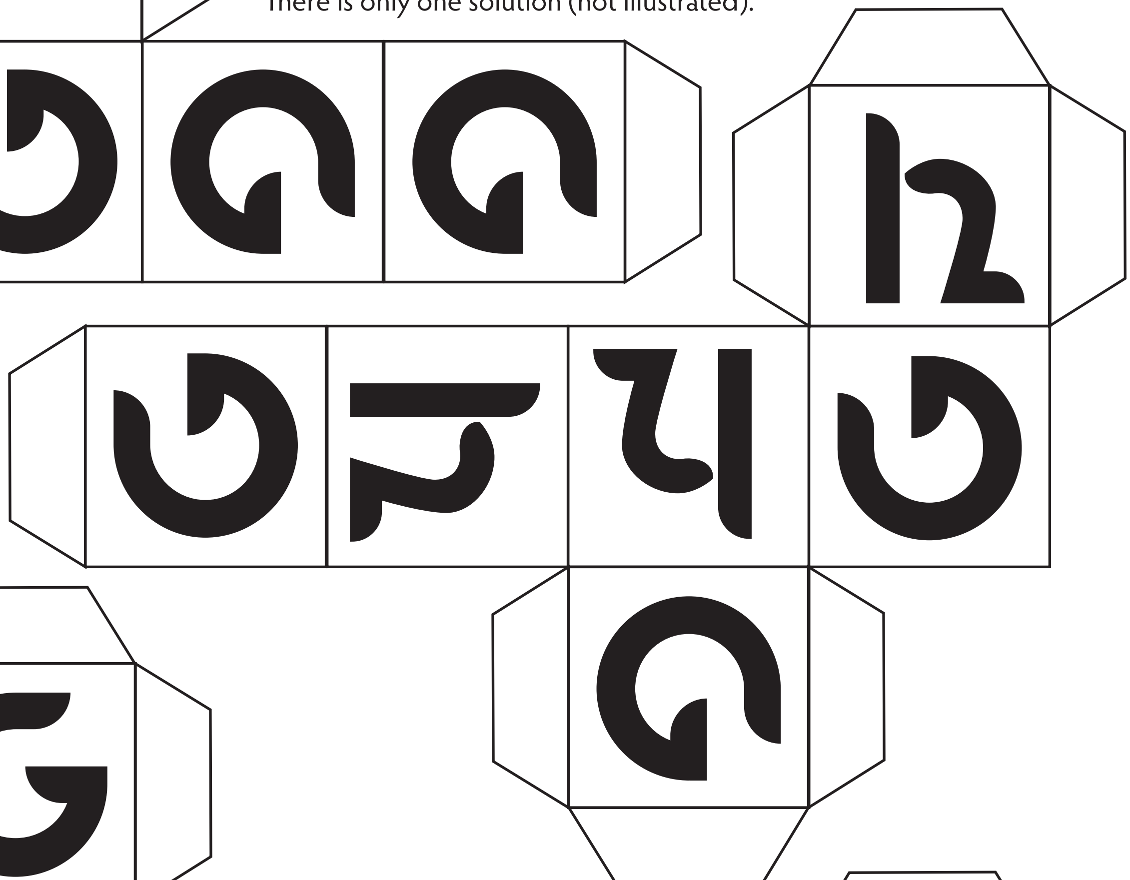
Cubes for 

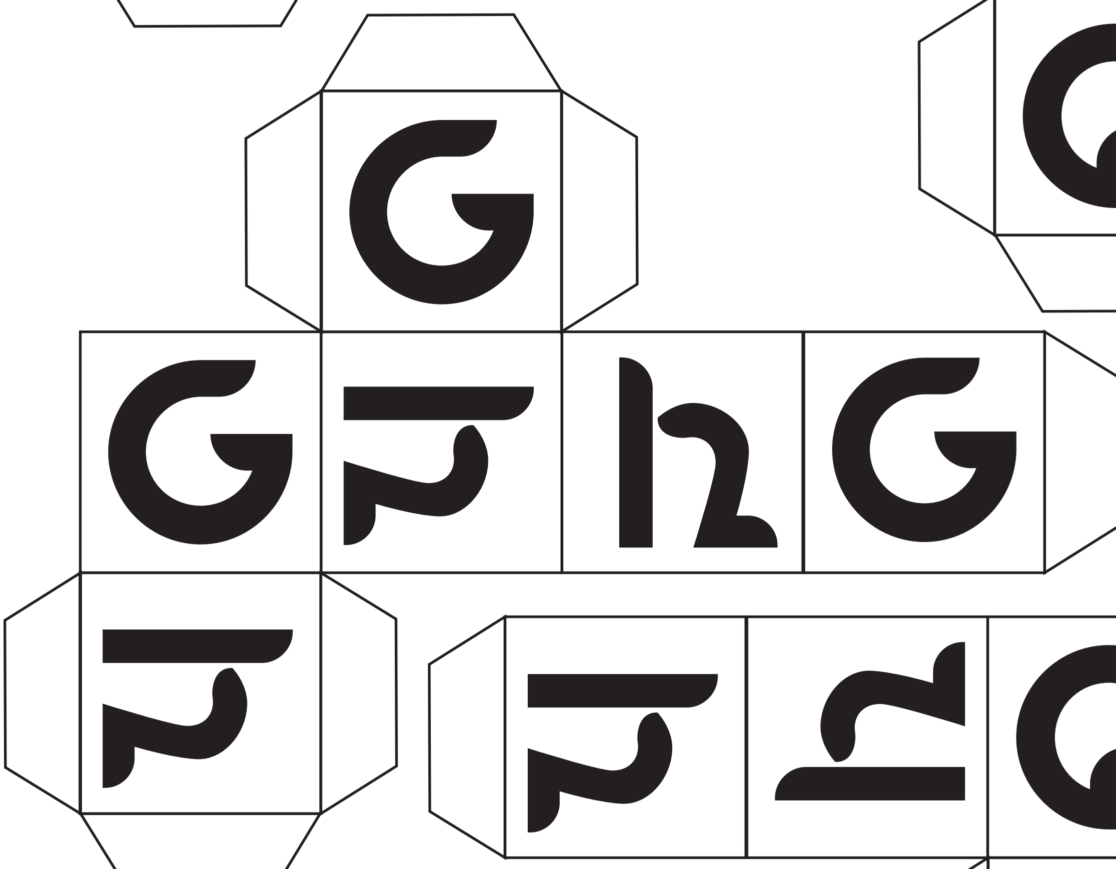


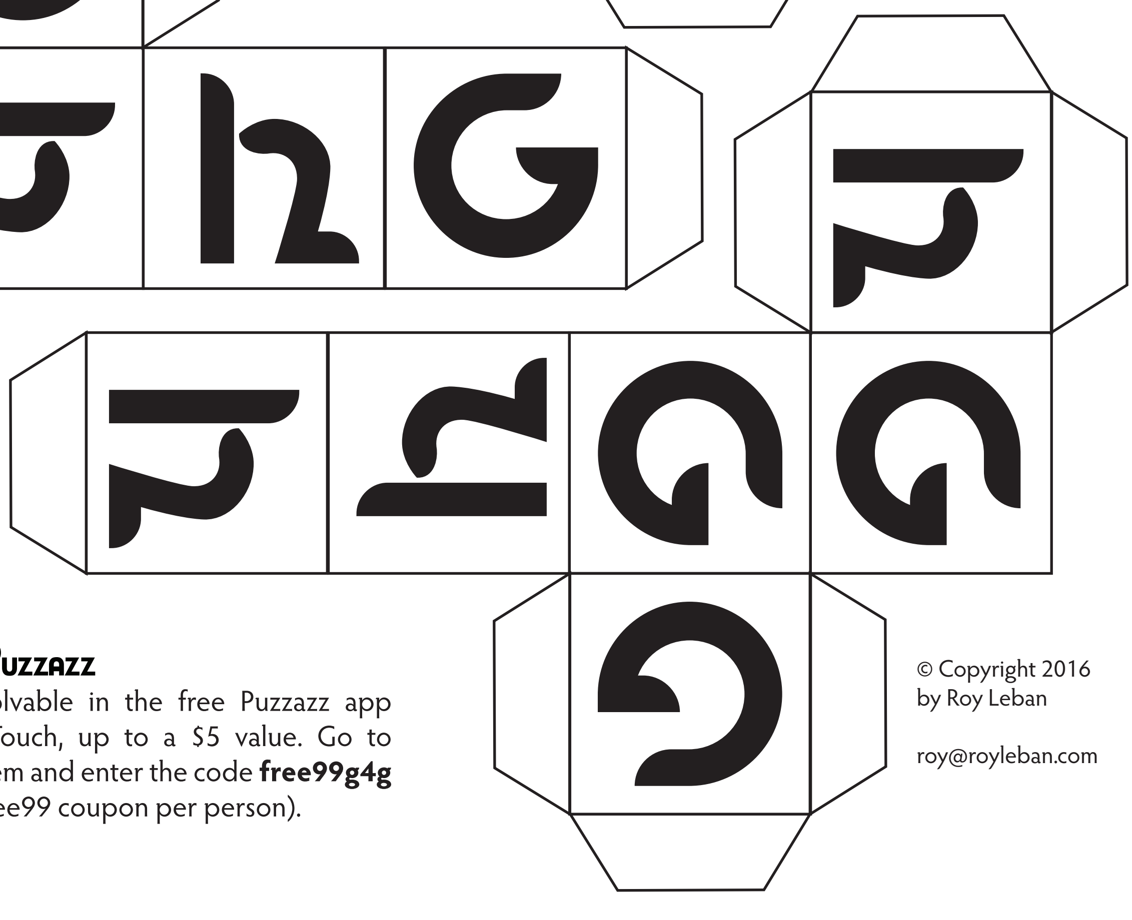
by Roy Leban

Cut out and assemble the four cubes, then arrange them in a row such that each of the four visible sides reads G4G12 in a different direction, as shown above and to the right. There is only one solution (not illustrated).





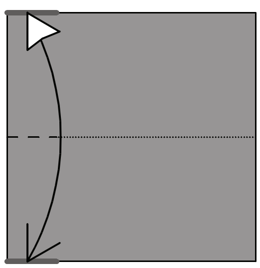




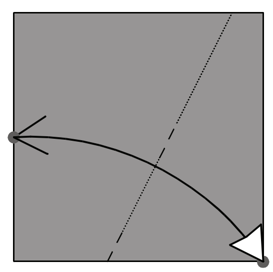
roy@royleban.com

DeZZ Unit

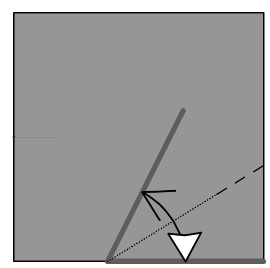
This is actually several units in one: a **De**ltahedron **Z**ig-**Z**ag unit, which can be used to fold any deltahedron (any polyhedron whose faces are equilateral triangles). A variation of the unit lets you fold a twisted-hole cube; another variation works for any deltahedrally elevated polyhedron; another variation folds a rhomboidal polyhedron that is the Wolfram Alpha logo. The units draw upon concepts identified and explored by Bob Neale, Lewis Simon, and Mitsunobu Sonobe, not to mention Tom Hull's famous PHiZZ unit, which provides, as well, the rationale for this module's name.



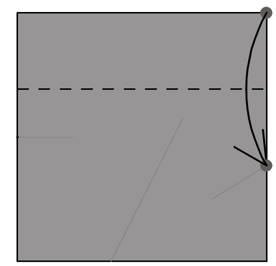
1. Begin with a square, colored side up. Fold in half vertically and unfold, making a pinch at the left.



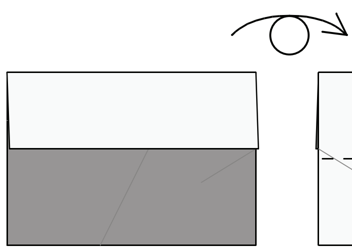
2. Fold the bottom left corner to the mark you just made, creasing as lightly as possible.



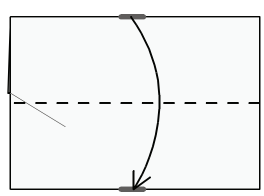
3. Fold and unfold along an angle bisector, making a pinch along the right edge.



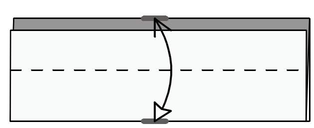
4. Fold the top left corner down to the crease intersection.



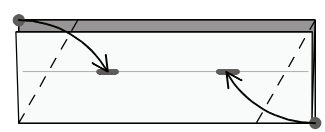
5. Turn the paper over.



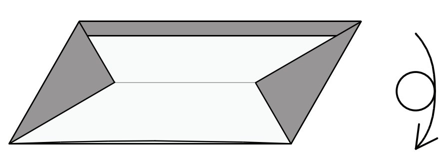
6. Fold the top folded edge down to the raw bottom edge.



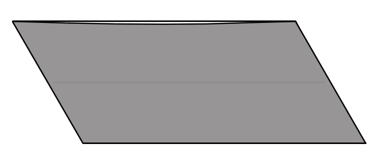
7. Fold the bottom folded edge (but not the raw edge behind it) up to the top and unfold.



8. Fold the top left corner and the bottom right corner to the crease you just made.



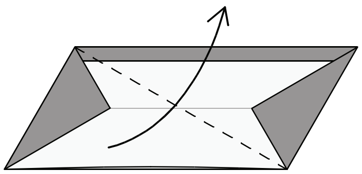
9. Here's the building block.



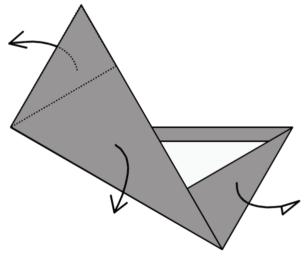
10. Here's the other side.

Deltahedra

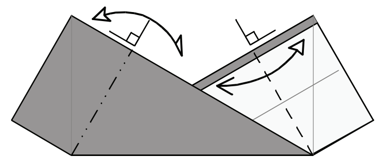
This unit can be used to make any polyhedron whose faces are equilateral triangles.



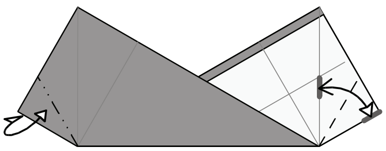
1. Begin with step 9 of the DeZZ building block. Fold the quadrilateral in half along the diagonal.



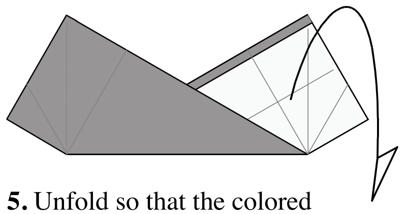
2. Unfold all the creases to 90° dhiedral angles. Make 12 units.



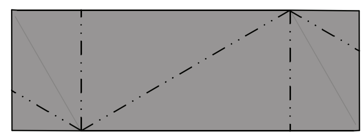
3. Fold and unfold. Repeat behind.



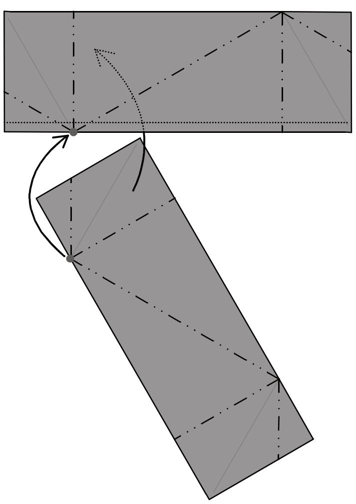
4. Fold and unfold. Repeat behind.



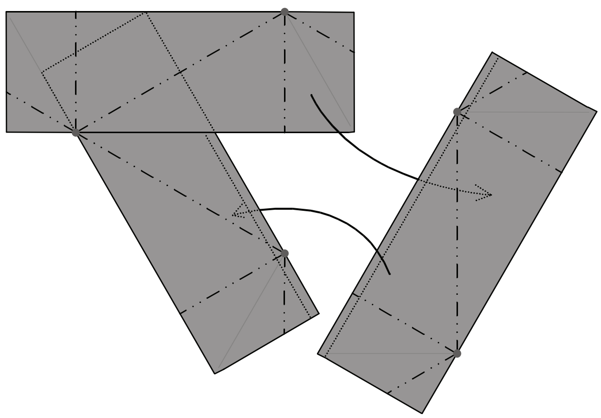
5. Unfold so that the colored side is visible.



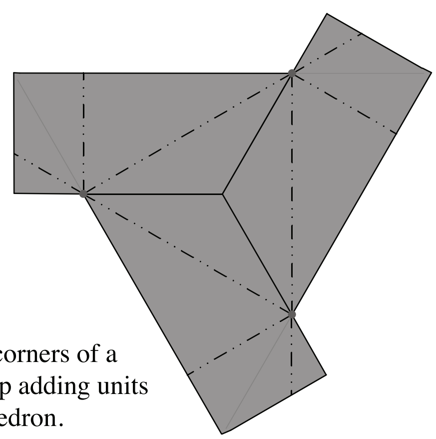
6. The mountain creases here show the folds that are used for the deltahedron. Fold $3N/2$ units for a deltahedron with N faces.



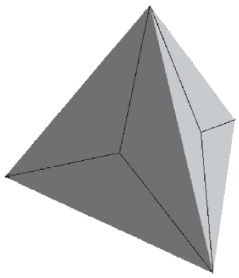
7. Here is how two units go together. The tab slips into the pocket on the back side, and the two points marked by dots come together.



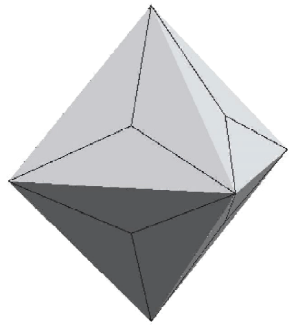
8. The third unit goes inside one pocket and outside one tab.



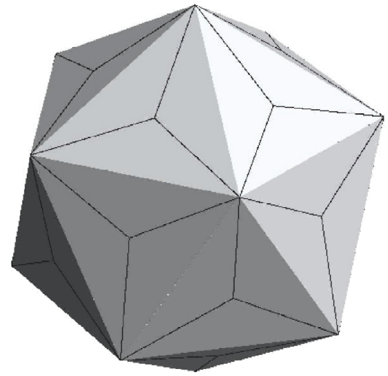
9. The dots are the corners of a triangular face. Keep adding units to create any deltahedron.



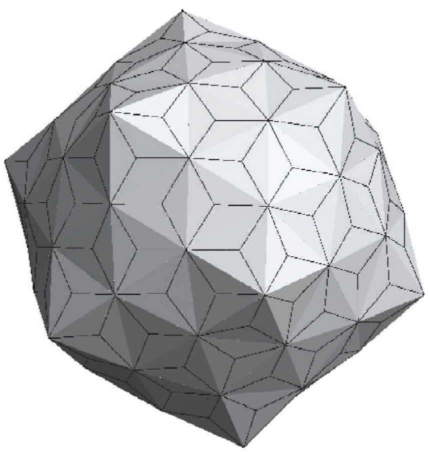
10. Here is a tetrahedron, from 6 units.



11. An octahedron takes 12 units.



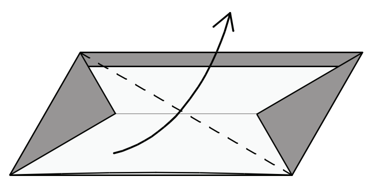
12. And an icosahedron takes 30 units.



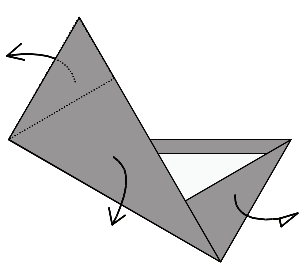
13. With 210 units, one can make a deltahedrified snub dodecahedron. However, the very shallow angles means that it doesn't hold together very well.

Plain Twisted-Hole Cube

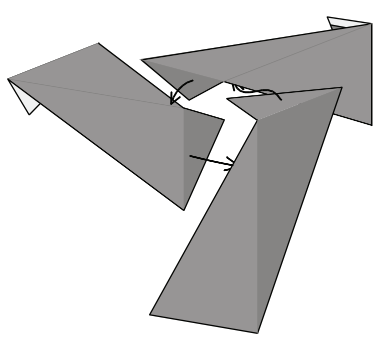
This structure is similar to Lewis Simon's many twist-hole cubes, but uses the assembly technique of Robert Neale's dodecahedron.



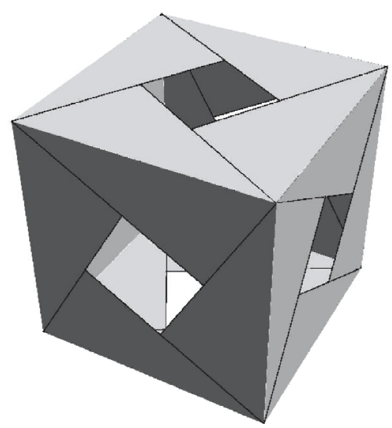
1. Begin with step 9 of the DeZZ building block. Fold the quadrilateral in half along the diagonal.



2. Unfold all the creases to 90° dhiedral angles. Make 12 units.



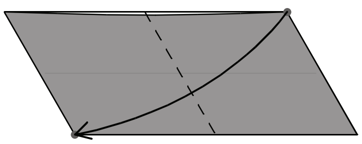
3. Join three units at a corner by sliding tabs into pockets.



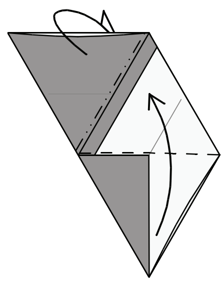
4. The finished cube.

Deltahedrally Elevated Polyhedra

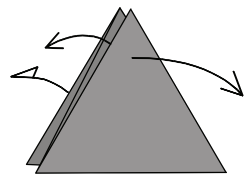
Elevation is the result of erecting a pyramid on each face. If the resulting new faces are equilateral triangles, then we can fold them from still another version of this unit that makes each face a seamless equilateral triangle.



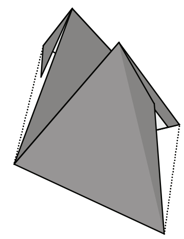
1. Begin with step 10 of the DeZZ building block. Bring two points together.



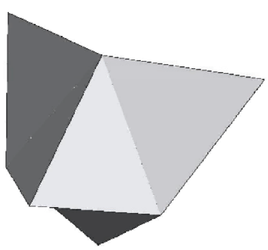
2. Fold the upper left triangle behind and the bottom triangle up.



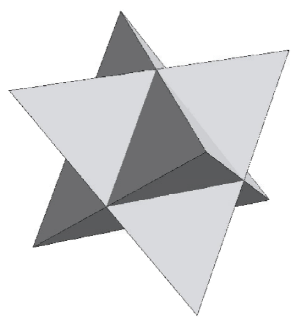
3. Partially unfold all of the folds along the strip.



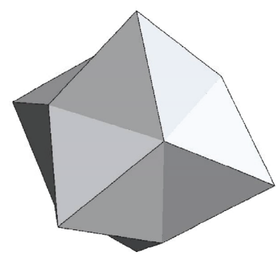
4. One unit makes a portion of a double pyramid.



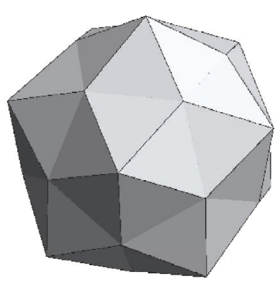
5. 4 units makes an elevated tetrahedron, which resembles a caltrop.



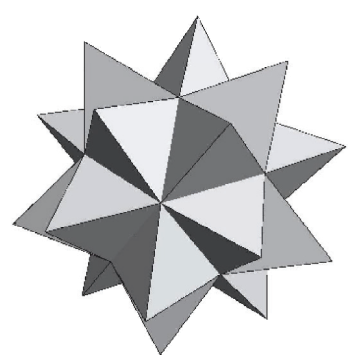
6. The 12-unit elevated octahedron is also a stellation of the octahedron; Kepler called it the *Stella Octangula*.



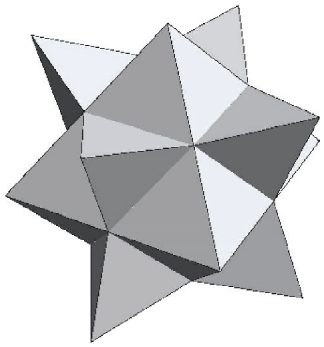
8. The elevated cube takes 12 units, and resembles a slightly stubbier version of the origami model called the *Jackstone*.



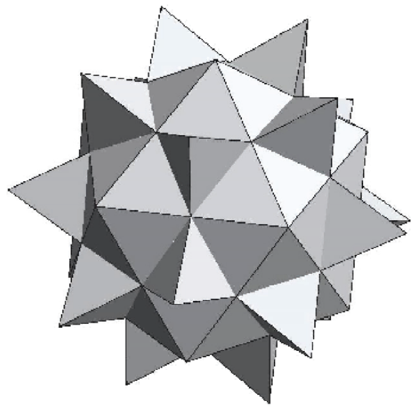
9. The 30-unit elevated dodecahedron is a slightly bumpy ball that is close to, but not exactly, a rhombic triacontahedron.



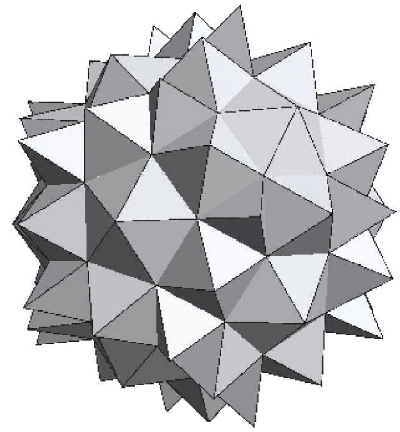
10. The 30-unit elevated icosahedron is considerably bumpier. It is close to, but not exactly, a stellation of the dodecahedron.



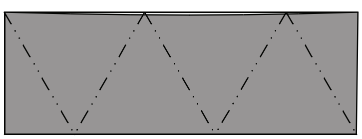
11. The elevated cuboctahedron takes 24 units.



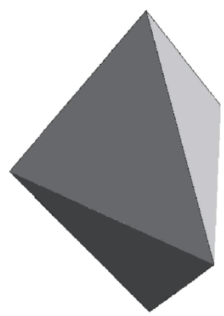
12. The elevated icosidodecahedron takes 60 units. Leonardo da Vinci described (and named) this solid.



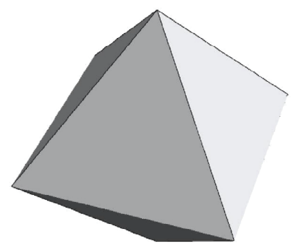
13. And finally, 200 units will build you the elevated small rhombicosidodecahedron.



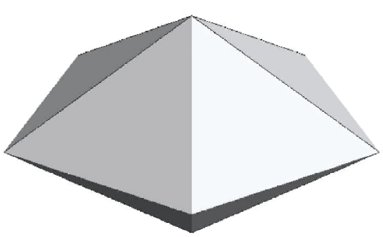
14. If you make 3 units with all mountain folds, they can be assembled into the deltahedral equivalent of *Takahama's Jewel*.



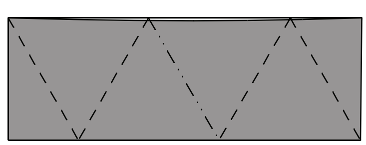
15 . Which is a deltahedrally elevated trigonal dihedron, or, more simply, a tetrahedral dipyramid.



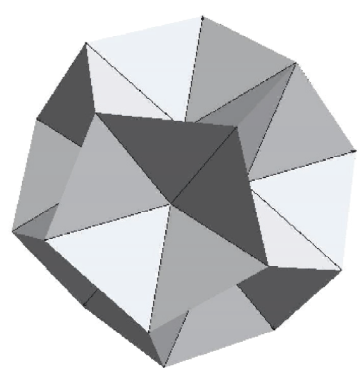
16 . Four units gives a deltahedrally elevated square dihedron, or a square dipyramid, or simply, an octahedron.



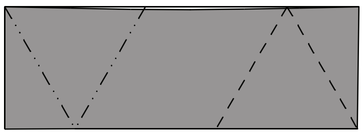
17. And 5 such units gives a pentagonal dipyramid.



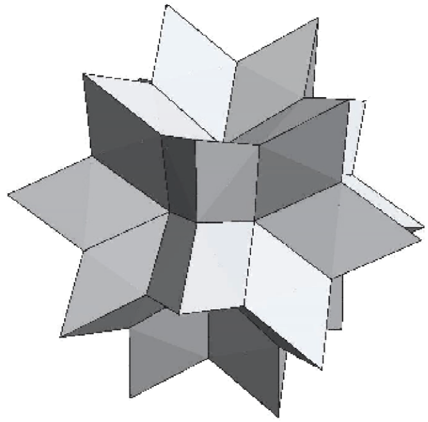
18. If a polyhedron is elevated with negative height, we call it "depressed." You can fold depressed polyhedra by changing the parity of some of the creases like this.



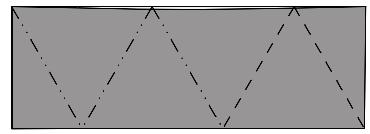
19 . This depressed dodecahedron requires 30 units, like its elevated kin.



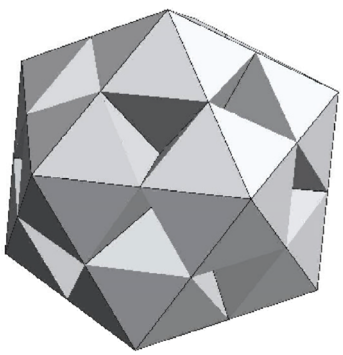
20. Leaving out the middle crease gives a unit that has an interesting application...



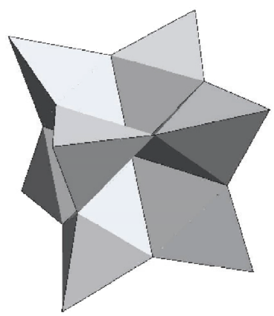
21. If you elevate the triangles and depress the pentagons of an icosidodecahedron, you get this shape, which also happens to be the logo of Wolfram | Alpha.



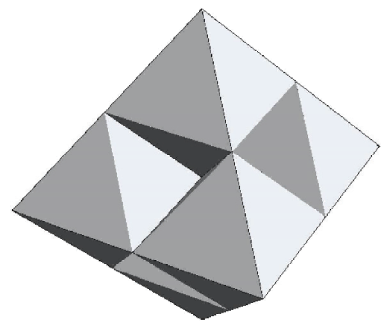
22. With the mountain fold back in place, we can make other mixed elevated/depressed polyhedra...



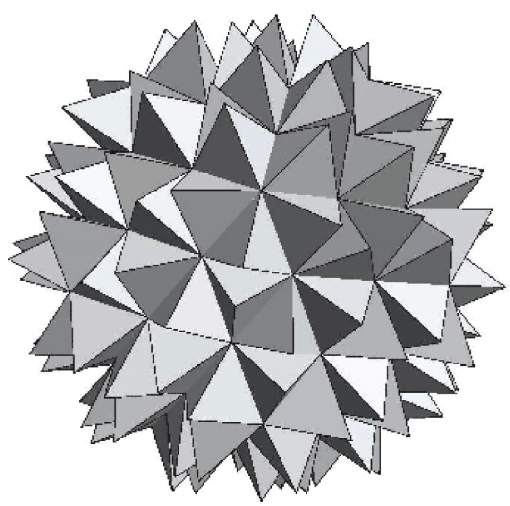
23. If you depress the triangles and elevate the pentagons of the icosidodecahedron, you get an icosahedron with holes.



24. We can treat the cuboctahedron similarly. Elevated triangles, depressed squares in a cuboctahedron.



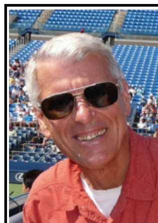
25. Elevated squares, depressed triangles in a cuboctahedron.



26. And finally, to wrap up, going back to the original elevated unit, 210 units give a deltahedrally-elevated deltahedrified snub dodecahedron. (Yes, that's double-deltahedrification!)

DOUBLE DUMMY PROBLEMS

BY DICK HESS



PUZZLES AND RECREATIONAL MATH

Dick Hess

E-mail: **rihess@cox.net**
Phone: 1-310-547-3637
4100 Palos Verdes Drive East
Rancho Palos Verdes, CA 90275-6462
Mobile: 1-310-227-6061

Introduction: These double dummy problems are prepared for the Gathering for Gardner 12 held in Atlanta from 30 March to 3 April 2016. Each problem asks you to show how south as declarer can make the indicated number of tricks. The problems come from various sources including the Encyclopedia of Bridge, Vanity Fair's Problem Book, and Bridge Squeezes Complete, by C. Love.

Happy puzzling from Dick Hess.

A

The contract is in No Trump.
South is on lead.
Take all 6 tricks.

North: ♠ AJ9 ♥ 432 ♦ - ♣ -
West: ♠ K106 ♥ - ♦ AKQ ♣ -
East: ♠ - ♥ Q85 ♦ - ♣ AKQ
South: ♠ 542 ♥ K107 ♦ - ♣ -

SOLUTION A

Alternate leading ♠'s from south and ♥'s from north, taking the trick as cheaply as possible each time to finesse your way to victory.

North: ♠ AJ9 ♥ 432 ♦ - ♣ -
West: ♠ K106 ♥ - ♦ AKQ ♣ -
East: ♠ - ♥ Q85 ♦ - ♣ AKQ
South: ♠ 542 ♥ K107 ♦ - ♣ -

B

Spades are trumps.
South is on lead.
Take all 6 tricks.

North: ♠ AJ10 ♥ 852 ♦ - ♣ -
West: ♠ 65 ♥ QJ ♦ AK ♣ -
East: ♠ 97432 ♥ - ♦ J ♣ -
South: ♠ KQ8 ♥ - ♦ 654 ♣ -

SOLUTION B

Alternate leading ♦'s and ♥'s, trumping the trick as cheaply as possible each time to cross ruff your way to victory.

North: ♠ AJ10 ♥ 852 ♦ - ♣ -
West: ♠ 65 ♥ QJ ♦ AK ♣ -
East: ♠ 97432 ♥ - ♦ J ♣ -
South: ♠ KQ8 ♥ - ♦ 654 ♣ -

C

Spades are trumps.
South is on lead.
Take 2 of 3 tricks.

♠ K32
♥ -
♦ -
♣ -

♠ AJ4 ♠ 976
♥ - ♥ -
♦ - ♦ -
♣ - ♣ -

♠ Q108
♥ -
♦ -
♣ -

SOLUTION C

Lead the ♠10 or the ♠Q. If west (a) ducks the lead: north plays the ♠2. (b) covers the ♠10 with the ♠J: north plays the ♠K and leads the ♠2. South covers east's play. (c) takes the ♠Q with the ♠A: west is end played.

♠ K32
♥ -
♦ -
♣ -

♠ AJ4 ♠ 976
♥ - ♥ -
♦ - ♦ -
♣ - ♣ -

♠ Q108
♥ -
♦ -
♣ -

D

Clubs are trumps.
North is on lead.
Take all 3 tricks.

♠ Q7
♥ -
♦ -
♣ K

♠ - ♠ K
♥ J ♥ -
♦ J ♦ 97
♣ 3 ♣ -

♠ -
♥ 9
♦ 10
♣ A

SOLUTION D

North leads the ♠7 and south trumps with the ♣A. If west plays (a) the ♣3: south trumps a ♥ or ♦ in north's hand. (b) a ♥ or ♦: south leads the promoted winner and north overruffs if west ruffs.

♠ Q7
♥ -
♦ -
♣ K

♠ - ♠ K
♥ J ♥ -
♦ J ♦ 97
♣ 3 ♣ -

♠ -
♥ 9
♦ 10
♣ A

E

The contract is in No Trump.
South is on lead.
Take all 4 tricks.

♠ -
♥ 9
♦ AK10
♣ -

♠ - ♠ -
♥ 3 ♥ 10
♦ 96 ♦ QJ5
♣ 8 ♣ -

♠ -
♥ 5
♦ 87
♣ 10

SOLUTION E

South leads the ♣10; north discards the ♥9 and east has no convenient discard.

♠ -
♥ 9
♦ AK10
♣ -

♠ - ♠ -
♥ 3 ♥ 10
♦ 96 ♦ QJ5
♣ 8 ♣ -

♠ -
♥ 5
♦ 87
♣ 10

F

Spades are trumps.
East is on lead.
Take 3 of 4 tricks.

♠ J2
♥ 10
♦ -
♣ 7

♠ KQ10 ♠ -
♥ 2 ♥ 8654
♦ - ♦ -
♣ - ♣ -

♠ A43
♥ -
♦ 5
♣ -

SOLUTION F

East plays the ♥4 and south plays the ♠3 to produce the position shown. South leads the ♦5, guaranteeing two tricks in ♠'s.

♠ J2
♥ -
♦ -
♣ 7

♠ KQ10 ♠ -
♥ - ♥ 865
♦ - ♦ -
♣ - ♣ -

♠ A4
♥ -
♦ 5
♣ -

G

The contract is in No Trump.
South is on lead.
Take all 4 tricks.

	♠ AJ ♥ 2 ♦ Q ♣ -	
♠ Q10 ♥ K ♦ A ♣ -		♠ K ♥ Q10 ♦ K ♣ -
	♠ 2 ♥ AJ ♦ - ♣ A	

SOLUTION G

South plays the ♣A and west must play the ♦A. North plays the ♥2 and east is squeezed in 3 suits.

	♠ AJ ♥ 2 ♦ Q ♣ -	
♠ Q10 ♥ K ♦ A ♣ -		♠ K ♥ Q10 ♦ K ♣ -
	♠ 2 ♥ AJ ♦ - ♣ A	

H

Hearts are trumps.
North is on lead.
Take all 4 tricks.

	♠ AK ♥ - ♦ A2 ♣ -	
♠ J5 ♥ - ♦ J9 ♣ -		♠ 8 ♥ Q4 ♦ 3 ♣ -
	♠ - ♥ KJ9 ♦ 5 ♣ -	

SOLUTION H

North plays the ♠K and south plays the ♥9 to produce the position shown. South leads the ♦5 to north's ♦A to take a ♥ finesse for the last two tricks.

	♠ A ♥ - ♦ A2 ♣ -	
♠ J ♥ - ♦ J9 ♣ -		♠ - ♥ Q4 ♦ 3 ♣ -
	♠ - ♥ KJ ♦ 5 ♣ -	

I

Spades are trumps.
South is on lead.
Take 3 of 4 tricks.

	♠ A ♥ - ♦ 876 ♣ -	
♠ K7 ♥ - ♦ 109 ♣ -		♠ - ♥ Q10 ♦ QJ ♣ -
	♠ QJ ♥ - ♦ A3 ♣ -	

SOLUTION I

South leads the ♦A and the ♦3 to put east on lead with the ♦Q to produce the position shown. Declarer will score two ♠'s after east's ♥ lead.

	♠ A ♥ - ♦ 8 ♣ -	
♠ K7 ♥ - ♦ - ♣ -		♠ - ♥ Q10 ♦ - ♣ -
	♠ QJ ♥ - ♦ - ♣ -	

J

Clubs are trumps.
South is on lead.
Take all 4 tricks.

	♠ - ♥ K6 ♦ - ♣ A5	
♠ - ♥ A4 ♦ K9 ♣ -		♠ J5 ♥ 3 ♦ - ♣ 8
	♠ - ♥ - ♦ Q2 ♣ J2	

SOLUTION J

Lead the ♣J. If west plays (a) a ♥: north plays the ♣A, leads a ♥, south ruffs and north's hand is good. (b) a ♦: north plays the ♣5. South leads the ♦2, north ruffs and south's hand is good.

	♠ - ♥ K6 ♦ - ♣ A5	
♠ - ♥ A4 ♦ K9 ♣ -		♠ J5 ♥ 3 ♦ - ♣ 8
	♠ - ♥ - ♦ Q2 ♣ J2	

K

Hearts are trumps.
North is on lead.
Take 3 of 4 tricks.

♠ -
♥ A
♦ 862
♣ -

♠ -	♠ Q10
♥ -	♥ -
♦ KJ103	♦ 95
♣ -	♣ -

♠ -
♥ 5
♦ AQ4
♣ -

SOLUTION K

Lead the ♥A. If west plays (a) the ♦3: north plays the ♦2 and south plays the ♦4. (b) the ♦J or ♦10: north leads the ♦6 or ♦8 and south covers with the ♦Q only if east plays the ♦9.

♠ -
♥ A
♦ 862
♣ -

♠ -	♠ Q10
♥ -	♥ -
♦ KJ103	♦ 95
♣ -	♣ -

♠ -
♥ 5
♦ AQ4
♣ -

L

The contract is in No Trump.
South is on lead.
Take 3 of 4 tricks.

♠ 2
♥ AJ
♦ -
♣ 2

♠ -	♠ -
♥ Q4	♥ -
♦ -	♦ 54
♣ AQ	♣ 94

♠ A
♥ K
♦ -
♣ K7

SOLUTION L

South leads the ♠A. If west plays (a) a ♥: north gets two ♥ tricks. (b) the ♣Q: south leads the ♣7. (c) ♣A: south cashes the ♣K and leads a ♥.

♠ 2
♥ AJ
♦ -
♣ 2

♠ -	♠ -
♥ Q4	♥ -
♦ -	♦ 54
♣ AQ	♣ 94

♠ A
♥ K
♦ -
♣ K7

M

Hearts are trumps.
West leads the ♣10.
Take 4 of 5 tricks.

♠ KJ102
♥ 10
♦ -
♣ -

♠ AQ8	♠ 765
♥ -	♥ -
♦ -	♦ -
♣ 107	♣ Q9

♠ 943
♥ Q9
♦ -
♣ -

SOLUTION M

North plays the ♥10 on the first trick and south plays the ♥Q to produce the position shown. South leads the ♠9 to finesse in ♠'s and scores another ♥.

♠ KJ102
♥ -
♦ -
♣ -

♠ AQ8	♠ 76
♥ -	♥ -
♦ -	♦ -
♣ 7	♣ Q9

♠ 943
♥ 9
♦ -
♣ -

N

The contract is in No Trump.
South is on lead.
Take all 5 tricks.

♠ K2
♥ K
♦ K
♣ 5

♠ 875	♠ QJ4
♥ 2	♥ A
♦ 5	♦ A
♣ -	♣ -

♠ A963
♥ -
♦ -
♣ A

SOLUTION N

Lead the ♣A. East must toss a red A (Shown for the ♥A in the position at right). Whichever he does south leads low to north's ♠K and north cashes the good red K, squeezing east for the remaining tricks.

♠ K2
♥ K
♦ K
♣ -

♠ 875	♠ QJ4
♥ -	♥ -
♦ 5	♦ A
♣ -	♣ -

♠ A963
♥ -
♦ -
♣ -

O

Spades are trumps.
South is on lead.
Take all 5 tricks.

	♠ -	
	♥ A108	
	♦ 9	
	♣ 6	
♠ 87		♠ -
♥ K6		♥ QJ
♦ Q		♦ J
♣ -		♣ J10
	♠ J10	
	♥ 4	
	♦ -	
	♣ Q9	

SOLUTION O

Lead the ♠J to produce the position shown. On the lead of the ♠10 north plays the ♣6 and east must toss the ♦J. South leads the ♣Q to squeeze west for the final tricks.

	♠ -	
	♥ A10	
	♦ 9	
	♣ 6	
♠ 8		♠ -
♥ K6		♥ Q
♦ Q		♦ J
♣ -		♣ J10
	♠ 10	
	♥ 4	
	♦ -	
	♣ Q9	

P

The contract is in No Trump.
South is on lead.
Take 4 of 5 tricks.

	♠ 8	
	♥ Q7	
	♦ 9	
	♣ 10	
♠ A		♠ -
♥ 1096		♥ -
♦ 8		♦ A7
♣ -		♣ Q96
	♠ -	
	♥ J	
	♦ K5	
	♣ A8	

SOLUTION P

Lead the ♦5. (a) If east wins, his best return is a ♦ to south's ♦K. West discards a ♥, north a ♣. South plays the ♣A; whichever suit west sheds, north keeps.
(b) If north holds the first trick he leads the 8♠, producing the position at right. West's ♥ return squeezes east.

	♠ -	
	♥ Q7	
	♦ -	
	♣ 10	
♠ -		♠ -
♥ 1096		♥ -
♦ -		♦ A
♣ -		♣ Q9
	♠ -	
	♥ -	
	♦ K	
	♣ A8	

Q

Clubs are trumps.
South is on lead.
Take all 5 tricks.

	♠ K32	
	♥ 102	
	♦ -	
	♣ -	
♠ Q75		♠ 1093
♥ -		♥ QJ
♦ 109		♦ -
♣ -		♣ -
	♠ AJ8	
	♥ K	
	♦ -	
	♣ A	

SOLUTION Q

South leads the ♣A and east must toss a ♠ to produce the position shown. South leads the ♠J. If west plays (a) the ♠Q: north plays the ♠K and south's hand is good.
(b) a low ♠: north ducks and south plays the ♠8 to north's ♠K and south's hand is good.

	♠ K3	
	♥ 102	
	♦ -	
	♣ -	
♠ Q75		♠ 109
♥ -		♥ QJ
♦ 10		♦ -
♣ -		♣ -
	♠ AJ8	
	♥ K	
	♦ -	
	♣ -	

R

The contract is in No Trump.
South is on lead.
Take all 5 tricks.

	♠ K3	
	♥ AJ3	
	♦ -	
	♣ -	
♠ J6		♠ Q94
♥ KQ		♥ 4
♦ A		♦ -
♣ -		♣ 6
	♠ A105	
	♥ -	
	♦ K	
	♣ A	

SOLUTION R

South plays the ♣A. If west plays (a) the ♠6: south leads to the ♠K and finesses the ♠10 on the lead of the ♠3.
(b) a ♥: north plays the ♠3 and his hand is good.
(c) the ♦A: north plays the ♥3 and south leads the ♦K to squeeze west.

	♠ K3	
	♥ AJ3	
	♦ -	
	♣ -	
♠ J6		♠ Q94
♥ KQ		♥ 4
♦ A		♦ -
♣ -		♣ 6
	♠ A105	
	♥ -	
	♦ K	
	♣ A	

S

Hearts are trumps.
North is on lead.
Take 4 of 6 tricks.

♠ 2
♥ -
♦ -
♣ A7642

♠ - ♠ Q
♥ Q ♥ J8
♦ Q10 ♦ -
♣ KJ10 ♣ Q53

♠ -
♥ K1097
♦ -
♣ 98

SOLUTION S

North plays the ♣A and then the ♠2. South plays the ♣9 to produce the position shown with east on lead. The defense gets only one more trick.

♠ -
♥ -
♦ -
♣ 7642

♠ - ♠ -
♥ Q ♥ J8
♦ Q ♦ -
♣ KJ ♣ Q5

♠ -
♥ K1097
♦ -
♣ -

T

The contract is in No Trump.
South is on lead.
Take 5 of 6 tricks.

♠ A864
♥ -
♦ KJ
♣ -

♠ KQ7 ♠ 53
♥ KQ6 ♥ 9832
♦ - ♦ -
♣ - ♣

♠ 2
♥ AJ10
♦ AQ
♣ -

SOLUTION T

Lead the ♦A. If west plays (a) a ♥: north plays the ♦K. South leads a low ♥ and north/south get the rest. (b) a ♠: north plays the ♦J. South leads the ♠2, west takes a ♠ and north/south get the rest.

♠ A864
♥ -
♦ KJ
♣ -

♠ KQ7 ♠ 53
♥ KQ6 ♥ 9832
♦ - ♦ -
♣ - ♣

♠ 2
♥ AJ10
♦ AQ
♣ -

U

$pades are trumps.
North is on lead.
Take 5 of 6 tricks.

♠ KJ
♥ 32
♦ A5
♣ -

♠ 854 ♠ -
♥ - ♥ Q
♦ KJ ♦ 10942
♣ 5 ♣ J

♠ A7
♥ -
♦ Q83
♣ 9

SOLUTION U

Lead the ♥2 and ruff with the ♠A. If west plays (a) the ♠4: declarer draws trumps and cashes winners in ♥'s and ♦'s. (b) the ♣5: north takes two ♠ tricks and leads the ♥3 to endplay west.

♠ KJ
♥ 32
♦ A5
♣ -

♠ 854 ♠ -
♥ - ♥ Q
♦ KJ ♦ 10942
♣ 5 ♣ J

♠ A7
♥ -
♦ Q83
♣ 9

V

The contract is in No Trump.
North is on lead.
Take 5 of 6 tricks.

♠ Q8
♥ 1063
♦ Q
♣ -

♠ 543 ♠ J1072
♥ A85 ♥ KQ
♦ - ♦ -
♣ - ♣ -

♠ AK96
♥J9
♦ -
♣ -

SOLUTION V

North plays the ♦Q and east must play a ♥. South plays the ♥9 and west tosses a ♠. North then plays the ♠Q and ♠8 to produce the position shown with south on lead. South plays the ♥J and whoever takes the trick is end-played.

♠ -
♥ 1063
♦ -
♣ -

♠ - ♠ J2
♥ A85 ♥ K
♦ - ♦ -
♣ - ♣ -

♠ A9
♥J
♦ -
♣ -

W

Hearts are trumps.
South is on lead.
Take all 6 tricks.

	♠ K9 ♥ K ♦ 4 ♣ AQ	
♠ QJ10 ♥ - ♦ 2 ♣ J6		♠ A8 ♥ - ♦ - ♣ K543
	♠ - ♥ AQJ ♦ 3 ♣ 97	

SOLUTION W

Lead the ♥A and ♥Q, to produce the position shown. South leads the ♦3 and west must play a ♠ or ♣. East must discard the other suit. If east plays (a) a ♠: north leads the ♠9, south trumps, and leads a ♣ to north's good hand. (b) the ♣5: north plays the ♠K, and south trumps if east covers.

	♠ K9 ♥ - ♦ 4 ♣ A	
♠ QJ ♥ - ♦ - ♣ J6		♠ A8 ♥ - ♦ - ♣ K5
	♠ - ♥ J ♦ 3 ♣ 97	

X

Hearts are trumps.
South is on lead.
Take all 6 tricks.

	♠ - ♥ 63 ♦ A9 ♣ 82	
♠ 73 ♥ - ♦ K10 ♣ 95		♠ 62 ♥ - ♦ 8 ♣ 743
	♠ 54 ♥ - ♦ Q ♣ J106	

SOLUTION X

Lead the ♣J and north plays the ♣8. Lead a ♠, which north trumps, producing the position shown. North leads the ♥6 and east must toss a ♦. South discards the ♦Q, and west must toss the ♠7. North now leads the ♦A and east is squeezed.

	♠ - ♥ 6 ♦ A9 ♣ 2	
♠ 7 ♥ - ♦ K10 ♣ 9		♠ 6 ♥ - ♦ 8 ♣ 74
	♠ 5 ♥ - ♦ Q ♣ 106	

Y

Hearts are trumps.
South is on lead.
Take all 7 tricks.

	♠ A ♥ A2 ♦ A1032 ♣ -	
♠ K3 ♥ 7 ♦ 964 ♣ 3		♠ 6 ♥ 8 ♦ J85 ♣ KJ
	♠ 754 ♥ 9 ♦ Q ♣ A2	

SOLUTION Y

South leads the ♣A; north throws the ♠A. South then leads the ♠7 from the positon shown. If west plays (a) the ♠K: north plays the ♥A and leads the ♥2. Then south takes two ♠ tricks, forcing east to un-guard ♦'s or ♣'s. (b) the ♠3: north plays the ♦2, south then leads the ♠4, north ruffs with the ♥A and leads the ♥2. South takes the ♠5 and east is squeezed.

	♠ - ♥ A2 ♦ A1032 ♣ -	
♠ K3 ♥ 7 ♦ 964 ♣ -		♠ 6 ♥ 8 ♦ J85 ♣ K
	♠ 754 ♥ 9 ♦ Q ♣ 2	

Z

Hearts are trumps.
South is on lead.
Take 6 of 7 tricks.

	♠ 83 ♥ J5 ♦ 642 ♣ -	
♠ J76 ♥ 9 ♦ 95 ♣ J		♠ 5 ♥ 6 ♦ Q10 ♣ K84
	♠ K4 ♥ - ♦ AJ ♣ Q76	

SOLUTION Z

South leads the ♣6 and north trumps with the ♥5, leads the ♥J and south plays the ♠K. North leads a ♦ to finesse east's ♦Q. South takes his second ♦ to produce the position shown. South leads the ♠4 to eventually give north the ♠8 and a third ♦.

	♠ 83 ♥ - ♦ 6 ♣ -	
♠ J76 ♥ - ♦ - ♣ -		♠ 5 ♥ - ♦ - ♣ K8
	♠ 4 ♥ - ♦ - ♣ Q7	

AA

Spades are trumps.
South is on lead.
Take all 7 tricks.

♠ K103
♥ A2
♦ 72
♣ -

♠ -
♥ Q43
♦ KJ
♣ 75

♠ 4
♥ 987
♦ Q94
♣ -

♠ 5
♥ J65
♦ A
♣ 94

SOLUTION AA

Lead the ♣4 and ruff with the ♠K. Cash the ♥A and lead the ♠3 to produce the position shown. Lead the ♣9. North plays the ♥2 and west must discard a ♦ and east a ♥. South leads the ♥J for the rest.

♠ 10
♥ 2
♦ 72
♣ -

♠ -
♥ Q4
♦ KJ
♣ -

♠ -
♥ 98
♦ Q9
♣ -

♠ -
♥ J6
♦ A
♣ 9

BB

Spades are trumps.
South is on lead.
Take all 7 tricks.

♠ -
♥ 10
♦ K85
♣ AK2

♠ -
♥ 8
♦ Q96
♣ Q86

♠ -
♥ K
♦ J107
♣ J95

♠ 98
♥ J
♦ A4
♣ 107

SOLUTION BB

Lead the ♠9. East must un-guard either ♣'s or ♦'s. The case for ♣'s is shown at right. South next leads the ♠8 and west must shed a ♦, north throws the ♣2 and east plays the ♣9. The lead of ♣AK squeezes east for the final tricks.

♠ -
♥-
♦ K85
♣ AK2

♠ -
♥ -
♦ Q96
♣ Q86

♠ -
♥ K
♦ J107
♣ J9

♠ 8
♥ J
♦ A4
♣ 107

CC

Hearts are trumps.
South is on lead.
Take all 7 tricks.

♠ AJ
♥ 5
♦ KQ
♣ 108

♠ K2
♥ -
♦ J98
♣ 97

♠ 43
♥ 63
♦ 7
♣ 65

♠ Q
♥ 84
♦ A1032
♣ -

SOLUTION CC

Lead the ♠Q and north plays the ♠A. North then plays the ♥5. If east plays (a) the ♥3: west plays a ♦. North's ♦K is covered by south's ♦A to produce the position shown. The ♥8 is lead and west is squeezed in 3 suits. (b) the ♥6: south plays the ♥8 and leads the ♥4 to squeeze west in 3 suits.

♠ J
♥ -
♦ Q
♣ 108

♠ K
♥ -
♦ J
♣ 97

♠ 4
♥ 6
♦ -
♣ 65

♠ -
♥ 8
♦ 1032
♣ -

DD

Hearts are trumps.
South is on lead.
Take all 7 tricks.

♠ -
♥ KQ
♦ 5
♣ Q1095

♠ 8765
♥ J5
♦ -
♣ J

♠ K10
♥ -
♦ 10
♣ 8764

♠ J93
♥ -
♦ J
♣ AK3

SOLUTION DD

South leads the ♣3 to north's ♣Q. North plays the ♥K and ♥Q to produce the position shown. The ♦5 lead from north squeezes east.

♠ -
♥ -
♦ 5
♣ 1095

♠ 8765
♥ -
♦ -
♣ -

♠ K
♥ -
♦ -
♣ 876

♠ J93
♥ -
♦ -
♣ A

EE

Hearts are trumps.
South is on lead.
Take all 7 tricks.

♠ -
♥ K103
♦ 72
♣ A2

West	East
♠ 75	♠ -
♥ -	♥ 4
♦ KJ	♦ Q94
♣ Q43	♣ 987

♠ 94
♥ 5
♦ A
♣ J65

SOLUTION EE

Lead the ♠4. North plays the ♥10 and east discards the ♦4. North plays the ♣A and the ♥3 to south's ♥5 to produce the position shown. South leads the ♠9 and north tosses the ♣2. If west plays (a) a ♣: south leads the ♣6 for north to trump, promoting south's hand. (b) a ♦: east sheds a ♣. South now leads the ♣J and west is helpless.

♠ -
♥ K
♦ 72
♣ 2

West	East
♠ -	♠ -
♥ -	♥ -
♦ KJ	♦ Q9
♣ Q4	♣ 98

♠ 9
♥ -
♦ A
♣ J6

FF

The contract is in No Trump.
South is on lead.
Take all 7 tricks.

♠ K5
♥ A9
♦ 43
♣ K

West	East
♠ J10	♠ Q
♥ J	♥ 8765
♦ K10	♦ J
♣ QJ	♣ 9

♠ 32
♥ K
♦ AQ
♣ 106

SOLUTION FF

South leads the ♥K and north overtakes to produce the position shown. North plays the ♥9 and south plays a ♠. If west plays (a) the ♠10: north plays the ♠K and the ♠5; west is squeezed in ♦'s and ♣'s. (b) the ♦10 or ♣J: north plays the ♣K and then ♦'s or ♣'s to squeeze west out of tricks.

♠ K5
♥ 9
♦ 43
♣ K

West	East
♠ J10	♠ Q
♥ -	♥ 876
♦ K10	♦ J
♣ QJ	♣ 9

♠ 32
♥ -
♦ AQ
♣ 106

GG

Hearts are trumps.
South is on lead.
Take 3 of 7 tricks.

♠ -
♥ A
♦ 86
♣ J1072

West	East
♠ AK92	♠ 10
♥ J9	♥ Q754
♦ 5	♦ 97
♣ -	♣ -

♠ 54
♥ K108
♦ 2
♣ A

SOLUTION GG

South leads the ♠4 and north plays the ♥A and ♦6, taken by east to produce the position shown. If east leads (a) the ♦9: south plays the ♠5. East leads the ♥4, south plays the ♥K and leads the ♥8, assuring he wins the ♥10. (b) the ♥4: south plays ♥K and leads the ♠5. East plays (b1) the ♥5: south plays the ♥8. West must take and lead a ♠ and south will score the ♥10. (b2) the ♦9: south plays the ♥8, west plays the ♥J and leads a ♠ to let south score the ♥10.

♠ -
♥ -
♦ 8
♣ J1072

West	East
♠ AK9	♠ -
♥ J9	♥ Q754
♦ -	♦ 9
♣ -	♣ -

♠ 5
♥ K108
♦ -
♣ A

HH

Hearts are trumps.
West leads the ♦5.
Take all 12 tricks.

♠ KQ
♥ K43
♦ -
♣ AKQ10732

West	East
♠ 1097532	♠ J6
♥ -	♥ J9762
♦ 875	♦ KQ10
♣ 864	♣ J9

♠ A84
♥ AQ1085
♦ J96
♣ 5

SOLUTION HH

North plays the ♥3 and leads the ♥K and then the ♥4, won by south's ♥8. The ♣5 from south and ♣'s from north. If east ruffs then south over-ruffs, draws trumps and reaches a good north hand in ♠'s. If east refuses to ruff we reach the position shown and East is helpless

♠ KQ
♥ -
♦ -
♣ 2

West	East
♠ 1097	♠ -
♥ -	♥ J97
♦ -	♦ -
♣ -	♣ -

♠ -
♥ AQ10
♦ -
♣ -

II

Hearts are trumps.
North is on lead.
Take all 12 tricks.

	♠ 5 ♥ 85 ♦ AK7 ♣ K86542	
♠ K107 ♥ 9 ♦ Q1083 ♣ J1097		♠ 86432 ♥ Q632 ♦ J62 ♣ -
	♠ AQJ9 ♥ AKJ1074 ♦ 95 ♣ -	

SOLUTION II

North plays the ♣K. If east plays (a) a ♥: south over-ruffs, plays the ♠A and takes a ruffing finesse in ♠'s to kill west's ♠K and a ♥ finesse for the rest.
(b) a ♠: south plays a ♥, ruffs out the ♠K as above, takes a ♥ finesse, plays his good ♠'s, crosses to the ♦A and ruffs a ♣. The ♦9 to north's ♦A ends with ♥AKJ over east's ♥Q63.
(c) a ♦: south plays the ♠9 and north leads the ♥8 to finesse east in ♥'s. South plays the ♦5 to north's ♦A and repeats the finesse in ♥'s if necessary. South then plays all his ♥'s, squeezing west in 3 suits.

JJ

The contract is in No Trump.
West leads the ♣10.
Take 10 of 13 tricks.

	♠ 765432 ♥ AK ♦ A ♣ KJ32	
♠ - ♥ - ♦ J1098765432 ♣ 1098		♠ QJ1098 ♥ QJ1098 ♦ Q ♣ 76
	♠ AK ♥ 765432 ♦ K ♣ AQ54	

SOLUTION JJ

Allow east to win the ♣10 and north takes the next two tricks with the ♦A and ♣J to produce the position shown. North plays the ♣3. If east plays (a) a ♥: south plays the ♣Q and works on ♥'s, using south's ♣A entry first.
(b) a ♠: south wins with the ♣A and works on ♠'s, using north's ♣K entry first. .

	♠ 765432 ♥ AK ♦ - ♣ K3	
♠ - ♥ - ♦ J109876543 ♣ 9		♠ QJ1098 ♥ QJ1098 ♦ - ♣ -
	♠ AK ♥ 765432 ♦ - ♣ AQ	

KK

The contract is in No Trump.
South is on lead.
Take all 10 tricks.

	♠ AQ86 ♥ AQ86 ♦ 86 ♣ -	
♠ KJ10 ♥ KJ10 ♦ - ♣ 8764		♠ 7542 ♥ 7542 ♦ J9 ♣ -
	♠ 93 ♥ 93 ♦ Q5 ♣ QJ105	

SOLUTION KK

South plays the ♣Q and east must play ♠'s or ♥'s (we will assume ♠'s). North plays the ♦6. South now finesses ♠'s to get two ♠ tricks by north to produce the position shown. North leads the ♦8 to squeeze west and he must discard a ♥. South continues with the ♣J and ♣10 and north plays his ♠'s to squeeze east out of any tricks.

	♠ 86 ♥ AQ86 ♦ 8 ♣ -	
♠ K ♥ KJ10 ♦ - ♣ 876		♠ 7 ♥ 7542 ♦ J9 ♣ -
	♠ - ♥ 93 ♦ Q5 ♣ J105	

LL

Spades are trumps.
West leads the ♣J.
Take 12 of 13 tricks.

	♠ K ♥ 1032 ♦ 10432 ♣ AKQ92	
♠ 432 ♥ - ♦ KJ987 ♣ J10876		♠ 765 ♥ AJ98654 ♦ - ♣ 543
	♠ AQJ1098 ♥ KQ7 ♦ AQ65 ♣ -	

SOLUTION LL

Take the ♣A and shed the ♦Q from south. Draw three rounds of trumps, shedding the ♦2 and ♣2 from north. Lead the ♥Q; east must duck to produce the position shown. Lead the ♦5 toward the ♦10. West must take or north/south take all but a ♦ loser. West's return gives north an entry with the ♦10.

	♠ - ♥ 103 ♦ 1043 ♣ KQ9	
♠ - ♥ - ♦ KJ987 ♣ 1087		♠ - ♥ AJ9865 ♦ - ♣ 54
	♠ 1098 ♥ K7 ♦ A65 ♣ -	

MM

Hearts are trumps.
West leads the ♠J.
Take all 13 tricks.

	♠ AK ♥ A108 ♦ AQ863 ♣ Q54	
♠ J10985 ♥ J94 ♦J ♣ J986		♠ Q64 ♥ 3 ♦ 1097542 ♣ K32
	♠ 732 ♥ KQ7652 ♦ K ♣ A107	

SOLUTION MM

North takes the ♠K and leads the ♦3. South cashes the ♣A. The next 6 tricks have south finessing west in ♥'s 3 times and north leading ♦'s for south to trump high twice. North takes the ♦A to give the position shown. North leads the ♦Q and south plays the ♣10. If east plays (a) the ♠6: west must play the ♣9; then the ♣Q from north spells the end. (b) the ♣3: north plays the ♣5 for south to ruff and north's hand is good.

	♠ A ♥ - ♦ Q ♣ Q5	
♠ 109 ♥ - ♦ - ♣ J9		♠ Q6 ♥ - ♦ - ♣ K3
	♠ 73 ♥ 7 ♦ ♣ 10	

NN

Spades are trumps.
West leads the ♠2.
Take 10 of 13 tricks.

	♠ QJ8 ♥ 8543 ♦ 632 ♣ A84	
♠ 62 ♥ 972 ♦ Q9 ♣ Q109752		♠ 754 ♥ KJ106 ♦ AKJ10 ♣ J3
	♠ AK1093 ♥ AQ ♦ 8754 ♣ K6	

SOLUTION NN

North wins the ♠8 and leads the ♥3 to finesse the ♥Q. Two rounds of ♣'s produces the position shown. North leads the ♣8. If east plays (a) a ♠: north gets an eventual ♦ ruff. (b) a ♥: south trumps with the ♠A, cashes the ♥A and leads the ♠9 to north, who leads a ♥ for south to trump with the ♠K. South leads the ♠10 to north, who cashes the ♥8. (c) a ♦: whatever east returns south can set up his 4th ♦.

	♠ QJ ♥ 854 ♦ 632 ♣ 8	
♠ 6 ♥ 97 ♦ Q9 ♣ Q1097		♠ 75 ♥ KJ10 ♦ AKJ10 ♣ -
	♠ AK109 ♥ A ♦ 8754 ♣ -	

David Leschinsky

Eureka!'s General Thinking Processing Chart

The brain is a complex bundle of neurons which hold memories and allows us to make many-layered connections between them. Different parts of the brain are tuned for different kinds of mental processes – language, creativity, logic thought, sequencing, music appreciation, physical dexterity, etc. All of these processes are strengthened through use, and can be exercised enjoyably with games and puzzles. The following chart highlights the connections between specific mental process and selected games.

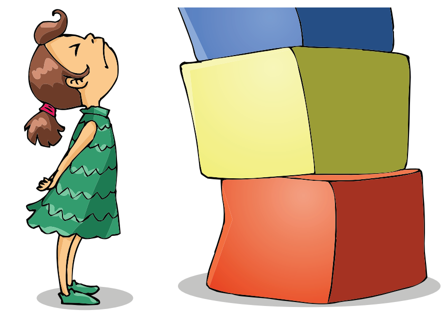
Thinking Categories	Types of thinking processes	Selected puzzles or games that can be played individually	Selected games for multiple players
Language	Spelling, Sentence, Word Association, Storytelling	Word Finds, Crossword puzzles, Word problems, Mystery puzzles, Bananagrams	Scrabble, Once Upon a Time, Disorder, Unspeakable Words, Quiddler, Pass the Bomb, Bananagrams, Gloom
Mathematics	Number sense, arithmetic, abstract representation, sequencing, pattern recognition, grouping, probability, statistics	Graduated Puzzles (Rush Hour, River Crossing, Tip Over, Tridio, Animal Logic) 15 Puzzle, mechanical puzzles, sliding puzzles, Game of Chips, Set, Reflection, Equi	Prime Number, Pass-the-Pigs, Yahtzee, Equate, Smath, Perudo, Rumikub, Puzzle books, Labyrinth, Gloom, Guillotine, Poker, Wizard, Gobblet, Go, Abalone
Spatial Ability	Geometric thinking in 2D and 3D, visualization, geometric transformation, geographic thinking	Lonpos, Knoodle, Rolling block puzzles (Hedghog Escape, Say Cheese), Soma cubes, jigsaw puzzles, spatial puzzle books, pentominoes, tangram type puzzles, clicko series, maze puzzles, mechanical puzzles (packing puzzles, burr puzzles)	Blokus, Ricochet Robot, Reflection, Ubongo
Memory	Concentration, recall	Concentration, Perplexus, Rubiks Cube	Gobblet, Concentration, Trivia or fact based games, Sherlock, Pengalo, Magic Labyrinth
Emotion	Recognition, identification, understanding	Grimaces, Trading Faces	Grimaces, Trading Faces, PsychoBox
Flexibility of Mind	Reframing situations, understanding rule sets, data exploration, making and changing hypotheses	Mechanical puzzles, optical illusions, mystery puzzle books, lateral thinking problems, science experiments	Fluxx, Clue, storytelling games (Once Upon a Time, Gloom)
Reasoning	Logic skills, lateral thinking skills, assessment, judgment, strategic thinking	Graduated Puzzles (Metaforms, Zoologic, Animal Logic) Sudoku, Logic and lateral thinking puzzle problem books, mystery puzzle books	Go, Abalone, chess, checkers, backgammon, Mindtrap, Clue, Spy Ally, Scotland Yard, Mr. Jack

 Compiled: March 2016

FIVE PROBLEMS

emrehan@halici.com.tr

These problems are chosen from puzzleup.com 2015 (weekly puzzle competition prepared by Emrehan Halici) for **G4G12**



01 - COLORED CUBES

You have red, blue and green unit cubes. You will create a 2x2x2 cube by using 8 of these unit cubes. What is the maximum number of different cubes you can create?

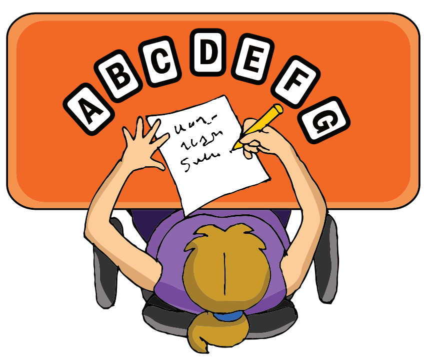
Note: If a cube can be obtained by rotating another one, these two cubes are not considered to be different.

If the problem was asked for only red and blue unit cubes, the answer would be 23..

02 - WINNING NUMBER

Five numbers will be randomly picked with replacement between 1 and 100 (1,2,...,99,100), and the largest of these five numbers will be named as the winning number. This process will be repeated many times and the average of all the winning numbers will be calculated.

What is the integer nearest to this average?



03 - SET OF CODES

You will produce a set of 7-letter codes using the the letters A, B, C, D, E, F and G.

-Two codes are called similar if they differ by just one letter.
-No two codes will be similar in the set.
-Letters can be used more than once in a code.

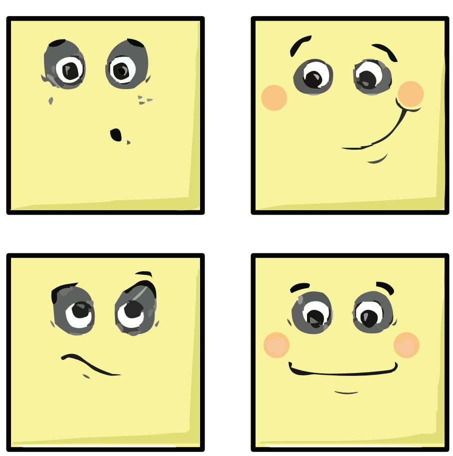
What can be the maximum number of codes in this set?

If the problem was asked for a set of 3-letter codes using the letters A, and B then the answer would be 4 (Example: AAA, ABB, BAB, BBA).

04 - HANDS OF A CLOCK

How many times at least two of the three hands (hour, minute, second) of a clock exactly overlap between 10:30 and 22:30?

Note: All hands move in a continuous motion with no discrete jumps.

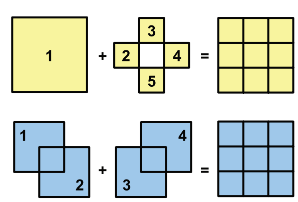


05 - SQUARES

What is the minimum number of squares to be drawn on a paper in order to obtain an 8x8 table divided into 64 unit squares.

Notes:
-The squares to be drawn can be of any size.
-There will be no drawings outside the table.

Two examples for a 3x3 table:



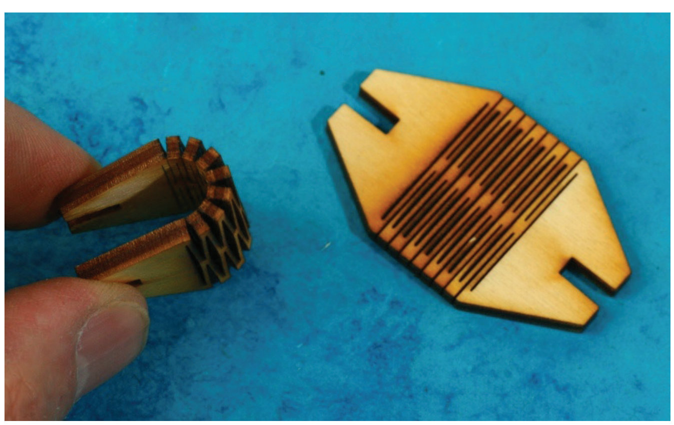
The second one with 4 squares is the solution for a 3x3 table.

Hinge-a-Tron

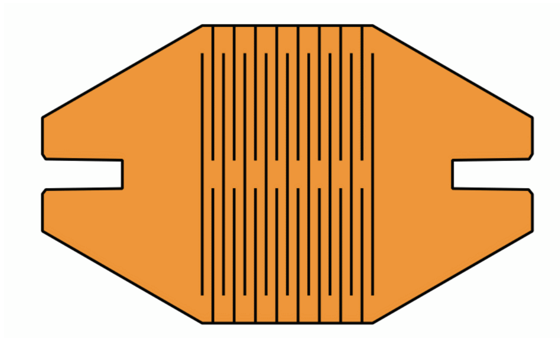
George Hart

Stony Brook University

This is a "Hinge-a-Tron," my exchange item for the G4G-12 conference. It's a variation on the classic rotating ring of tetrahedra (which goes back at least to W. W. Rouse Ball's 1939 book *Mathematical Recreations and Essays*). Although it is made from solid plywood, the parts flex and it is fun to rotate it around and around into itself. This version is made of six laser-cut plywood pieces and is a modification of an earlier version designed by Henry Segerman and me, shown in the video. The living hinge can easily bend back on itself as shown below, but **be careful not to stretch or twist the pieces**, as they will snap.



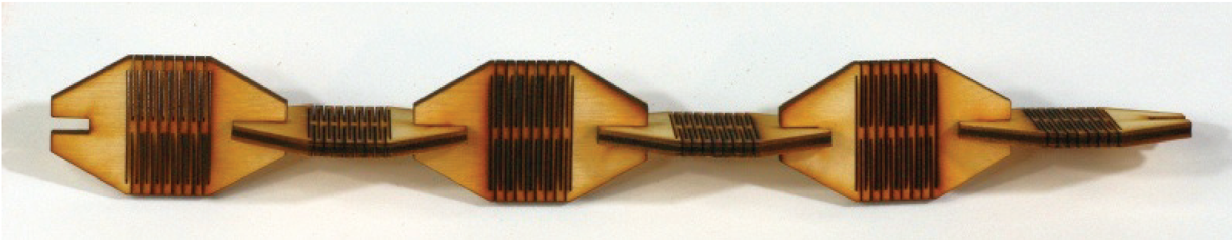
It works because the long thin strips of plywood that remain between the laser-cut lines can twist a few degrees each without snapping and there are enough of them to total to 180 degrees of bend.



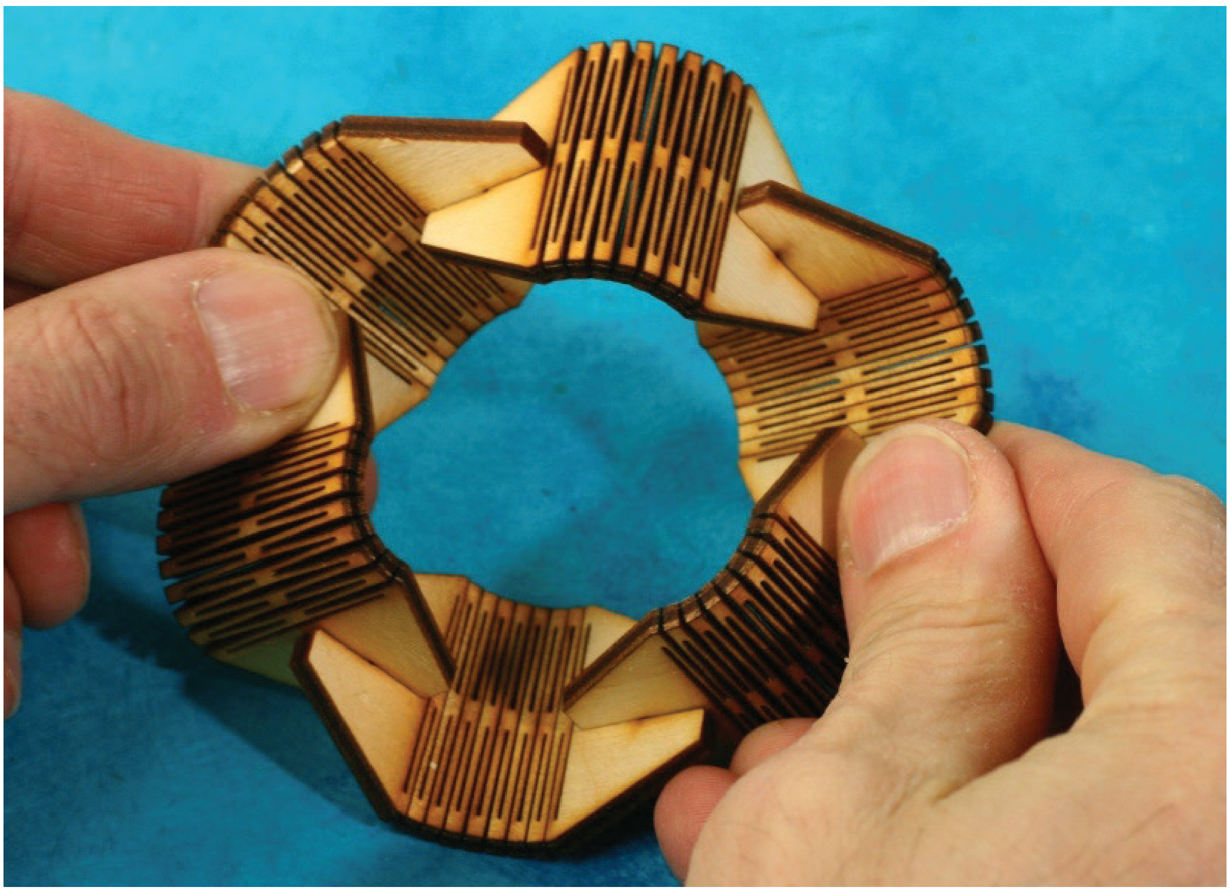
For the G4G-12 conference, I packaged up little kits of six parts in a plastic bag to give each participant. You can make your own parts if you have access to a laser-cutter. Just cut [this template] from 1/8 thickness (3 mm) plywood.

I made 400 kits, which is more work than you might think:

When assembling the parts, **be careful never to stretch or twist them**. They can easily tear. (If you received a G4G-12 kit and broke a piece, let me know; I can mail you a replacement.) First make a straight chain, wedging the slots together:



Then close the chain into a cycle. They should hold together without glue, but if a joint comes undone, you can add a dot of wood glue. You will discover how fun it is to rotate it around and around.



If you don't have a kit or access to a laser-cutter, all I can suggest is to watch the video:
`https://www.youtube.com/watch?v=GEJgrbiGETo`

How Puzzles Made Us Human

Pradeep Mutalik

Here's a simple mathematical puzzle. Multiply together the numbers of fingers on each hand of all the human beings in the world—approximately 7 billion in all. Is the answer approximately: A) $5^{7,000,000,000}$ B) $10^{7,000,000,000}$ C) $5^{14,000,000,000}$ D) Something else entirely?

While solving the above puzzle, did you get a flash of insight that led you to the correct answer without any trace of doubt whatsoever—a mini *Aha!*, or insight, moment? If you did not, read the hint at the bottom of the page and try again. The goal of this exercise is not so much to get the right answer, but to give you a small taste of the emotions of joy and certainty that accompany the *Aha!*, or insight, phenomenon that characterizes the cognitive act of solving some puzzles.

Here's another example from a completely different realm of thought. Make sense of the following sentence: "The haystack was important because the cloth ripped."

The answers to both puzzles are at the end of the article. If you solved one or both of these puzzles, or even if you just looked at and understood the answers, you may have experienced the sense of rightness or certainty—"Of course!"—and the positive emotion—"Cool!"—that accompanies the *Aha!* experience.

It is the contention of this article that this intrinsic emotional reward you may have experienced, linked to the cognitive act that you just performed, is an extremely important human characteristic. This cognitive-emotional link in the solving of puzzles, I contend, is one of the most important things that evolutionarily made us what we are today. We are different from other animals in many ways, but each of those differences requires or presupposes this cognitive-emotional link.

Hint: Some people may be missing some fingers.

To judge whether this seemingly grandiose claim is tenable, we need to isolate what characteristics of humans are *qualitatively* different from other intelligent animals and especially from our close ape relatives.

Does the difference lie in what we term our complex social human emotions—love, empathy, shame, jealousy, political intrigue, and the like? Not at all, as any pet lover knows—pets regularly exhibit such emotions, and

> We seek out puzzles and learning for fun.

political intrigue is well known in apes. We share many behaviors with animals, and although we execute them with greater complexity and sophistication as a result of our greater intelligence, they do not define us.

Is it tool use or problem solving that makes us different? No. The use of simple tools and the ability to solve problems to obtain food or other extrinsic rewards is well known in animals.

What is different about human beings is *our underlying emotional attitude to problem solving*. We seek out puzzles and learning for fun. This makes us learning machines in the area of our choice, whether it be tracking prey or navigating difficult terrain. *Aha!* experiences help us master an area of learning unique to our species: spontaneous syntactic language. We enjoy art, music, and humor: cognitive experiences that seem to be without any short-term practical purpose. And we can form models of the world and understand it. "The most incomprehensible thing about the universe it that it is comprehensible," Albert Einstein famously declared. As we shall see, it is the cognitive-emotional links in our brains, of which the *Aha!* experience is the most dramatic manifestation, that makes all this possible.

Our brains have cognitive modules for language, face

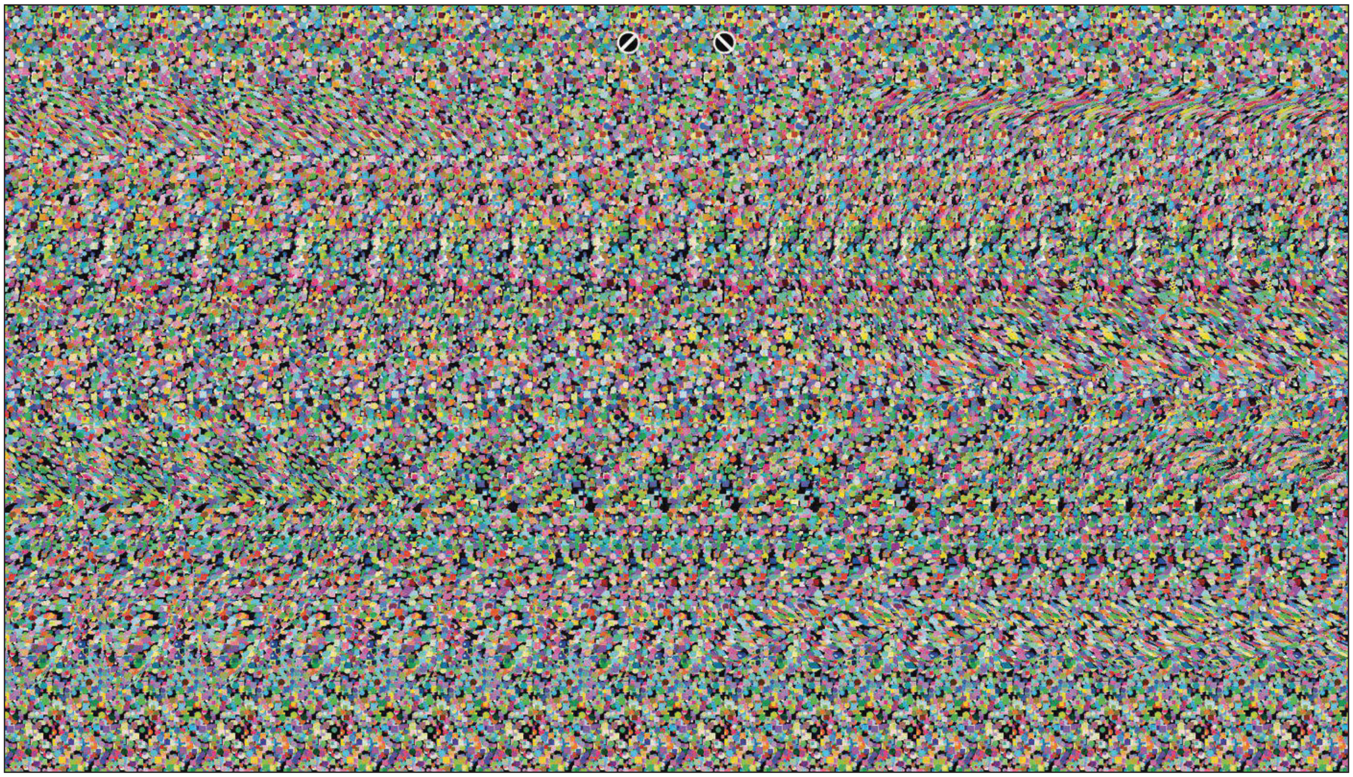


Figure 1. What mathematical objects do you see in this picture? (The answer is at the end of the article.)

recognition, social interaction, numerical manipulations, motor planning, and so on. But as we just saw, even disparate cognitive processes have the same emotional concomitants when a solution is found. The modules all use the same reward mechanism.

What exactly is this unifying *Aha!* experience? At its strongest, it is a flash of insight that instantly shifts our worldview. It is accompanied by intense pleasure and the confident realization that the answer is right: No external validation is needed. There is a sense of rightness, of things falling into place, like a puzzle piece that can fit only one way. There is a strong memory of the insight, and the feeling is somewhat addictive: You want to come back for more.

Another important characteristic is that this feeling is an intrinsic, impersonal reward—it is not related to the utility of the result. This is perhaps most extremely illustrated in a statement made by the Cambridge mathematician G. H. Hardy to a friend, the philosopher Bertrand Russell: "If I could prove by logic that you would die in five minutes, I should be sorry you were going to die, but my sorrow would be very much mitigated by pleasure in the proof!"

Math enthusiasts know that puzzle solving is intrinsically fun, but seeking out puzzles is not a universal activity by any means. What relevance does the *Aha!* experience have to the vast number of human beings who don't care for puzzles, mathematical or otherwise? Here's the kicker: The same emotional reaction of joy and certainty is experienced when the brain solves a puzzle that is subconscious—when a person is not even aware that he or she has solved a puzzle!

Such puzzles are constantly being solved by the cognitive, visual, and auditory systems of all humans in day-to-day activities. The cognitive puzzles we need to solve all the time require abstraction, pattern recognition, generalization, the solving of equations, and rule-based induction—things that mathematicians do consciously. And when these puzzles are solved, our brains reward themselves by a similar positive emotional reaction.

As *Gestalt* psychology has shown, some functions of the brain are global: common across modules. The brain has general algorithms that can recognize good solutions to any kind of problem. Let's look at some examples to try to understand what these are.

Figure 1 shows a stereogram puzzle of the type popularized by the *Magic Eye* book series. When you relax your eyes, allowing the two guide circles at the top to come together, and staying focusing on the pattern, some hidden three-dimensional objects emerge. Finding this image elicits the same emotional elements as the *Aha!* experience—positive reinforcement with no doubts at all.

In fact, every act of recognition—whether visual, auditory, or conceptual—is an *Aha!* experience. Cognitively, it is triggered by a change in an initially disordered internal

Gabriel Calderón, *https://www.flickr.com/photos/searingheat*

Figure 2. Beautiful woodwork on the ceiling of the Alhambra in Granada, Spain.

representation to one that makes sense. Order is created out of disorder; the new representation is more compact and coherent. It is much easier to have a bunch of splotches coherently organized into the shape of a recognized object than to account for them individually.

Thus, what brings on the *Aha!* experience is something that can be termed a *decrease in cognitive entropy*. Our brains appear to have a built-in algorithm that triggers the familiar emotional *Aha!* reaction whenever a simple coherent explanation fits disorderly input. The famous principle of parsimony in problem solving—Occam's razor—is apparently built in to our brains.

This powerful principle also helps us learn language. When a child learns to speak, the number of words he or she knows grows slowly at first, and then at around 18 months, suddenly takes off at an exponential rate. The reason seems to be that every child inductively discovers the rule that every object has a name. From then on, the child hounds its parents into feeding it names . . . and the rest is history.

The experience of discovering the name rule occurs too early for most of us to remember, but Helen Keller had it at the age of seven and here's how she described it: "I knew then that 'w-a-t-e-r' meant the wonderful cool something that was flowing over my hand. That living word awakened my soul, gave it light, hope, joy, set it free!"

The certainty and joy she describes clearly identify this as a true *Aha!* experience. *This certitude and pleasure is extremely important to learning language because the child cannot turn to anyone else for validation of its conclusions: It still has to learn language!* Cognitively, the unification of independent representations caused by this induced rule represents a large decrease in cognitive entropy quite similar to the visual case. Mini *Aha!* experiences continue to guide language learning and, in fact, all independent learning throughout childhood.

This emotional reaction that favors low cognitive entropy in the solution of unconscious problems gives a natural explanation for those uniquely human aesthetic pursuits: art and music. We find regular visual patterns like the one in figure 2 pleasing. We love symmetry. Our visual system makes recognized patterns pop out. Symmetry and observed patterns reduce the representational requirement of a visual object, triggering pleasurable reactions.

Music is pleasurable for the same reason. Musical scales consist of notes in simple integer ratios: 1:2, 1:3, 5:4, and so on. The pleasure associated with such ratios is based on the fact that sound-makers in the environment essential to our survival, such as predators, prey, and vibrating inanimate objects, give out resonant frequencies in integer ratios.

To parcel out environmental sounds accurately, the brain has to be able to identify integer ratios in the mishmash of frequencies that we hear. So in effect, our auditory system tries to solve Diophantine equations. When it does so, *Aha!* There is a reduction of cognitive entropy and we feel pleasure. Also, musical rhythm is a compact organization of time intervals, creating, essentially, symmetric patterns in time. Of course, there is a lot more to aesthetics than these basic elements, but the underlying intrinsic pleasure of low cognitive entropy motivates us to follow these pursuits.

The same drive to detect existing patterns in aesthetics extends to finding hitherto unknown patterns in humor and creativity. As Arthur Koestler outlined in his brilliant book *The Act of Creation*, humor and creativity are linked because they both arise from finding new patterns of reasoning that are intrinsically appealing: those that decrease cognitive entropy. Once we find such new patterns, we can celebrate those that are valid and weed out those that don't quite work in the real world and are therefore funny.

It is a signal achievement of human brain evolution that it has managed to link the results of our most sophisticated cognitive processes with our most primitive pleasure centers.

Koestler tells the joke about the man who came home to find his wife in bed with a priest and, instead of reacting angrily, went out onto the balcony and pretended to bless an imaginary congregation. His explanation to the priest was "You are doing my job, so let me do yours." This creative pattern of thinking—reciprocity—is valid in many situations, but not in this one. So we find it funny: Humor is the brain's way of saying, "Nice try, but you are reasoning on thin ice here."

Neuroimaging studies confirm that both cognition and emotion are involved in the *Aha!* effect. There is increased brain activity in the more recently evolved brain structures of the cerebral cortex—specifically, the anterior superior temporal gyrus and the right hemisphere—during the *Aha!* effect. But there is also increased activation of the right hippocampus, which is involved in memory, and of more primitive brain structures that are powerfully involved in emotion, motivation, and even addiction, such as the amygdala.

It is a signal achievement of human brain evolution that it has managed to link the results of our most sophisticated cognitive processes with our most primitive pleasure centers. It makes evolutionary sense: if you were to make an animal with no imposing physical traits that had to live off its wits, you would provide it an internal reward when it solved a problem. And that's exactly what evolution has done.

All primitive human societies have experts that excel in particular fields of knowledge: language, reckoning, navigating by the stars, tracking, and so on. Unlike, say, insect societies, this expertise is not innate but self-cultivated. *Aha!* experiences in childhood in a particular field can accentuate variations in intrinsic ability, leading the child to seek problems in, and master, a particular field. The almost addictive nature of the *Aha!* experience can set a child's course for life. This phenomenon likely gave human societies the specialists that helped them survive and thrive. In the words of Jacob Bronowski, "The most powerful drive in the ascent of man is his pleasure in his own skill. He loves to do what he does well and, having done it well, he loves to do it better."

We are finally in a position to respond to Einstein's observation that the universe is comprehensible to us. Occam's razor is a part of our conscious and subconscious problem solving: We experience joy in finding simple elegant representations of complexity. *This is adaptive because the universe has evolved by self-assembly and natural selection, gradually growing more complex from simple beginnings. In such a process, the simplest mechanisms of complexity are encountered first and hence are the most probable.* We conceptually run this process of complexification in reverse when we find simple explanations. Hence, the patterns we find attractive are likely to correspond to the workings of the world. That's all there is to it, Albert.

Although it is heartening to know that the quest for mathematical elegance is hard-wired in our brains, it is humbling—and satisfying—to know that it is not unique to mathematicians. ■

Pradeep Mutalik, a research scientist at the Yale Center for Medical Informatics, has been addicted to the Aha! *experience since early childhood and has sought it everywhere: in mathematics, animal behavior, neuroscience, radiology, computer science, artificial intelligence, and consciousness. He founded the* New York Times *puzzle blog "Numberplay" and authored it from 2009 through 2012. He would love to hear from readers about their memorable* Aha! *moments and whether they influenced their choice of major or career.*
Email: pradeep.mutalik@yale.edu

http://dx.doi.org/10.4169/mathhorizons.22.1.10

Answers to the puzzles:
1. The product is zero, because there is at least one person in the world who has no fingers on one hand.
2. A parachute ripped.
3. The Platonic solids: the tetrahedron, the octahedron, the icosahedron, the cube, and the dodecahedron.

Incredible 18 Piece Burrs

by Frans de Vreugd

0) Introduction

Interlocking puzzles have fascinated me for almost 30 years. Having a set of wooden pieces that will only assemble in a certain way is intriguing. One of the most well-known puzzles is the ***18 Piece Burr***. It was designed in 1952 by Dutch mathematician Willem van der Poel. Ever since that moment this puzzle has been inspiring puzzle designers worldwide. Despite the fact that it has been around for almost seven decades there have been interesting developments recently, in both the classic design and puzzles derived from it. In this article I will show some incredible recent designs and some very interesting ***Pseudo 18 Piece Burrs***.

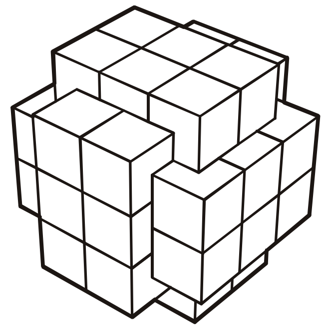


Figure 1.
Van der Poel Puzzle

1) Design criteria for 18 piece burrs

When designing interlocking puzzles, there are a number of design criteria that are used. Having a unique solution for a puzzle (the puzzle pieces will fit together in one way only) is very desirable, both for interlocking puzzles and for packing puzzles. Not every design has a unique solution, but there are many different tricks to make a solution unique (we will look into this later). The shape of the pieces is also important. Ideally the pieces are notchable (which means that they can be made by a tablesaw alone (without having to chisel out blind corners by hand). Second best are millable pieces. No hand chiseling is allowed, but more complicated pieces can be made with a router. Another (practical) characteristic is having as many identical pieces as possible. For practical purposes it is a lot easier to make two or three different pieces rather than 18 different ones. An additional design criterion for the 18 piece burr might be to have identical pieces for the 12 pieces of the outer cage.

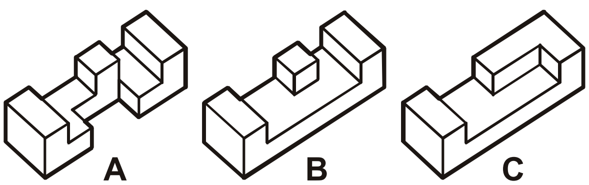


Figure 2. notchable (A), millable (B), non-notchable (C) pieces

2) The 18 piece Burr - unbreakable by computer?

Shown in Figure 1 is the iconic shape of this puzzle. It consists of an outer cage of 12 pieces combined with an internal locking mechanism of 6 pieces. The puzzles are grouped in a 2x3 array in each of its main axis. This first design was not a very difficult puzzle (at least not compared to today's standards), it only needed a few moves to remove the first piece. The principle of having an outer cage with the internal lock proved to be a very fertile starting point for other (more complicated) puzzle designs though. If the number of pieces of a puzzle is small enough (e.g. six or eight pieces) it is possible to do a full analysis by computer. Bill Cutler for instance analysed the infamous ***Six Piece Burr*** (aka ***Chinese Cross***) this way. What you do is determine all the possible different pieces, then find every single combination of six of these and use a brute force attack to calculate all of the different sets by computer. Bill Cutler did his research on the six piece burr in the 1980s. Since then, computer capacity has improved immensely, supercomputers have been developed and initiatives to have thousands of computers work together to do large and complicated computations (like SETI) were introduced. All of this is no help at all though for doing a full analysis of the 18 piece burr! The number of combinations to be calculated is mind-boggling. With today's technology, cracking the 18 pieces seems far out of reach. Maybe the development of quantum computers might help. To give an example, on a modern (2012) home computer it might take several weeks of computing a *single set* of 18 pieces. This is probably part of the success of this puzzle: the fact that computers cannot crack it has contributed to its almost mythical proportions.

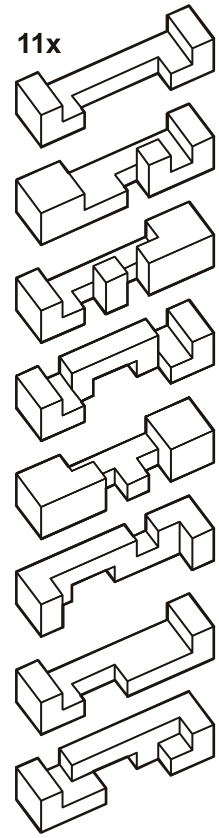


Figure 3. Pieces of *Lovely*

3) The race for the highest level

In the world of interlocking puzzles there seems to be an ongoing race between puzzle designers worldwide to find the puzzle with the highest number of moves to take the first piece out (which is how the level of a puzzle is defined). The original 18 pieces burr only needed three moves and is therefore a level 3 puzzle. Bruce Love from Australia designed a puzzle with the same shape, but using different pieces. This was a great improvement on the original design. His design, called ***Lovely***, was a level 18 puzzle. No less than 18 consecutive moves were needed to get the first piece out. Incredible as this sounds, this was only the start of the race for finding high levels. In the days before computers, designing a puzzle like that was a painstaking and incredible labour intensive job. Finding an interesting combination of 18 pieces out of the trillions of combinations was far from easy. Finding a set of pieces with just one more move was considered a big step forward in these days. Brian Young (also from Australia) found a level 19 puzzle, called ***Coming of age II***, which came out in the 1990s and held the record for quite a while.

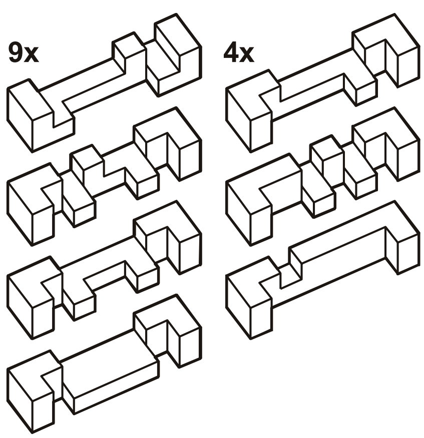


Figure 4. Pieces of *Burrloon*

When the computer entered the world of puzzle designers, this was a major improvement. Although it was not possible to calculate all the possibilities, checking a certain set of puzzle pieces for solutions had become a lot easier. A puzzle called ***Burrloon*** was designed by computer programmer Pit Khiam Goh from Singapore. It beat the record of 19 moves from Brian Young, and not just by one or two moves. To remove the first piece from the Burrloon puzzle no less than a staggering number of 33 moves was required. This puzzle has two other very interesting characteristics, the solution is unique and all the pieces are notchable. The pieces of Burrloon are shown in Figure 4.

In 2003, a puzzle collector from the Netherlands named Jack Krijnen improved the record again. His puzzle ***Tipperary*** has a wonderfully unique level 43 solution. The pieces are shown in Figure 5. At the time when I wrote an article on high-level puzzles in 2004, this was the highest level known for an 18-piece burr. Jack used a combination of techniques by hand and computer to find his high-level designs.

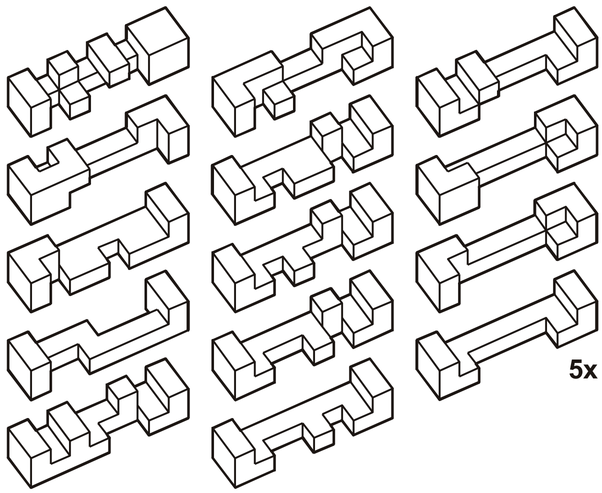


Figure 5. Pieces of *Tipperary*

4) Using new techniques

Finding a new and higher-level solution was still not easy, even with the help of a computer. Dic Sonneveld from the Netherlands came up with a new technique to find high-level puzzles. He used a system called 'burr-growing'. The basic idea is that you analyse an existing puzzle, find out where the vacant units are inside this puzzle, add a single cube to one of these voids and recalculate the puzzle. You can do this for each of the vacant units and then use this technique iteratively. Although this mechanism worked, it was a painstaking process of taking an existing puzzle, building a model of centicubes, figuring out the vacant spaces, adding a cube and then trying to disassemble the puzzle by hand and see if the level had gone up. Dic soon realised that it would be far easier to do this burrgrowing by computer. He wrote a program that started as a simple Word macro, but soon developed in a very complicated Visual Basic program, including many different options for effective burrgrowing. Although Dic Sonneveld did not use this program for 18-piece burrs, puzzle designer Pit Khiam Goh also started using this technique in his own software.

5) Higher and higher

With three different ways to design puzzles (studying mechanisms by hand, trying out many different sets of pieces and burrgrowing), new puzzle designs saw the light very frequently. Although it seems to be a competitive race to find the levels, puzzle designers share ideas and designs. In 2005 Jack Krijnen and Pit Khiam Goh joined forces and came up with ***Burrserk***, taking the record from 43 moves to 50.

Alphons Eyckmans from Belgium (one of the main designers of 18 piece puzzles) came up with a puzzle called ***Conder***, with a level 59 solution. Later that year Jack Krijnen designed a puzzle called ***Condor's Peeper***. Using three colours, the solution is unique, with a level 62 solution. All the pieces are notchable. Generally speaking, the higher the level, the harder it gets to use notchable pieces only.

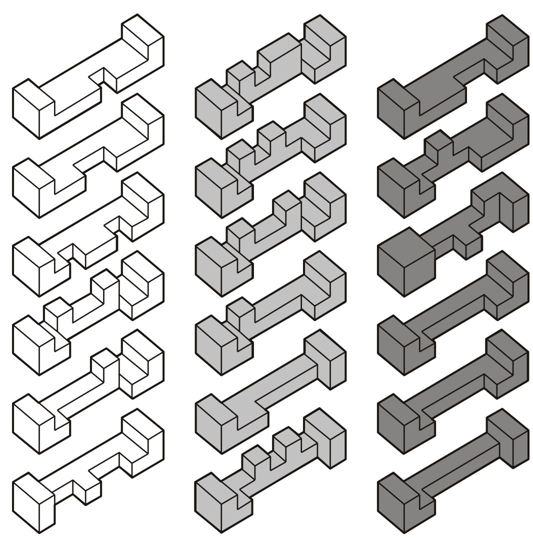


Figure 6. Pieces of *Condor's Peeper*

Using several different techniques and with puzzle designers worldwide focusing on these puzzles, bigger and bigger steps were made. An incredible leap forward was made by Alfons Eyckmans. His Phoenix puzzle almost doubled the number of moves from the previous design. It needed no less than 109 moves for the first piece, which is 47 moves more than the previous record! A small improvement to this design was made by Jack Krijnen who was able to crank the level up to 111 by changing two of the pieces. Alphons Eykmans once more took the lead with his Phoenix Cabracan design, needing 113 moves.

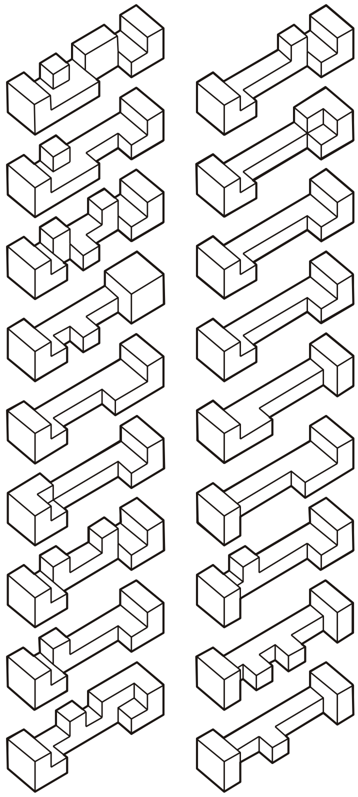


Figure 7. Pieces of *Phoenix*

Compared to the design of Lovely almost 100 moves were added! And still, this was not the end.

It was very hard to predict what more would happen. Sometimes minor improvements were made, raising the level with just one or two moves, but every now and then giant steps were reported. In early 2010 there was another one of these incredible leaps. The record of 113 moves seemed hard to beat, yet Alphons Eyckmans did it again. His design of ***Tiros*** has a level 150 solution. That is another 37 moves added. In regular interlocking puzzles, 37 moves is quite an achievement in itself, but in this case the previous record was impoved by 37 steps! A few months later Jack Krijnen came up with three different designs. One of these, called ***Burrly Sane for Extreme Puzzlers*** broke the record again by taking it to 152. In 2012 Alphons Eyckmans designed ***The Barones***, also with level 152.

In early 2013 Eyckmans and Krijnen joined forces and were able to raise the bar even further. Their joined design ***Excelsior*** had a level 156 solution. Their cooperation continued and led to the current record, which they set later that year. The design of ***Supernova*** is the current record holder with 166 moves. This number is truly mind-boggling. If you look at the puzzle in Figure 1 and tell people, that it will need 166 consecutive moves to get one piece out of the puzzle, they will think you are mad! And that piece will only come out if you make all the moves correctly! If you make a mistake somewhere on the way, it is very difficult to get back on track again!

The 166-move record is truly amazing, but there is no way of knowing what other designs with higher levels might be possible. Theoretically anything is possible, as we cannot do a full search. We will never know. Who knows, we might hit the 200 mark somewhere in the future.

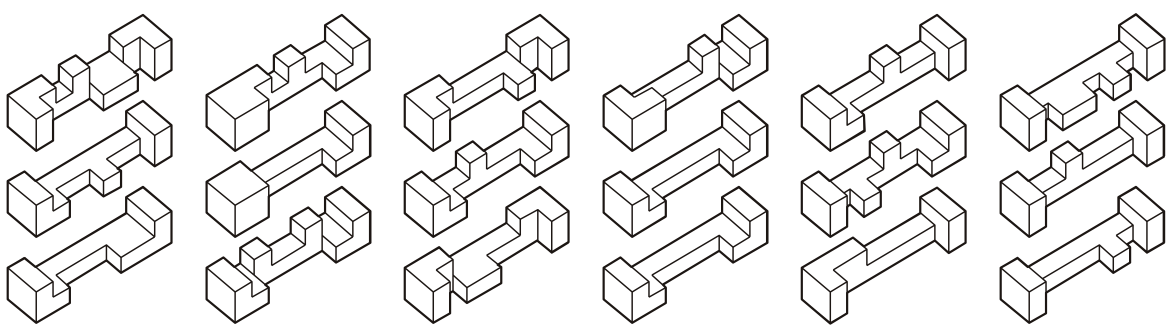


Figure 8. Pieces of *Supernova,* the current record holder

6) Solving puzzles by hand or by computer

The first 18 piece burr designed by Willem van der Poel was an interesting puzzle. With enough perseverance, it was possible to solve this puzzle by hand. This became a lot harder in the case of Bruce Love's Lovely. Taking the puzzle apart is doable (although not easy), but finding the right position and orientation of the pieces from scratch is close to impossible.

This puzzle, and many of its followers are therefore only of theoretical interest. It is virtually impossible to solve these by hand (although some are slightly easier than others). Despite the fact that it is almost impossible to solve these puzzles by hand, puzzle collectors are very keen to get a physical copy of such puzzles (me included). It is hard to determine whether these puzzle designs should be called inventions or discoveries. Surely, the help of a computer has played a vital role in the design of these puzzles, but that is just one part of the equation. If you buy a very expensive table saw, that does not automatically make you a good carpenter! Having good tools is important, but the experience and ingenuity of the puzzle designer is much more important.

7) Making 18 piece burr solutions unique

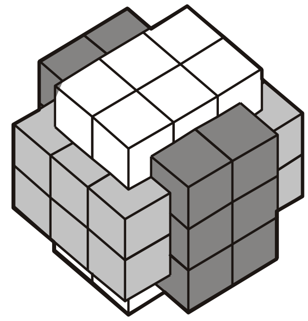


Figure 9.
X-/Y-/Z-colouring

There is a direct relationship between the number of vacant units inside the assembled puzzle and the level of difficulty. If there are no holes, the puzzle has a level one solution by definition. As there is no room for internal movements, one or more pieces have to be taken out in the first step. The more holes you introduce in the puzzle, the more room there is for pieces moving without being taken out of the puzzle. If you study the level of the puzzle compared to the number of holes, you will notice that usually the higher the level, the more holes are present.

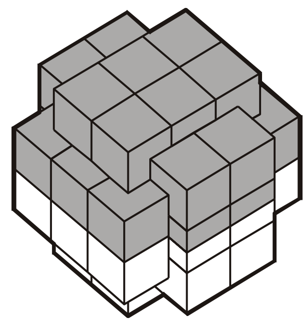


Figure 10.
'Chocolate-dip 'colouring

Having many holes inside the puzzle also has a major downside. The number of different ways that the pieces can fit toghether goes up very rapidly. In some cases a certain set of pieces might fit together in millions of different ways. As discussed earlier, you preferable want a solutoin to be unique (only one way to fit the pieces together). This means that you have to use one or more tricks to force certain pieces in certain positions.

Using different colours for the pieces is an easy way to force pieces into a certain position. Using three different colours for each of the main axis is an easy way to do this. Apart from that, it will also add to the aesthetics of the puzzle. Although this helps a lot, it does not work in every case, so different ways of colouring or marking the pieces are needed. A second way to make a puzzle unique is to use a two colour scheme, comparable to a chocolate dip on an ice cream. Apart from using different colours for the pieces, you can also add markings on the pieces to force them into certain orientations and/or locations. A technique often used is to use a router to make an decorative edge along the ends of the pieces. This is very restricting. If you combine this with three different colours, almost any 18 piece burr can be made unique.

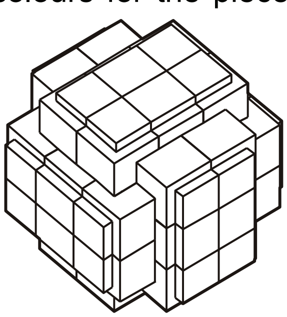


Figure 11.
Using decorative edges to make a puzzle unique

8) Pseudo 18-piece burrs

Some puzzles seem to be identical to an 18 piece burr, but they are actually different puzzles, disguised as the classic puzzle. Different type of puzzles have been designed like this. The simplest variation is to add an extra piece that is hidden inside the puzzle, as ***Save the Gorilla*** (by Alfons Eyckmans), where a monkey shaped piece complements the 18 other pieces. Strictly speaking this is a 19 piece puzzle, but you cannot tell from the outside. Eyckmans designed six different puzzles like this. Stephan Baumegger also designed one, called ***Beware of the Snake***.

If you want to stick to the number of 18 pieces, there are other ways to make variations. Two puzzle designs called ***Hunchback 12*** and ***Hunchback 32*** were designed by Alphons Eyckmans. In these designs some of the pieces have extra cubes protruding their usual envelope. Some of the pieces are not 2x2x8, but 2x3x8. This is not visible from the outside though.

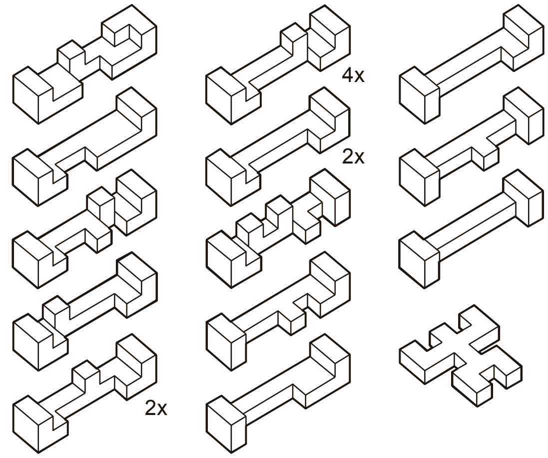


Figure 12. Pieces of *Save the Gorilla*

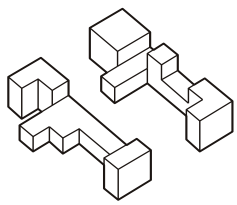


Figure 13. 2x3x8 pieces

Adding extra pieces or making subtle adjustments is relatively easy, but you can also do things the hard way. Jan Naert from Belgium came up with a puzzle called ***Delerium***. This puzzle has 18 pieces and looks just like the standard puzzle when assembled, but the pieces are a complete mess! Most of the pieces do not look like the original pieces at all. Alfons Eyckmans also designed two of these puzzles, called ***Mayhem*** and ***Nightmare***.

9) Other variations

The pseudo 18-piece burrs all look identical to the original puzzle on the outside, but piece numbers or pieces' shapes are different. There are other variations though. Several designs are known where one or more of the pieces are offset. Other variations are also possible, like framed or boxed versions. There are so many of these, that I can fill an entirely new article with those.

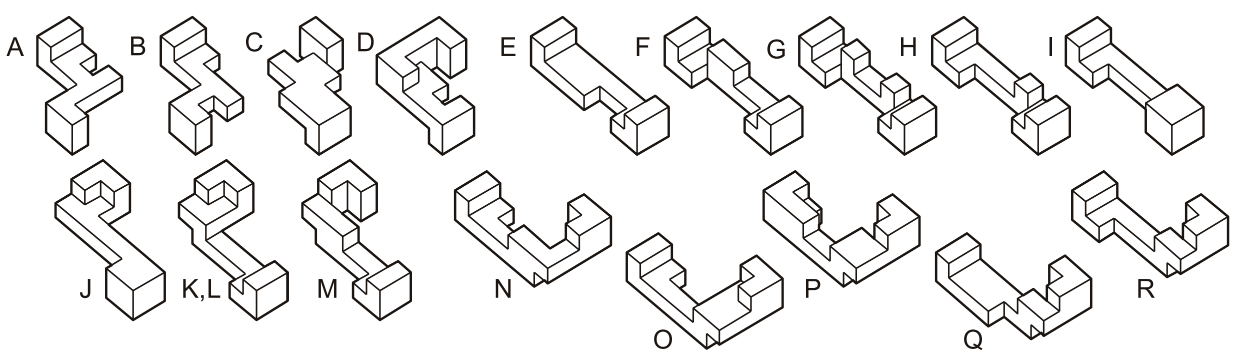


Figure 14. Pieces of *Delerium*

10) Conclusions

What seemed like a relatively simple puzzle has puzzled people for many decades. The introduction of the computer resulted in the race for high-level solutions. The results so far are incredible, but there is no way of knowing what other designs are possible. Doing a full analysis of all the possible pieces and their combinations seems to be well out of reach of today's technology. This probably adds to the myth of the 18 piece burr.

References

[1] http://www.puzzlewillbeplayed.com/-/burr-18t.xml#burr-18t

Gift 1: Jangrams next to Tangram and Sei Shonagon Chie no Ita

Gift 2: Simple Polygons Folding to Three Boxes

Ryuhei Uehara
School of Information Science
Japan Advanced Institute of Science and Technology
http://www.jaist.ac.jp/~uehara
uehara@jaist.ac.jp

1 Jangrams

A *silhouette puzzle* is a game where, given a set of polygons, one must decide whether they can be placed in the plane in such a way that their union is a target polygon. Rotation and reflection are allowed but scaling is not, and all polygons must be internally disjoint.[1]

The *tangram* is the set of polygons illustrated in Figure 1(left). Of anonymous origin, their first known reference in literature is from 1813 in China [Slo04]. The tangram has grown to be extremely popular throughout the world; now, over 2000 silhouette and related puzzles exist for it [Slo04, Gar87].

Much less famous is a quite similar Japanese puzzle called *Sei Shonagon Chie no Ita*. Sei Shonagon was a courtier and famous novelist in Japan, but there is no evidence that the puzzle existed a millennium ago when she was living (966?-1025?). *Chie no ita* means *wisdom plates*, which refers to this type of physical puzzle. It is said that the puzzle is named after Sei Shonagon's wisdom. Historically, the Sei Shonagon Chie no Ita first appeared in literature in 1742 [Slo04]. Even in Japan, the tangram is more popular than Sei Shonagon Chie no Ita, though Sei Shonagon Chie no Ita is common enough to have been made into ceramic dinner plates (Figure 1(right), [Tak14]).

Wang and Hsiung considered the number of possible convex (filled) polygons formed by the tangram [WH42]. They first noted that, given sixteen identical isosceles right triangles, one can create the tangram

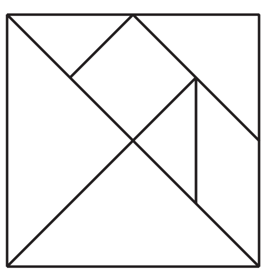


Figure 1: Left: the tangram in square configuration. Right: a set of traditional ceramic plates in the form of Sei Shonagon Chie no Ita pieces, crafted by Tomomi Takeda in Kanazawa, Japan.

[1] Sometimes this puzzle is also called "dissection puzzle." However, dissection puzzle usually indicates the puzzles that focus on finding the cutting line itself. The most famous one is known as the Haberdasher's Puzzle by Henry Dudeney that asks to find cut lines of a regular triangle such that the resulting four pieces can be rearranged to form a square [Dud58].

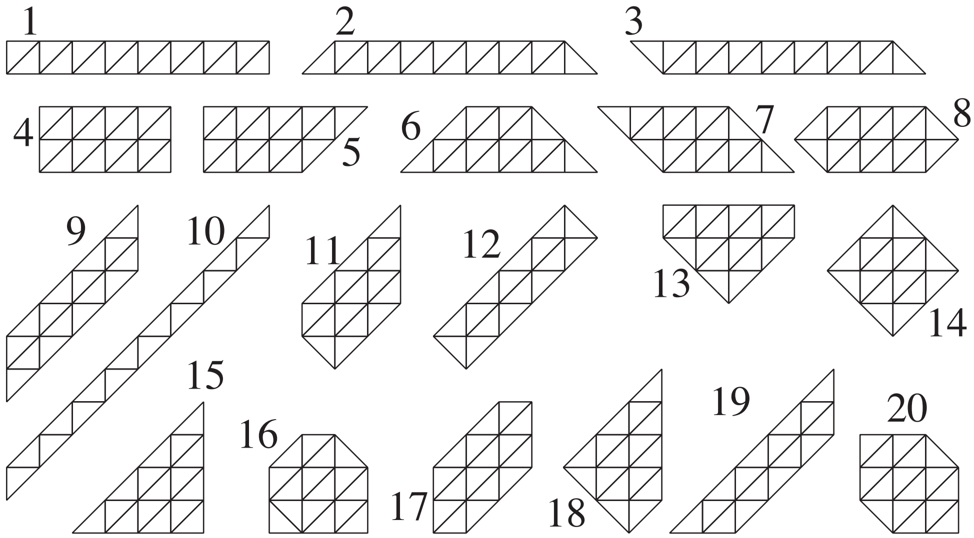


Figure 2: All 20 potential convex polygons that can be formed from 16 identical isosceles right triangles.

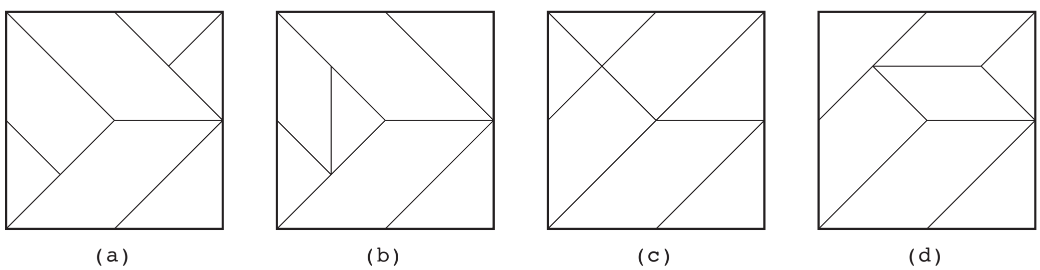


Figure 3: Jangrams: Four patterns that can form nineteen convex polygons out of 20.

pieces by gluing some edges together. So, clearly, the set of convex polygons one can create from the tangram is a subset of those that sixteen identical isosceles right triangles can form. Embedded in the proof of their main theorem, Wang and Hsiung [WH42] demonstrate that sixteen identical isosceles right triangles can form exactly 20 convex polygons. These 20 are illustrated in Figure 2. The tangram can realize 13 of those 20. Also Sei Shonagon Chie no Ita achieves 16 out of 20, which is folklore in the puzzle society in Japan [Aki14]. Therefore, in a sense, we can conclude Sei Shonagon Chie no Ita is more expressive than the tangram. while both the tangram and Sei Shonagon Chie no Ita contain seven pieces made from sixteen identical isosceles right triangles, Sei Shonagon Chie no Ita can form more convex polygons than the tangram.

One might next wonder if this can be improved with different shapes. We demonstrate a set of seven pieces that can form nineteen convex polygons among twenty candidates, and that to realize all twenty convex polygons, it is necessary and sufficient to have eleven pieces. We investigate all possible cases and conclude that there are four sets of seven pieces that allow to form nineteen convex polygons as shown in Figure 3. Based on this result, we also show that no set of six pieces can form nineteen convex polygons. That is, our results for general silhouette puzzles can be summarized as the following theorem:

Theorem 1 *(1) There are only four patterns of seven pieces (Figure 3) that can form nineteen convex polygons among twenty candidates in Figure 2. (2) To form all twenty polygons in Figure 2, eleven pieces are necessary and sufficient. (3) Any six pieces in the same manner cannot form nineteen convex polygons among twenty candidates.*

The proof of this theorem can be found [FEKU16]. Enjoy our Jangrams!

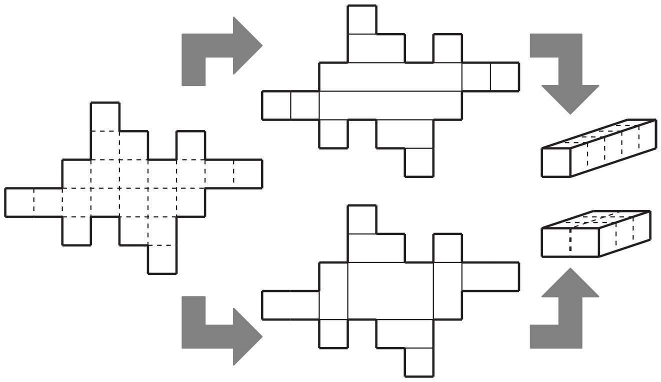


Figure 4: A polygon folding into two boxes of size $1 \times 1 \times 5$ and $1 \times 2 \times 3$ in [MU08].

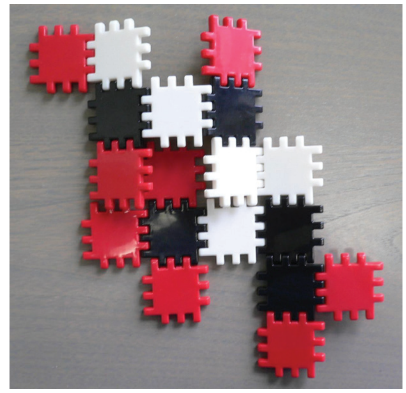


Figure 5: Cubigami.

2 Common development of three different boxes

At G4G9, the author prepared some gift, a set of polygons that can fold into two boxes in 2010. This folding problem is very natural but quite counterintuitive; for a given polygon that consists of unit squares, and the problem asks if there are two or more ways to fold it into simple convex orthogonal polyhedra (Figure 4). Some similar idea can be found in a nice puzzle "cubigami" (Figure 5), which is a common development of all tetracubes except one (since the last one has surface area 16, while the others have surface area 18) developed by Miller and Knuth. Some related results can be found in the books on geometric folding algorithms by Demaine and O'Rourke [DO07, O'R11].

Biedl et al. first gave two polygons that fold into two incongruent orthogonal boxes [BCD+99] (see also Figure 25.53 in the book by Demaine and O'Rourke [DO07]). Later, Mitani and Uehara constructed infinite families of orthogonal polygons that fold into two incongruent orthogonal boxes [MU08], that is the base of the gift at G4G9. Recently, Shirakawa and Uehara extended the result to three boxes in a nontrivial way; they showed infinite families of orthogonal polygons that fold into three incongruent orthogonal boxes [SU13]. One example is shown in Figure 6. However, the smallest polygon by this method contains 532 unit squares so far, and it is still open if there exists much smaller polygon of several dozens of squares that folds into three (or more) different boxes.

First of all, two boxes of size $a \times b \times c$ and $a' \times b' \times c'$ share a common development only if they have the same surface area, i.e., when $2(ab+bc+ca) = 2(a'b'+b'c'+c'a')$ holds. We can compute small surface areas ($1 \le a \le b \le c \le 50$) that may admit to fold into two or more boxes by a simple exhaustive search (Table 1). From the table, we can say that the smallest surface area is at least 22 to have a common development of two boxes, and then their sizes are $1 \times 1 \times 5$ and $1 \times 2 \times 3$. In fact, Abel et al. have confirmed that there exist 2,263 common developments of two boxes of size $1 \times 1 \times 5$ and $1 \times 2 \times 3$ by an exhaustive search [ADD+11]. On the other hand, the smallest surface area that may admit to fold into three boxes is 46, which may fold into three boxes of size $1 \times 1 \times 11$, $1 \times 2 \times 7$, and $1 \times 3 \times 5$. However, the number of polygons of area 46 seems to be too huge to search. This number is strongly related to the enumeration and counting of polyominoes, namely, orthogonal polygons that consist of unit squares [Gol94]. The number of polyominoes of area n is well investigated in the puzzle society, but it is known up to $n = 45$, which is given by Shirakawa (see the OEIS (`https://oeis.org/A000105`) for the references). Therefore, it seems to be quite hard to enumerate all common developments of three boxes of size $1 \times 1 \times 11$, $1 \times 2 \times 7$, and $1 \times 3 \times 5$ since their common area consists of 46 unit squares.

One natural step is the next one of the surface area 22 in Table 1. The next area of 22 in the table is 30, which admits to fold into two boxes of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$. We employed nontrivial algorithmic tricks, and completed the analysis. As a result, we finally obtained all common development of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$ which is 1,080.

$2(ab+bc+ca)$	$a \times b \times c$
22	$1 \times 1 \times 5$, $1 \times 2 \times 3$
30	$1 \times 1 \times 7$, $1 \times 3 \times 3$
34	$1 \times 1 \times 8$, $1 \times 2 \times 5$
38	$1 \times 1 \times 9$, $1 \times 3 \times 4$
46	$1 \times 1 \times 11$, $1 \times 2 \times 7$, $1 \times 3 \times 5$
54	$1 \times 1 \times 13$, $1 \times 3 \times 6$, $3 \times 3 \times 3$
58	$1 \times 1 \times 14$, $1 \times 2 \times 9$, $1 \times 4 \times 5$
62	$1 \times 1 \times 15$, $1 \times 3 \times 7$, $2 \times 3 \times 5$
64	$1 \times 2 \times 10$, $2 \times 2 \times 7$, $2 \times 4 \times 4$
70	$1 \times 1 \times 17$, $1 \times 2 \times 11$, $1 \times 3 \times 8$, $1 \times 5 \times 5$
88	$1 \times 2 \times 14$, $1 \times 4 \times 8$, $2 \times 2 \times 10$, $2 \times 4 \times 6$

Table 1: A part of possible size $a \times b \times c$ of boxes and its common surface area $2(ab+bc+ca)$.

Based on the obtained common developments, we next change our scheme. In [BCD$^+$99], they also considered folding along 45 degree lines, and showed that there was a polygon folding into two boxes of size $1 \times 2 \times 4$ and $\sqrt{2} \times \sqrt{2} \times 3\sqrt{2}$ (Figure 7). In this context, we can observe that the area 30 may admit to fold into another box of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ by folding along the diagonal lines of rectangles of size 1×2. This idea leads us to the problem that asks if there exist common developments of three boxes of size $1 \times 1 \times 7$, $1 \times 3 \times 3$, and $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ among these 1,080 common developments of two boxes of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$.

We checked if these common developments of two boxes of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$ could also fold into the third box of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$, and obtained an affirmative answer. We found that nine of 1,080 common developments of two boxes of size $1 \times 1 \times 7$ and $1 \times 3 \times 3$ folded into the third cube of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ (Figure 8). Moreover, one of the nine common developments of three boxes had another way of folding. Precisely, the last one (Figure 8(9)) admits to fold into the third box of size $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ in two different ways! These four ways of folding are depicted in Figure 9.

We summarize the recent results about this topic:

Theorem 2 *(1) There are 1,080 polyominoes of area 30 that admit to fold (along the edges of unit squares) into two boxes of size* $1 \times 1 \times 7$ *and* $1 \times 3 \times 3$. *(2) Among the above 1,080, nine polyominoes can fold into the third box of size* $\sqrt{5} \times \sqrt{5} \times \sqrt{5}$ *if we admit to fold along diagonal lines (Figure 8). (3) Among these nine polyominoes, one can fold into the third box in two different ways (Figure 9).*

The details of the algorithms used to prove this theorem can be found [XHSU15]. The folding ways are very nice puzzles. Enjoy folding!

References

[ADD+11] Z. Abel, E. Demaine, M. Demaine, H. Matsui, G. Rote, and R. Uehara. Common Development of Several Different Orthogonal Boxes. In *23rd Canadian Conference on Computational Geometry (CCCG)*, pages 77–82, 2011.

[Aki14] Hisayoshi Akiyama. Personal communication. 2014.

[BCD+99] T. Biedl, T. Chan, E. Demaine, M. Demaine, A. Lubiw, J. I. Munro, and J. Shallit. Notes from the University of Waterloo Algorithmic Problem Session. September 8 1999.

[DO07] E. D. Demaine and J. O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra.* Cambridge University Press, 2007.

[Dud58] Henry Ernest Dudeney. *The Canterbury Puzzles.* Dover Publications, Inc., 1958.

[FEKU16] Eli Fox-Epstein, Kazuho Katsumata, and Ryuhei Uehara. The Convex Configurations of "Sei Shonagon Chie no Ita," Tangram, and Other Silhouette Puzzles with Seven Pieces. *IEICE Trans. on Inf. and Sys.*, accepted, 2016.

[Gar87] Martin Gardner. *Time Travel and Other Mathematical Bewilderments.* W. H. Freeman and Company, 1987.

[Gol94] Solomon W. Golomb. *Polyominoes.* Princeton University Press, 1994.

[MU08] J. Mitani and R. Uehara. Polygons Folding to Plural Incongruent Orthogonal Boxes. In *Canadian Conference on Computational Geometry (CCCG 2008)*, pages 39–42, 2008.

[O'R11] J. O'Rourke. *How to Fold It: The Mathematics of Linkage, Origami and Polyhedra.* Cambridge University Press, 2011.

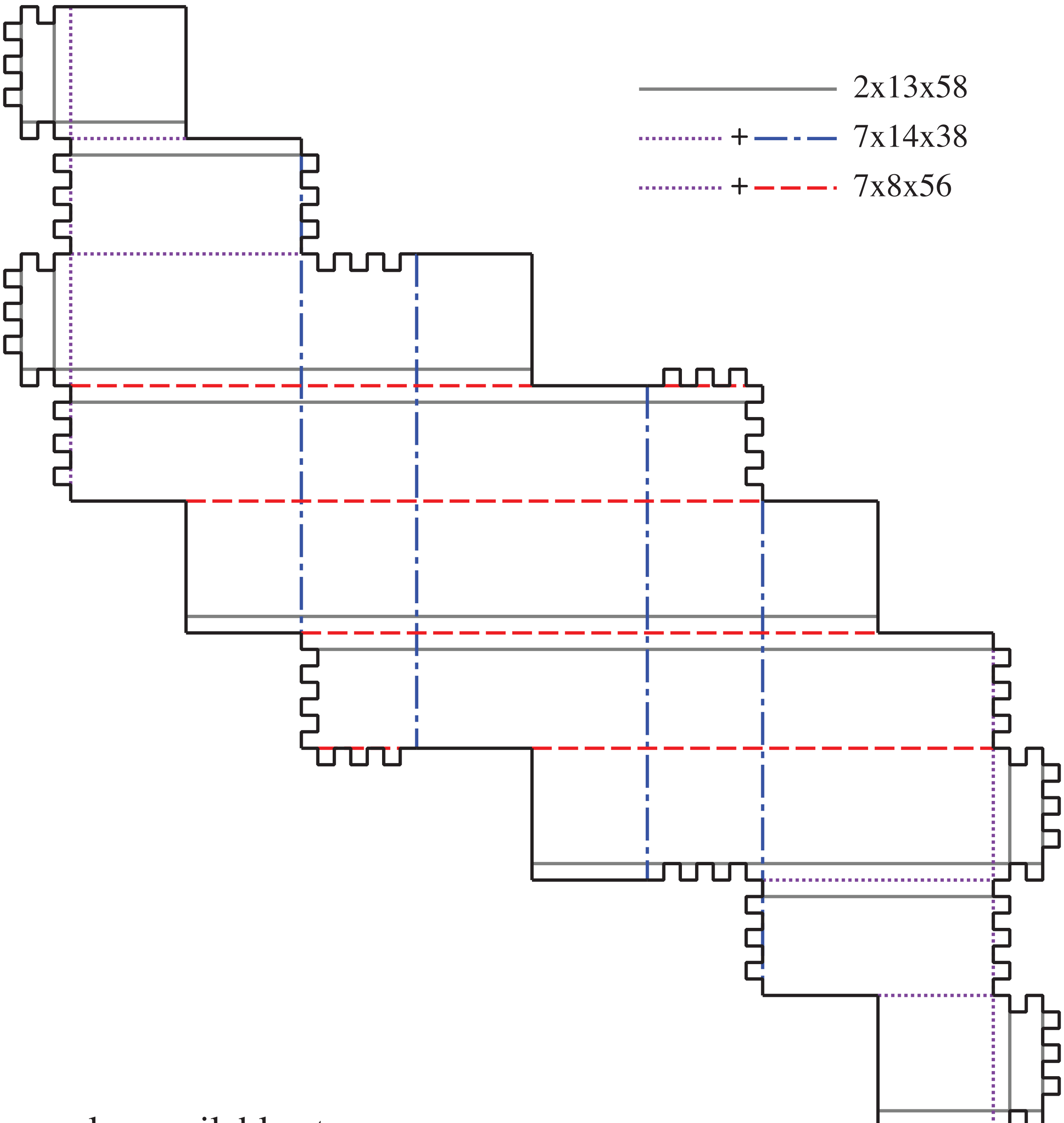
[Slo04] Jerry Slocum. *The Tangram Book: The Story of the Chinese Puzzle with Over 2000 Puzzle to Solve.* Sterling Publishing, 2004.

[SU13] Toshihiro Shirakawa and Ryuhei Uehara. Common Developments of Three Incongruent Orthogonal Boxes. *International Journal of Computational Geometry and Applications*, 23(1):65–71, 2013.

[Tak14] Tomomi Takeda. `http://www.matsusaki.jp/local-event/20140310/1024/`. (in Japanese; accessed 22 Feb 2016), March 2014.

[WH42] Fu-Traing Wang and Chuan-Chih Hsiung. A Theorem on the Tangram. *The American Mathematical Monthly*, 9(49):596–599, 1942.

[XHSU15] Dawei Xu, Takashi Horiyama, Toshihiro Shirakawa, and Ryuhei Uehara. Common Developments of Three Incongruent Boxes of Area 30. In *The 12th Annual Conference on Theory and Applications of Models of Computation (TAMC 2015)*, pages 236–247. Lecture Notes in Computer Science Vol. 9076, Springer-Verlag, 2015.



also available at
http://www.jaist.ac.jp/~uehara/etc/origami/nets/3box.pdf

Figure 6: A polygon folding into three boxes of size $2 \times 13 \times 58$, $7 \times 14 \times 38$, and $7 \times 8 \times 56$.

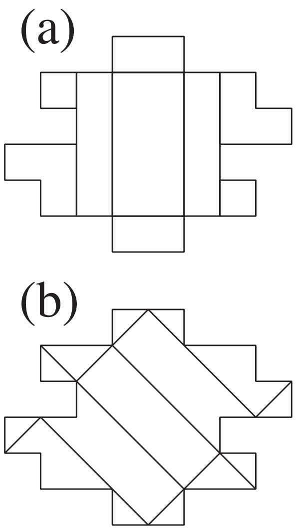


Figure 7: The common development shown in [BCD+99]. (a) It folds into a box of size $1 \times 2 \times 4$ and (b) it also folds into a box of size $\sqrt{2} \times \sqrt{2} \times 3\sqrt{2}$.

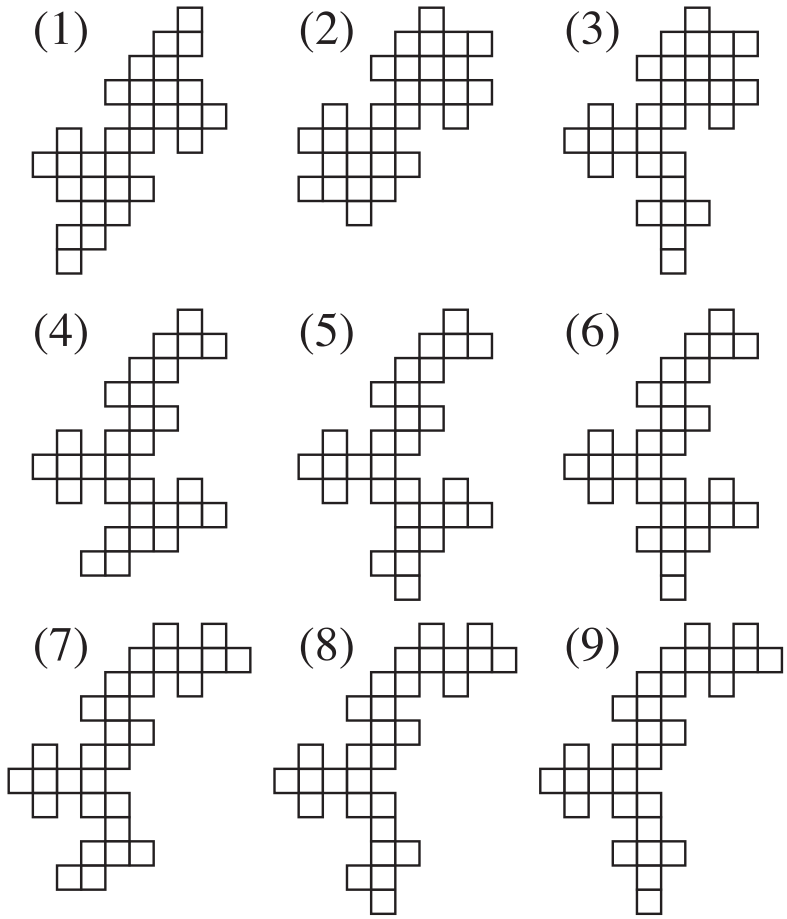


Figure 8: Nine polygons that fold into three boxes of size $1\times1\times7$, $1\times3\times3$, and $\sqrt{5}\times\sqrt{5}\times\sqrt{5}$. The last one can fold into the third box in two different ways (Figure 9).

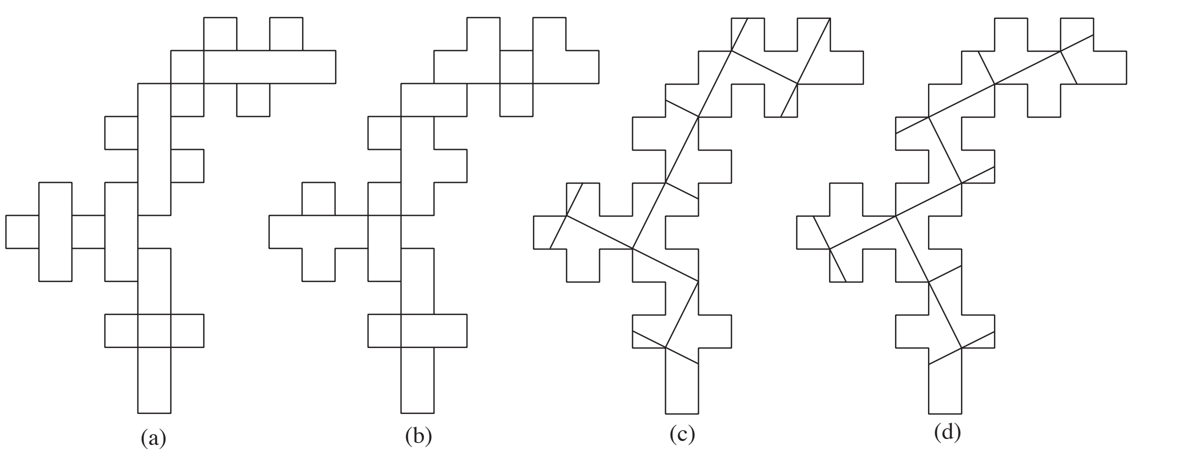


Figure 9: The *unique* polygon folds into three boxes of size (a) $1\times1\times7$, (b) $1\times3\times3$, and (c)(d) $\sqrt{5}\times\sqrt{5}\times\sqrt{5}$ in four different ways.

Martin Gardner Modular Origami G4G12 Rhombic Dodecahedron By Peter Knoppers

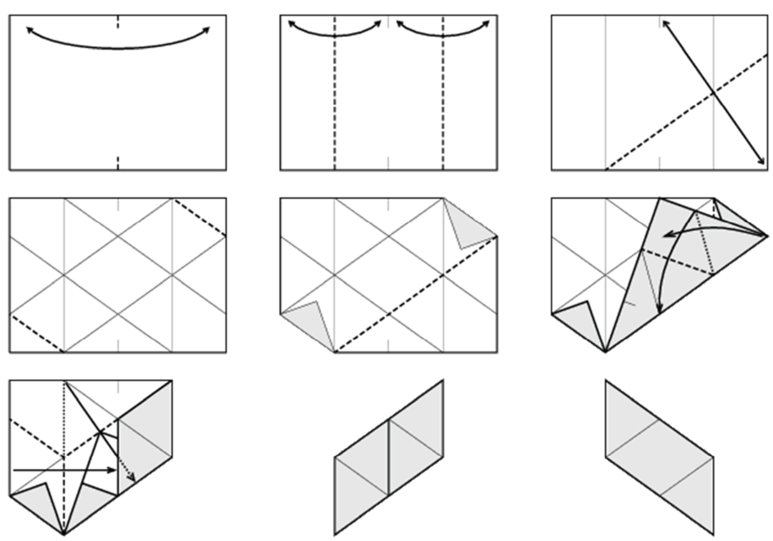
Description

This artwork was designed for G4G6 to cover the 6 faces of a cube. As most of you will know, a Rhombic Dodecahedron can be constructed from a cube by adding a pyramid with square base and height 0.5 to each face of the cube. Faces of a pyramid on adjacent cube faces are then joined to create 12 Rhombuses. To match the artwork, it needed to be stretched by a factor √2 along the long diagonal of the Rhombus. The (very charming) property of this artwork on a cube that all faces show essentially the same image is not maintained. The Rhombic Dodecahedron requires three different clippings.

Folding instructions

There are 12 numbered pages. Each page must be folded the same way.

Solid lines (as printed on the sheets) become (initially) hill folds; dotted lines valley folds. The printed lines are indications; you get a better result by making fold markings (step 1 below) and folding the various corners of the paper up to those markings in steps 2..7 as indicated below.



The image shown here is © by Ole Arntzen who has a web site that generates pages for a 12 month calendar using this folding and assembly method (image reused with permission). You can find this calendar generator at http://folk.uib.no/nmioa/kalender/.

1. Put the printed side up. Mark the center of the long edges by folding the A4 page in half; Only make the fold near the edges to mark the halfway points; unfold.
2. Fold a short edge up to the marking points made in step 1; unfold.
3. Fold the other short edge up to the marking points made in step 1; unfold
4. Printed side down. Place a corner onto the marked center of the opposite long edge; fold; unfold.
5. Place the next corner onto the marked center of the opposite long edge; fold; unfold.
6. Place the next corner onto the marked center of the opposite long edge; fold; unfold.
7. Place the last corner onto the marked center of the opposite long edge; fold; unfold.
8. Fold the corner in; the fold line should connect the points where fold lines 4 and 5 meet the edges of the paper; do not unfold. (The folding lines are numbered on, what at this time should be, the bottom side of the paper.)
9. Fold the opposite corner in; the fold line should connect the points where fold lines 4 and 5 meet the edges of the paper; do not unfold.
10. Fold along line 4; pull up fold line 6 and push it down to align on top of fold line 4; flatten the result.
11. Fold along line 5; pull up fold line 7 and push it down to align on top of fold line 5; flatten the result.
12. Lift the flap with fold lines 3-4-6 of step 11 and tuck the square protrusion of step 11 underneath it.
    You should now have a nice symmetrical shape
13. Fold up along line 4; leave that fold about 90 degrees up.
14. Fold up along line 5; leave that fold about 90 degrees up.
    You should now have a rhombus with slots along two opposite edges (near folds 6 and 7) and triangular tabs sticking up along folds 4 and 5.

Assembly instructions

After folding all 12 pieces (or, if you want to see a partial result before doing all that folding, after folding sheets 1 and 2) you can start assembly. Each piece has two numbered tabs and two numbered slots. To view the number of a slot you have to open it slightly. A tab with number N should go into a slot with that same number. If you want to make it more of a puzzle, ignore the tab numbers and use only the artwork to figure out which tabs should go into which slots.

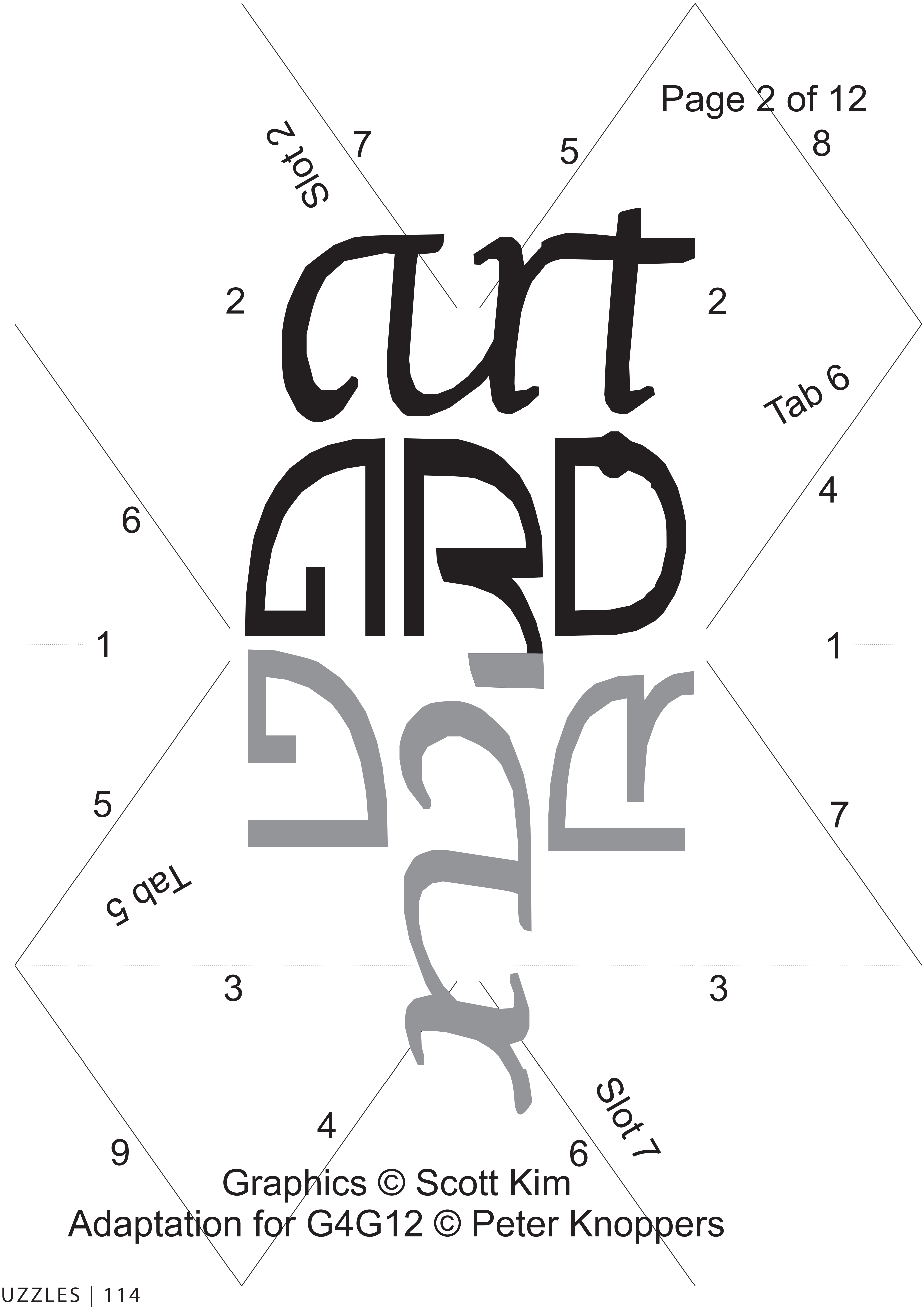
Display suggestions

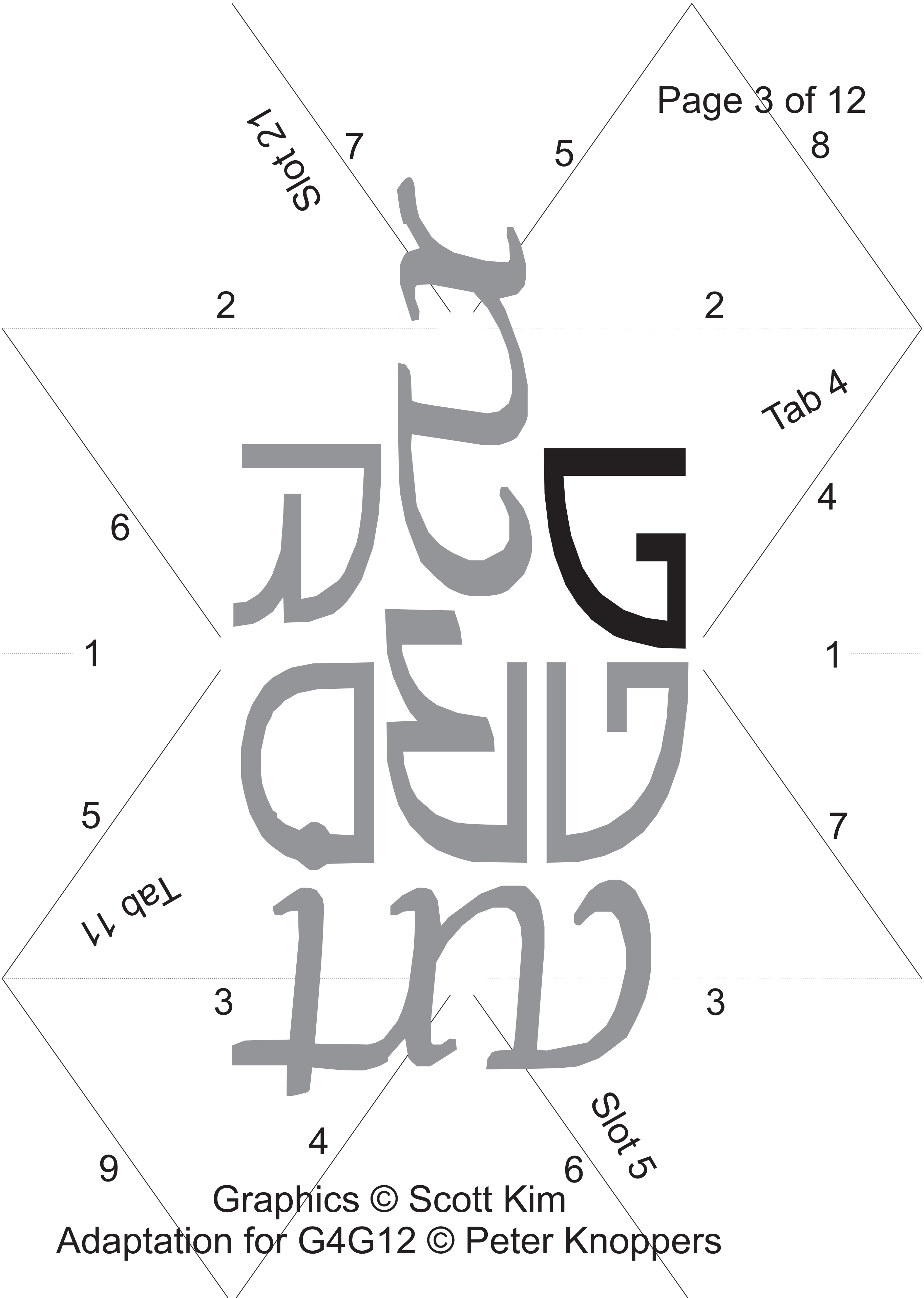
For optimal readability of the Martin Gardner graphics the object should be positioned on one of its 4-edge vertices. You can place the object with this vertex down on a big curtain ring, or a glass, or hang it from a string. (Tie a large paper clip to the end of a string; open up the top vertex of the Rhombic Dodecahedron, insert the paper clip, close the vertex.)

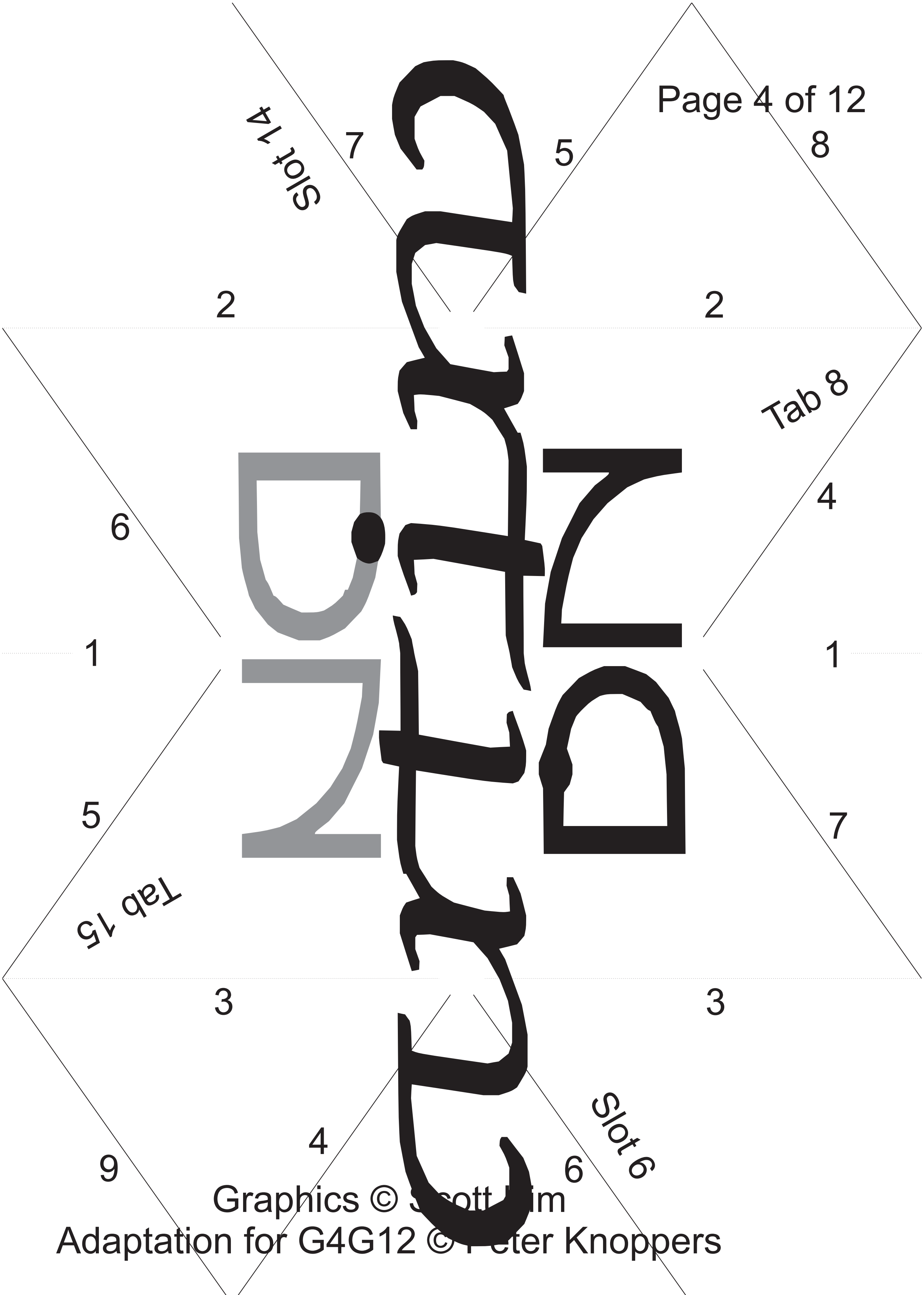
Addendum

If you copy / print your own copy, ensure that the 12 pages of the puzzle get copied / printed onto single sided A4 size paper.

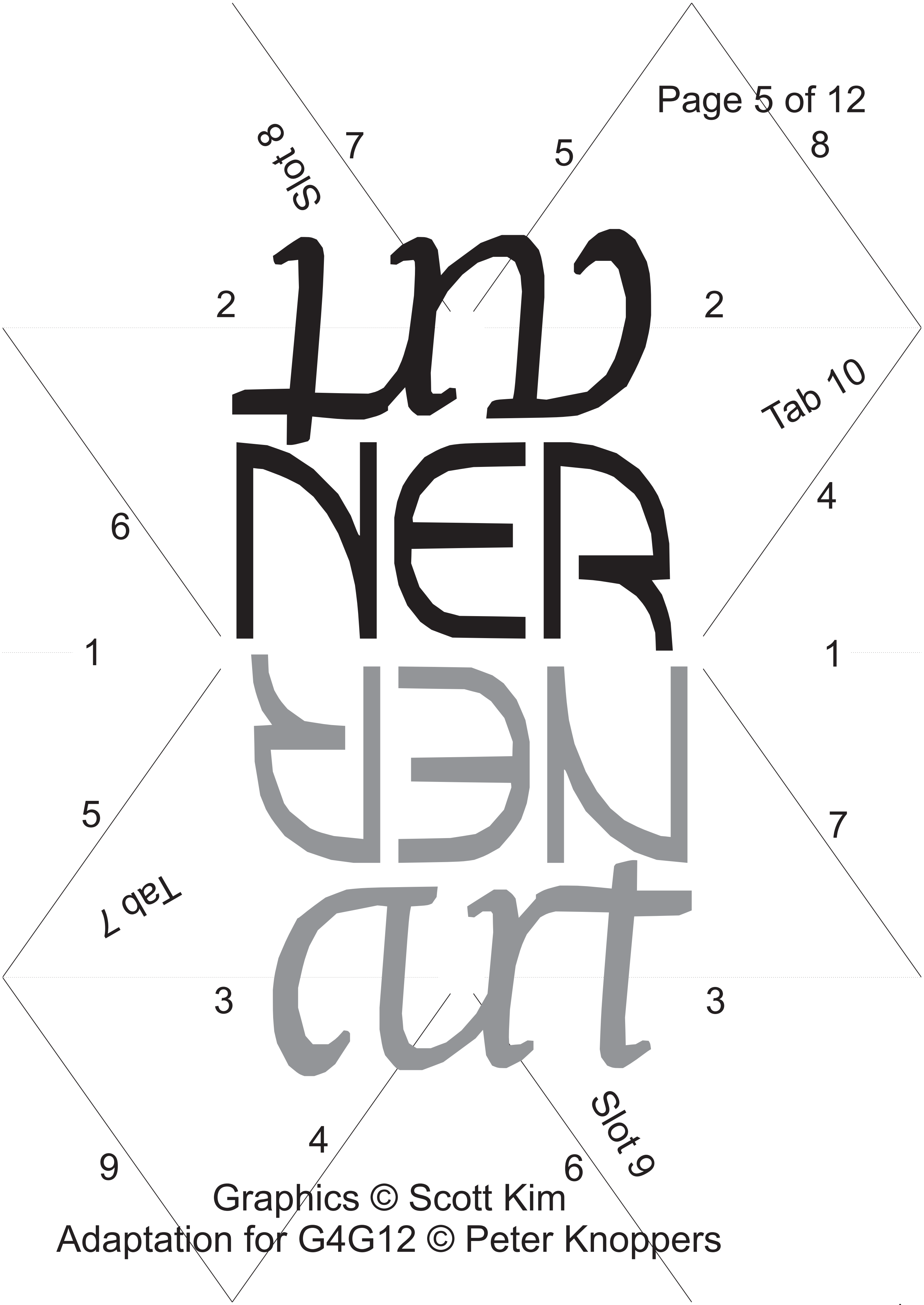
Graphics © Scott Kim
Adaptation for G4G12 © Peter Knoppers

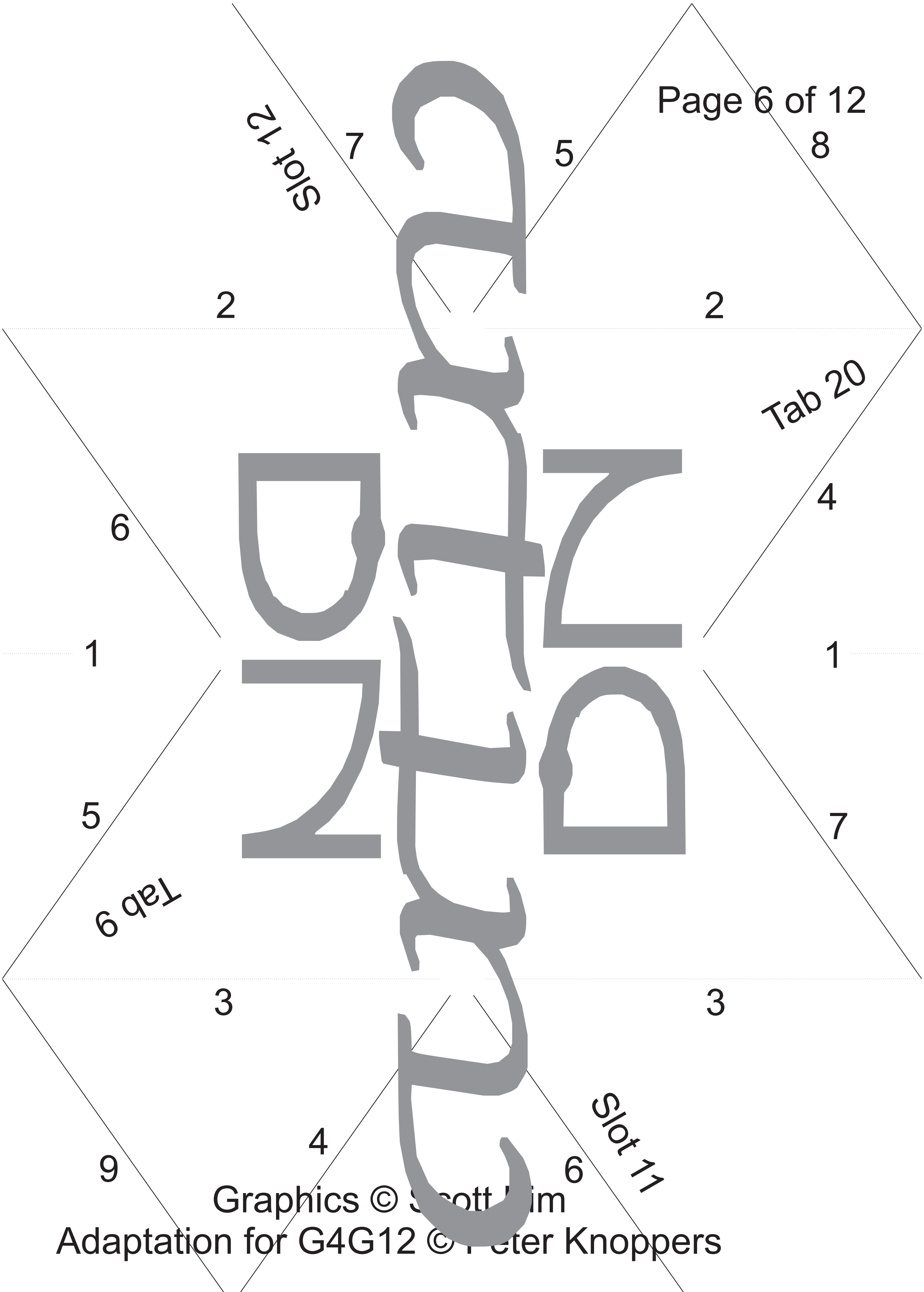

Page 2 of 12
Slot 2
7
5
8
2
2
Tab 6
6
4
1
1
5
7
Tab 5
3
3
Slot 7
4
9
6

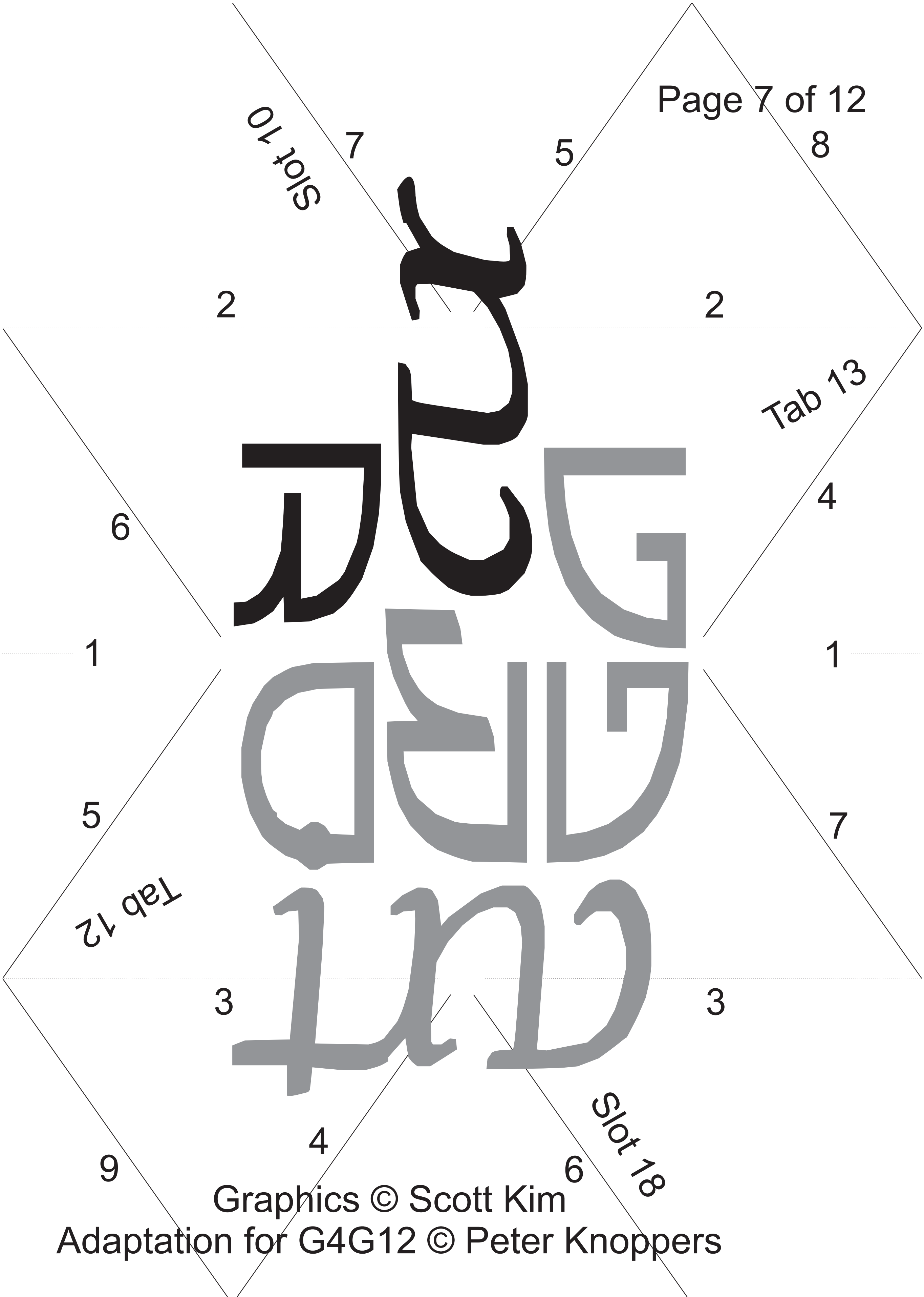

Page 3 of 12
Slot 27
7
5
8
2
2
Tab 4
6
4
1
1
5
7
Tab 11
3
3
4
9
6
Slot 5



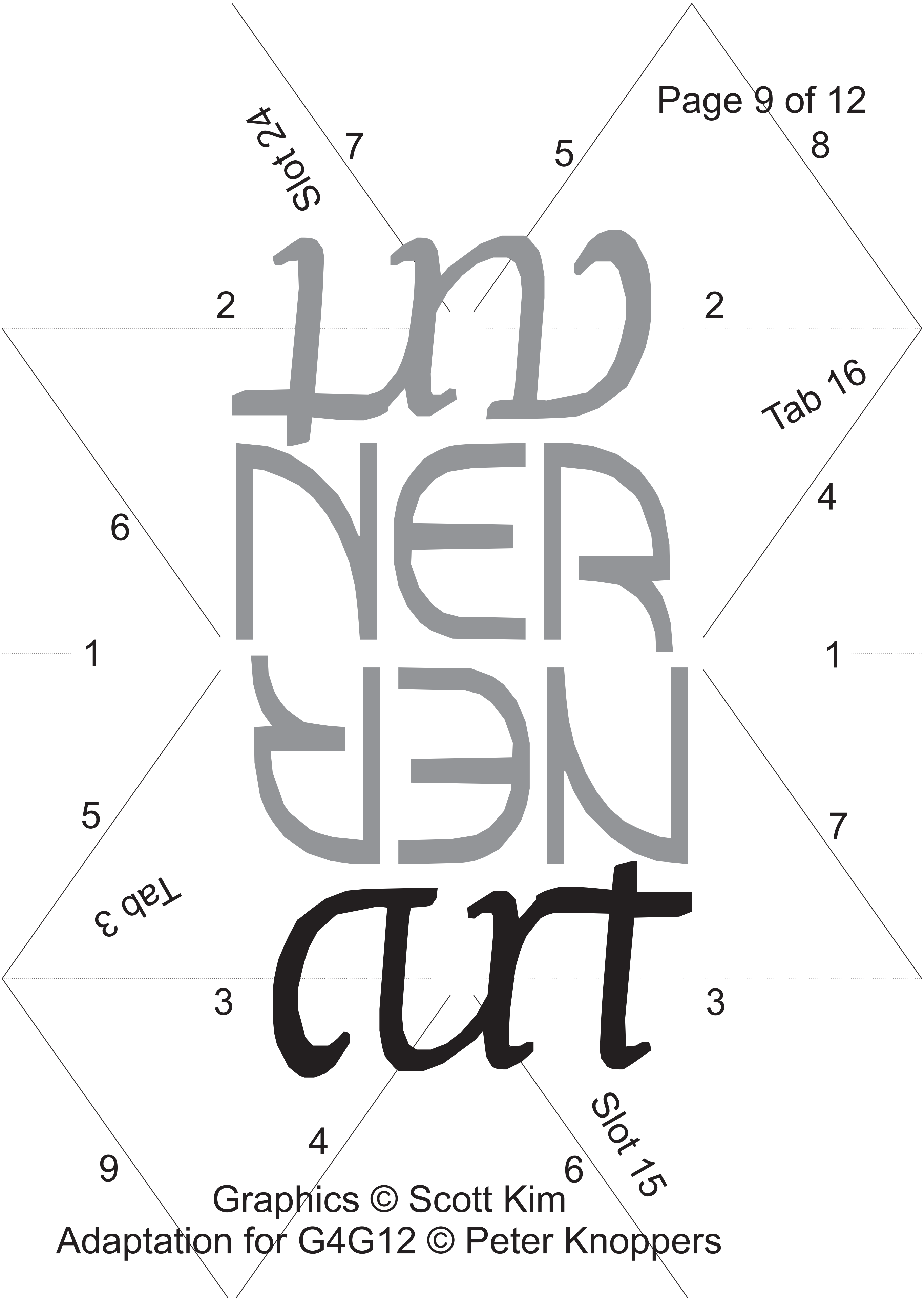
Graphics © Scott Kim
Adaptation for G4G12 © Peter Knoppers

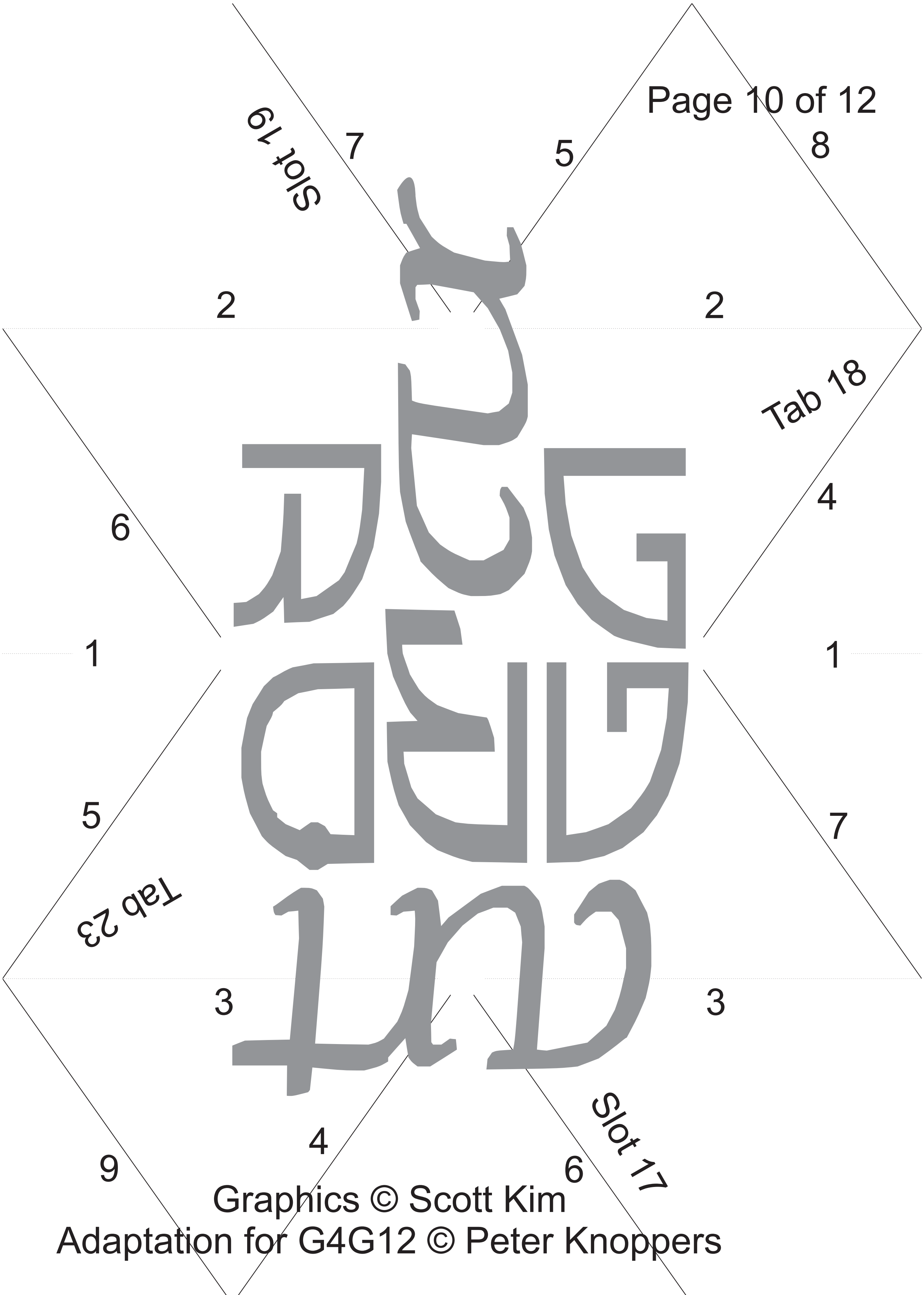

Slot 8
7
5
8
2
2
Tab 10
NER
6
4
1
1
5
7
Tab 7
3
3
Slot 9
4
9
6

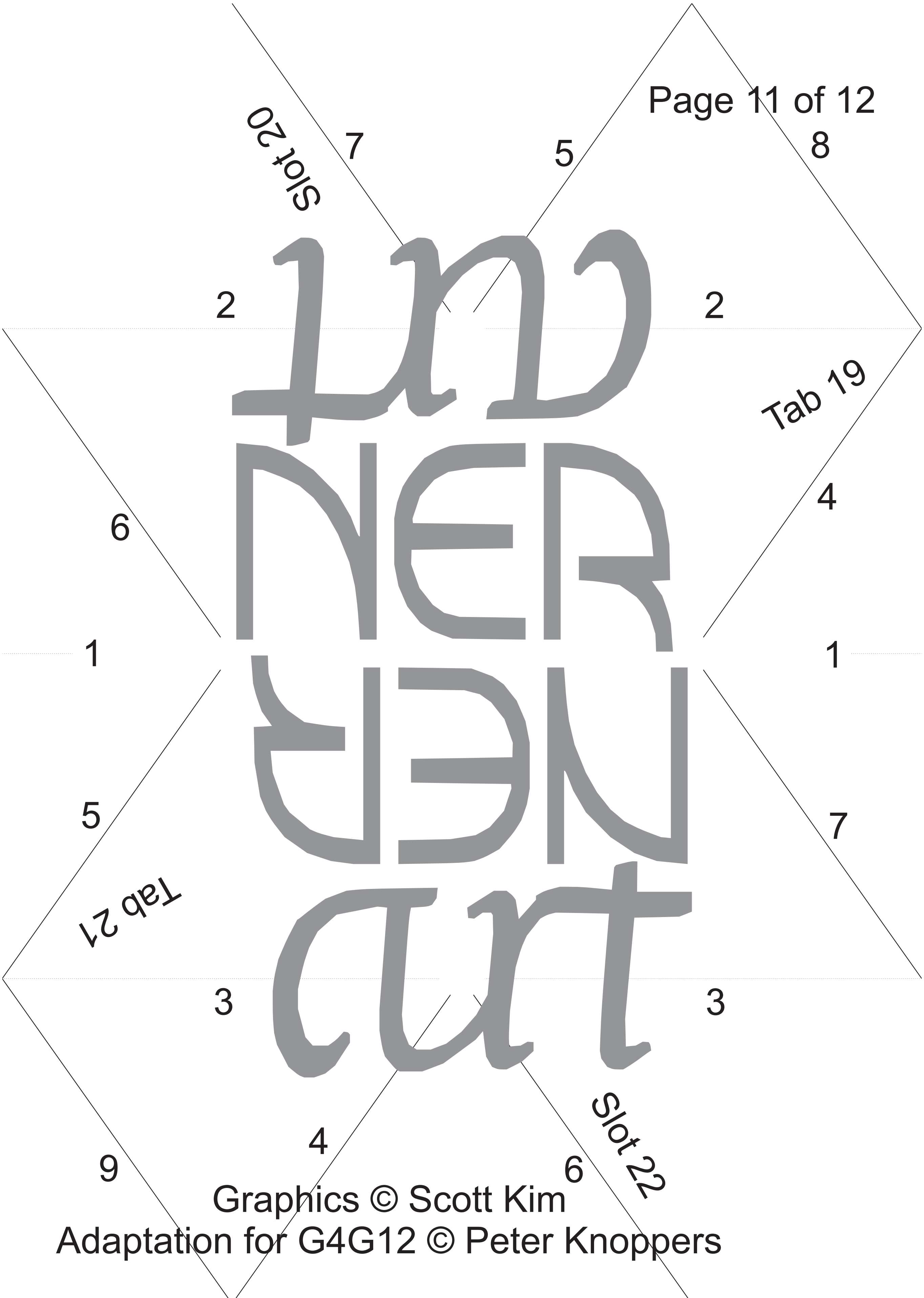

Page 6 of 12
Slot 12
7
5
8
2
2
Tab 20
4
6
1
1
5
7
Tab 9
3
3
4
6
Slot 11
9
Graphics © Scott Kim
Adaptation for G4G12 © Peter Knoppers


7
5
8
2
2
Tab 13
6
4
1
1
5
7
3
3
Slot 18
4
9
6

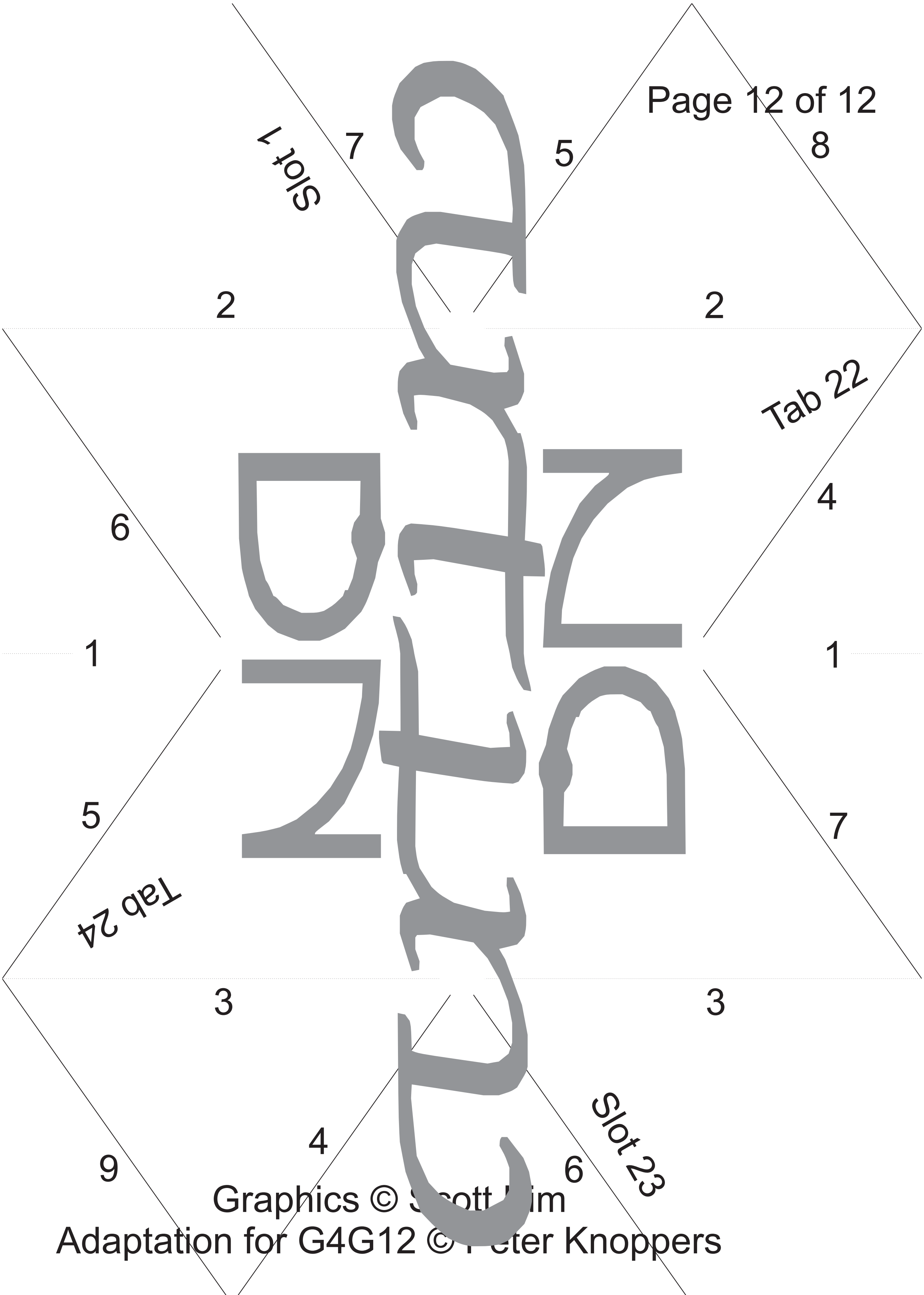

Page 8 of 12
Slot 16
7
5
8
2
2
Tab 17
4
6
1
1
5
7
Tab 14
3
3
Slot 13
4
9
6


Slot 24
7
5
8
2
2
Tab 16
4
6
1
1
5
7
Tab 3
art
3
3
4
9
6
Slot 15


Page 10 of 12
Slot 19
7
5
8
2
2
Tab 18
4
6
1
1
5
7
Tab 23
3
3
4
9
6
Slot 17



Graphics © Scott Kim
Adaptation for G4G12 © Peter Knoppers


Page 12 of 12
Slot 1
7
5
8
2
2
Tab 22
6
4
1
1
5
7
Tab 24
3
3
4
9
6
Slot 23

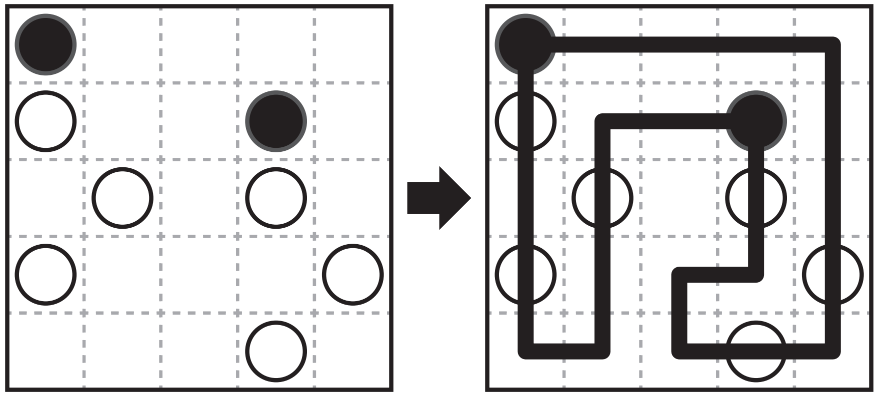
Masyu

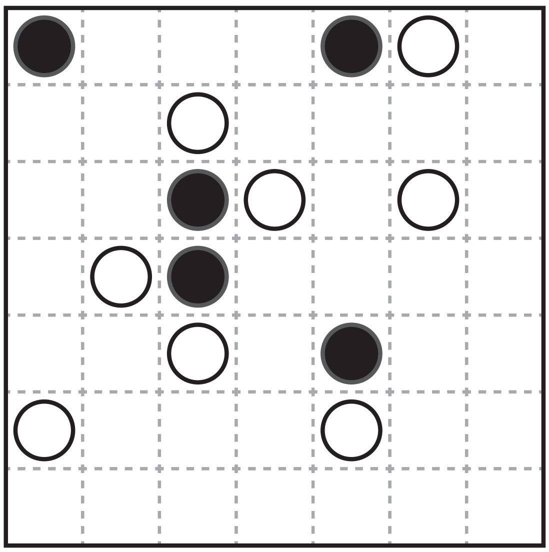
by Wei-Hwa Huang

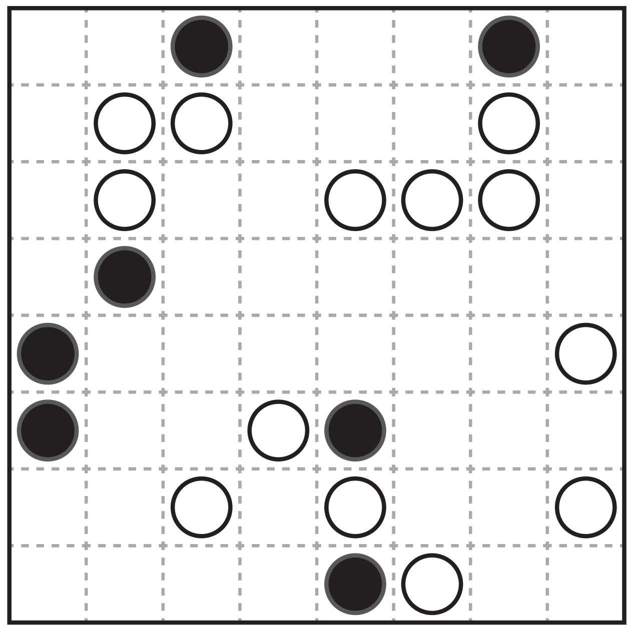
(onigame@gmail.com)

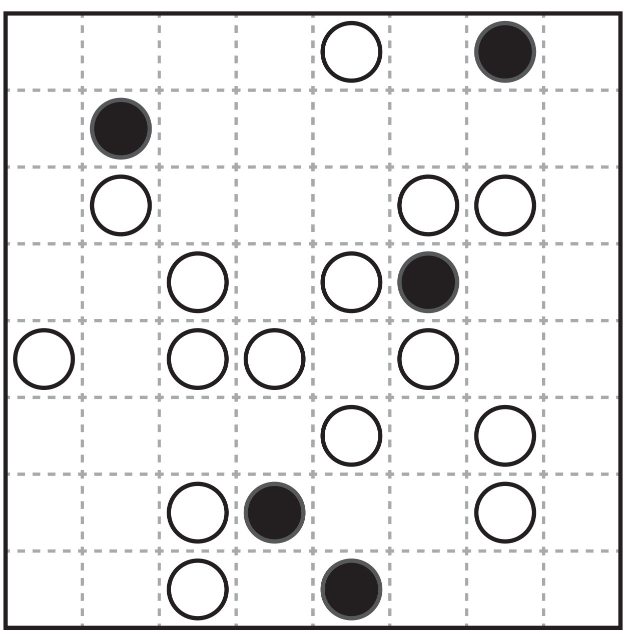
Gathering 4 Gardner 2016 Gift Exchange

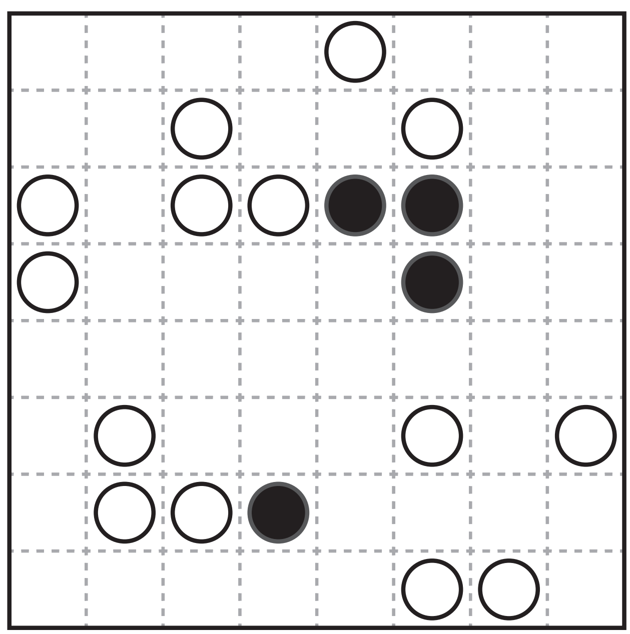
Draw a single, non-intersecting loop that passes orthogonally through all circled cells. The loop must go straight through the cells with white circles, with a turn in at least one of the cells immediately before or after each white circle. The loop must make a turn in all the black circles, but must go straight in both cells immediately before and after each black circle.











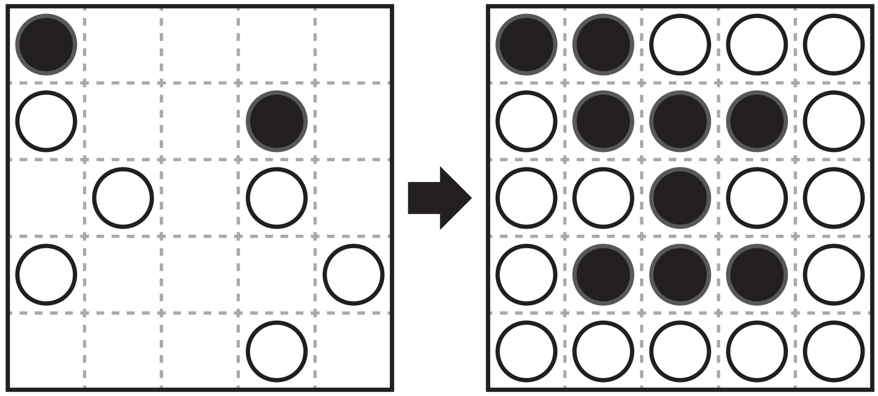
Yin-Yang

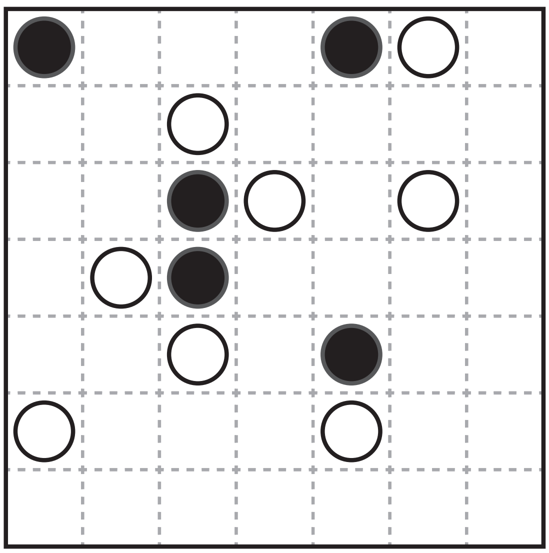
by Wei-Hwa Huang

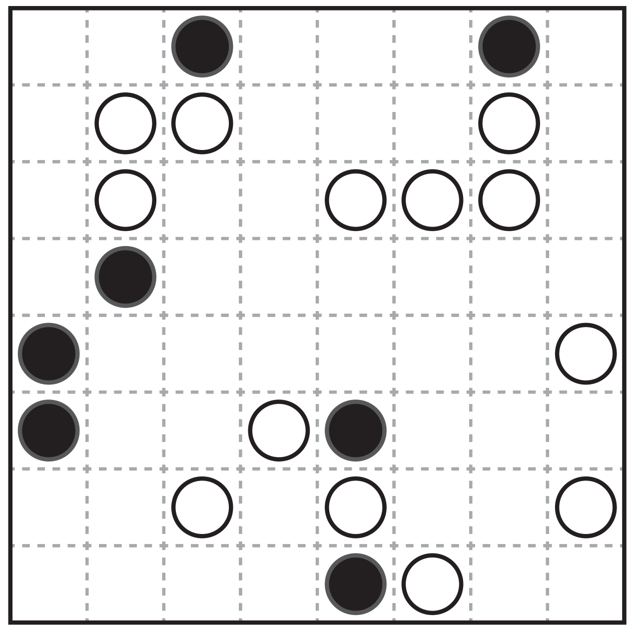
(onigame@gmail.com)

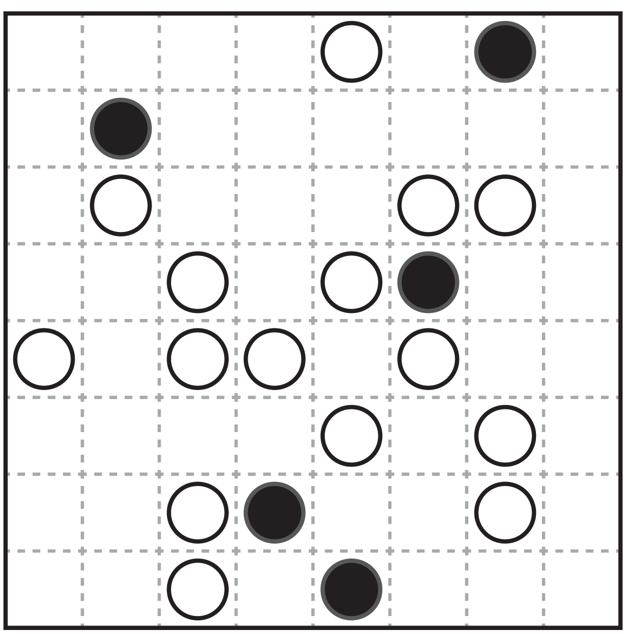
Gathering 4 Gardner 2016 Gift Exchange

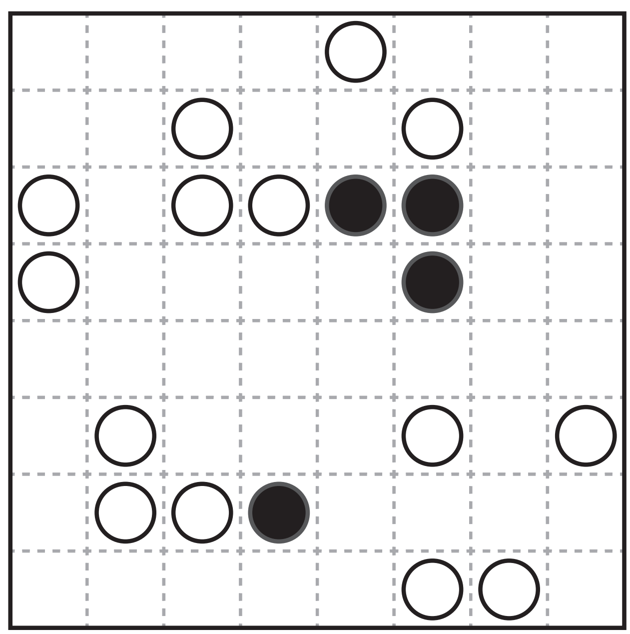
Fill each cell with either a black or a white circle. All cells with black circles must be connected orthogonally, and all cells with white circles must be connected orthogonally. Every 2×2 group of cells must contain at least one black circle and at least one white circle. Some cells are already filled in for you.











Mini Folding Puzzles

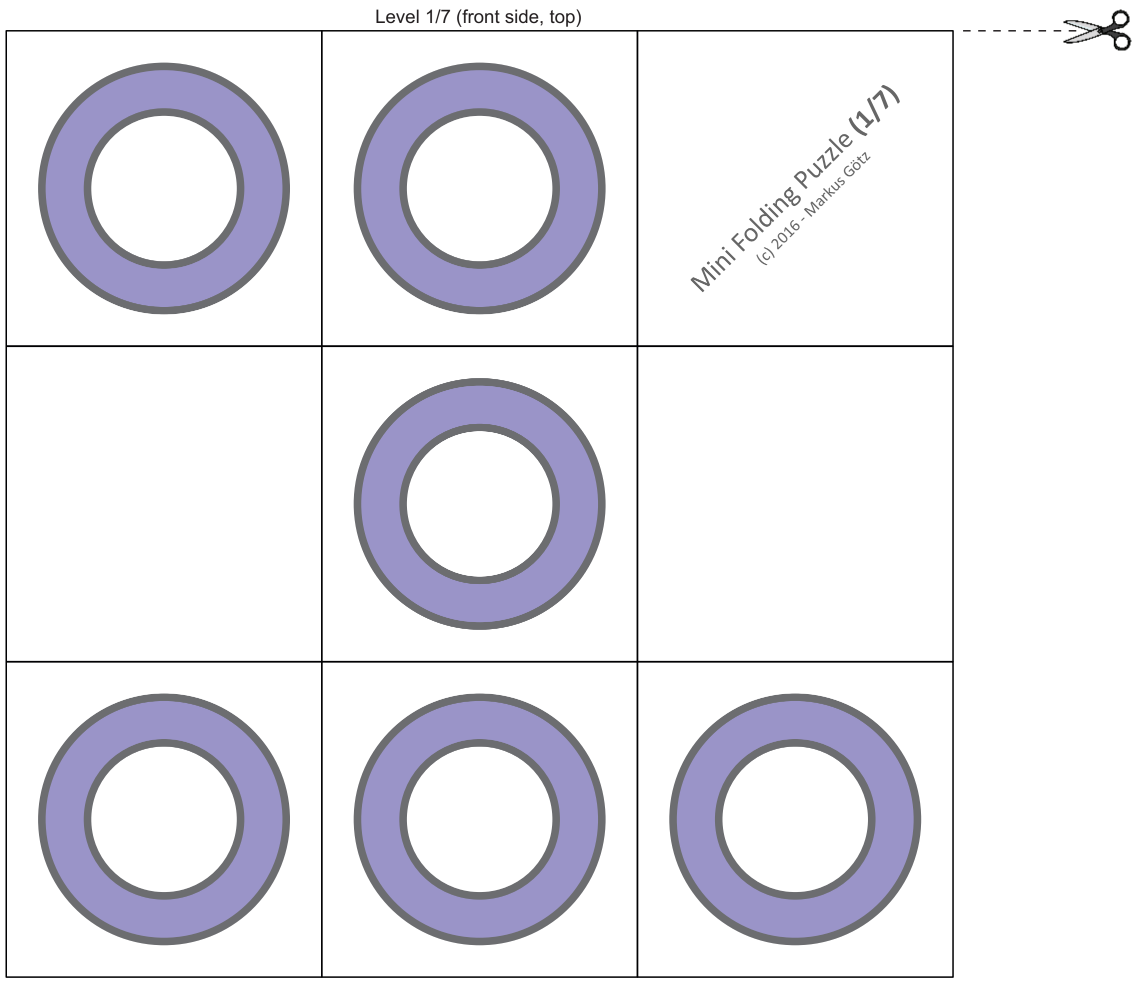
Markus Götz
mail@markus-goetz.de

Although there are already a lot of folding puzzles out there, I made a set of seven new and simple looking folding puzzles. Each one is based on a square piece of paper of 3x3 unit squares with some symbols and/or parts of a picture printed within the unit squares. The first objective is to fold this sheet into a smaller square of size 2x2 with only four circles visible on both sides. How difficult can it be? Well, the first level starts to be easy, but for sure the next ones are getting harder and harder. The objective of each puzzle is printed on the puzzle itself. For all levels the final structure (=solution) has to be flat. And it is not allowed to cut or tear the paper - only folding is allowed.

On the next pages you find the seven folding puzzles. Print them (double-sided) and cut them out. Then start accepting the challenge. Can you solve them all?

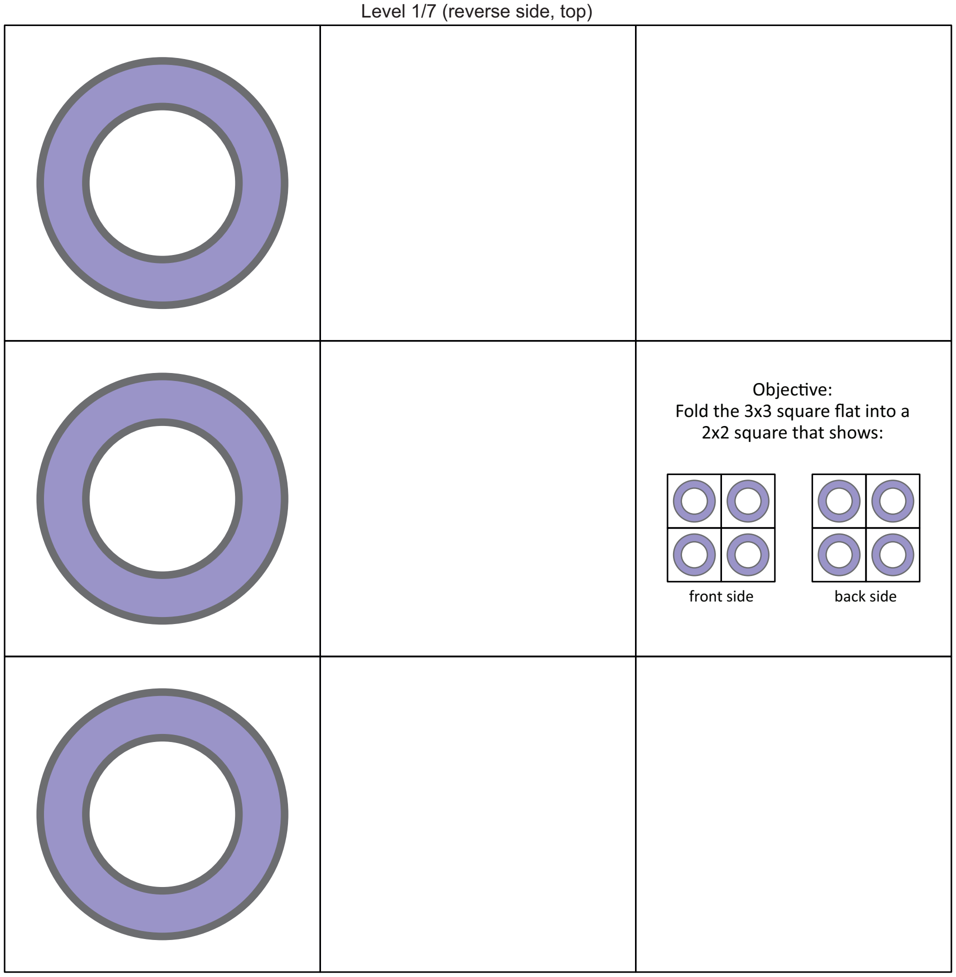
Happy puzzling. Markus

Level 1/7 (front side, top)



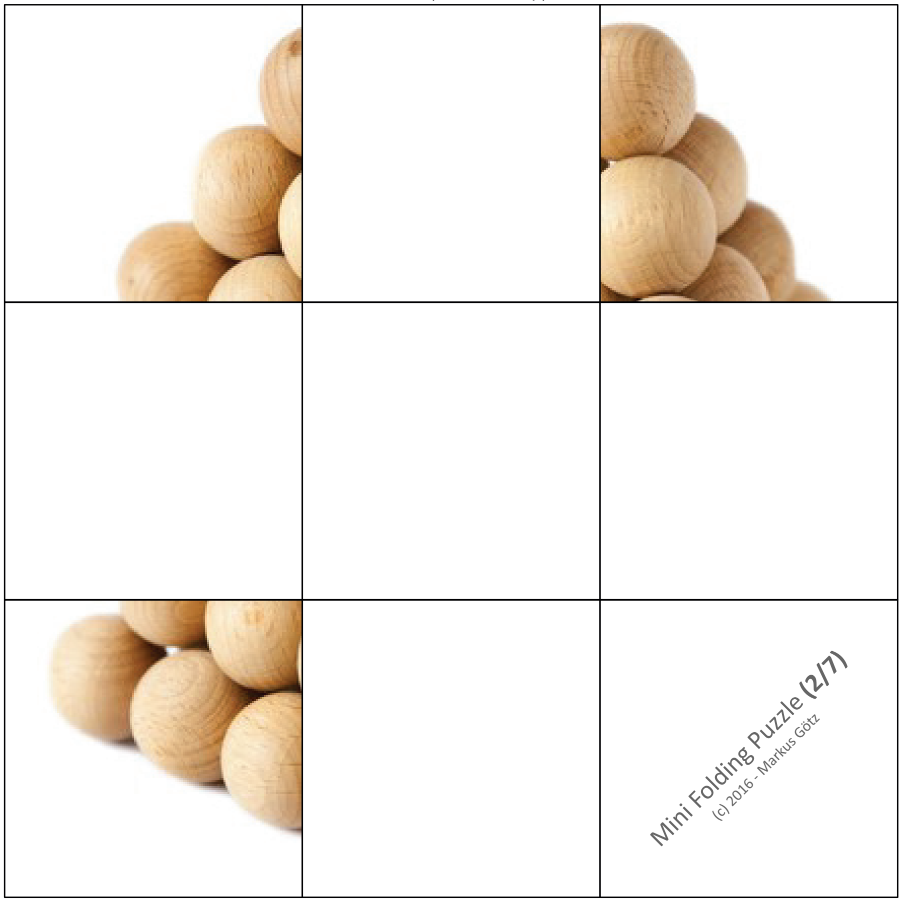
Level 1/7 (front side, bottom)

Level 1/7 (reverse side, top)



Level 1/7 (reverse side, bottom)

Level 2/7 (front side, top)



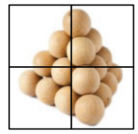
Level 2/7 (front side, bottom)

Level 3/7 (front side, top)

Level 3/7 (front side, bottom)

Level 2/7 (reverse side, top)

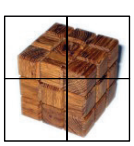
Objective:
Fold the 3x3 square flat into a 2x2 square that shows the following ball pyramid puzzle picture:



Level 2/7 (reverse side, bottom)

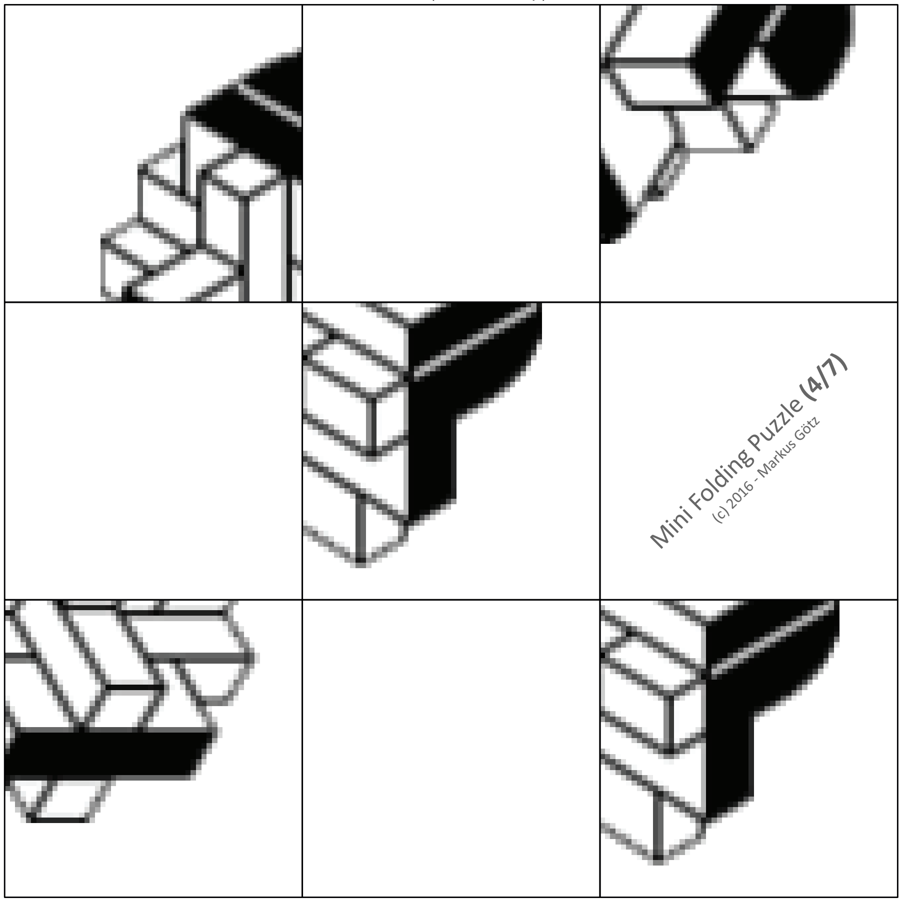
Level 3/7 (reverse side, top)

Objective:
Fold the 3x3 square flat into a 2x2 square that shows the following cube puzzle picture:



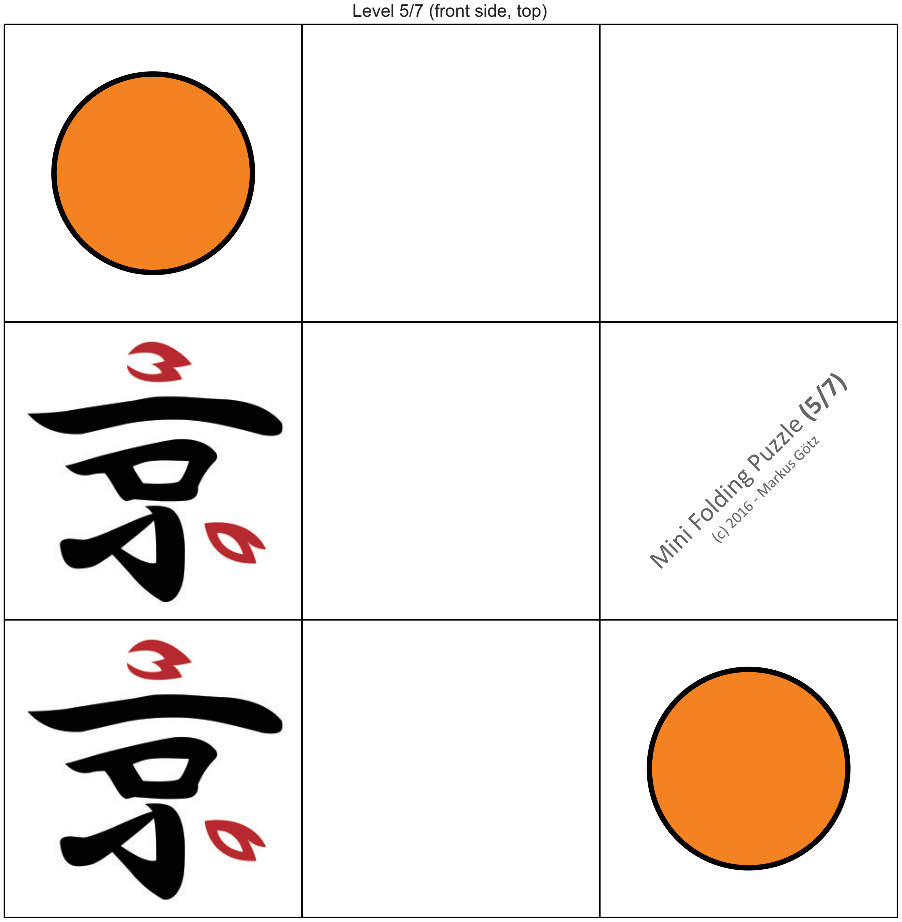
Level 3/7 (reverse side, bottom)

Level 4/7 (front side, top)



Level 4/7 (front side, bottom)

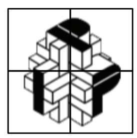
Level 5/7 (front side, top)



Level 5/7 (front side, bottom)

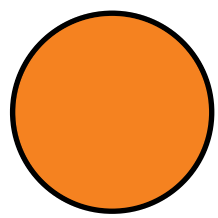
Level 4/7 (reverse side, top)

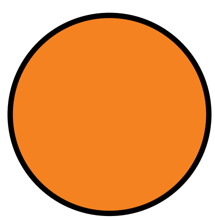
Objective:
Fold the 3x3 square flat into a 2x2 square that shows the following IPP puzzle picture:

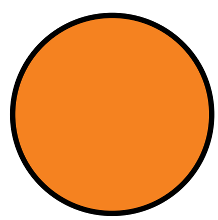


Level 4/7 (reverse side, bottom)

Level 5/7 (reverse side, top)

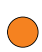






Objective:
Fold the 3x3 square into a flat structure that shows:

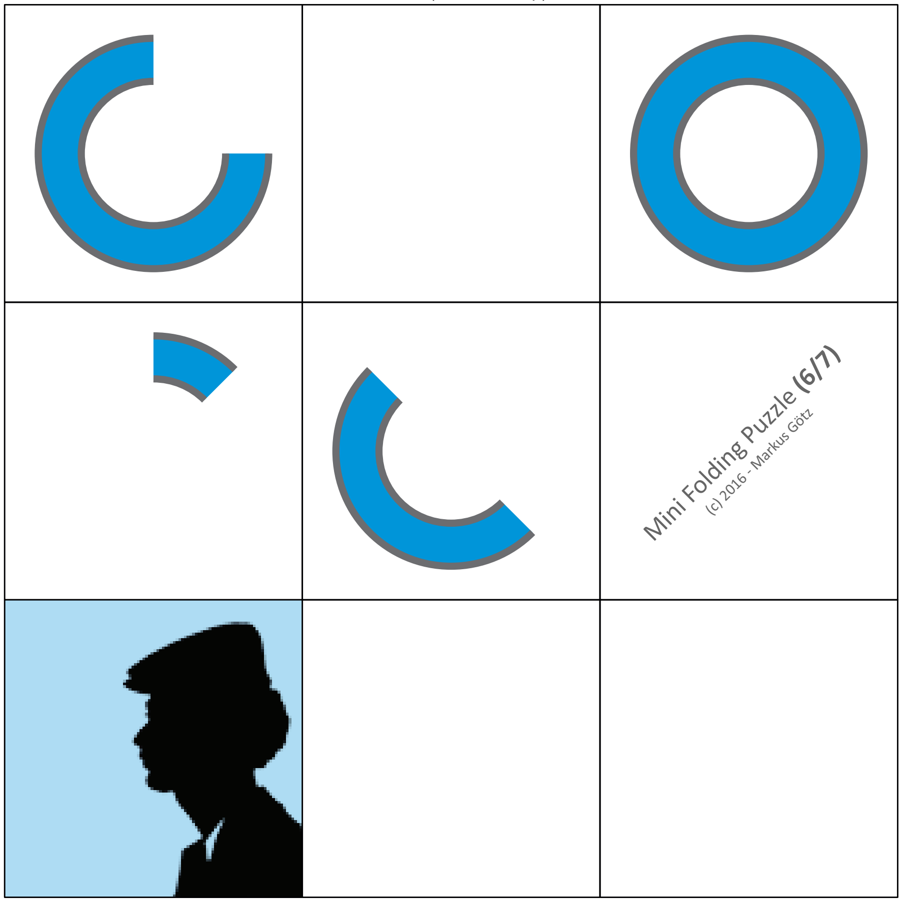
+

2 x 
in total
(front + back)

The area with the questionmark marks „not predefined structures“.

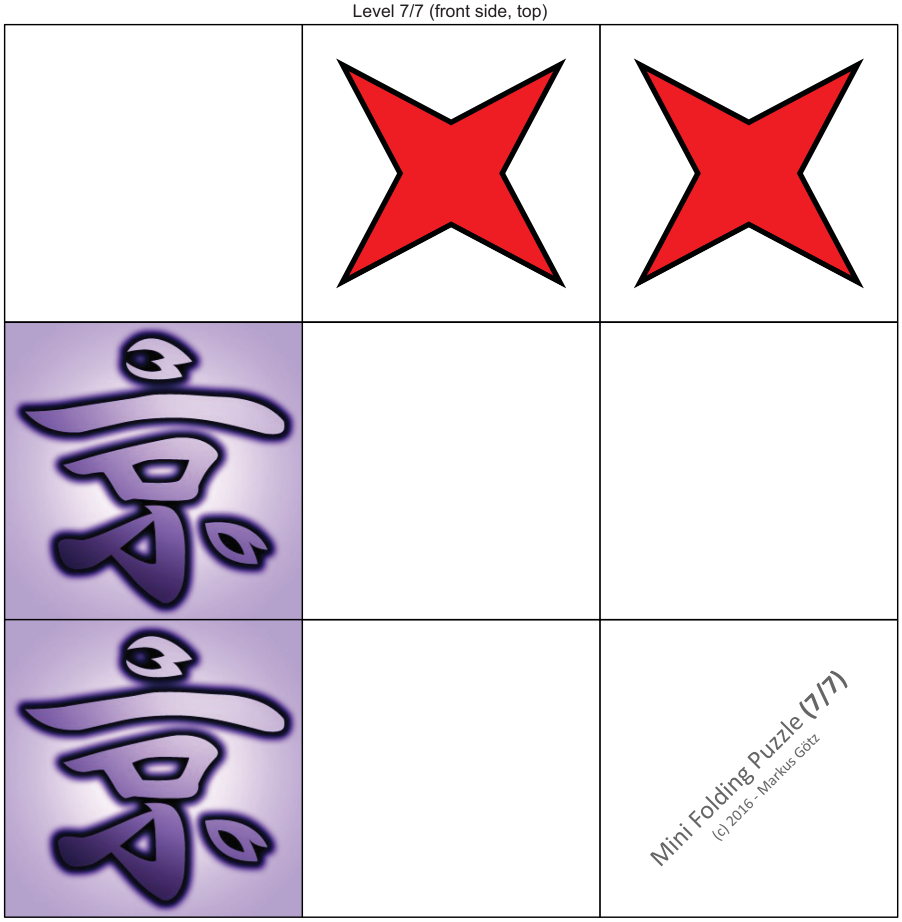
Level 5/7 (reverse side, bottom)

Level 6/7 (front side, top)



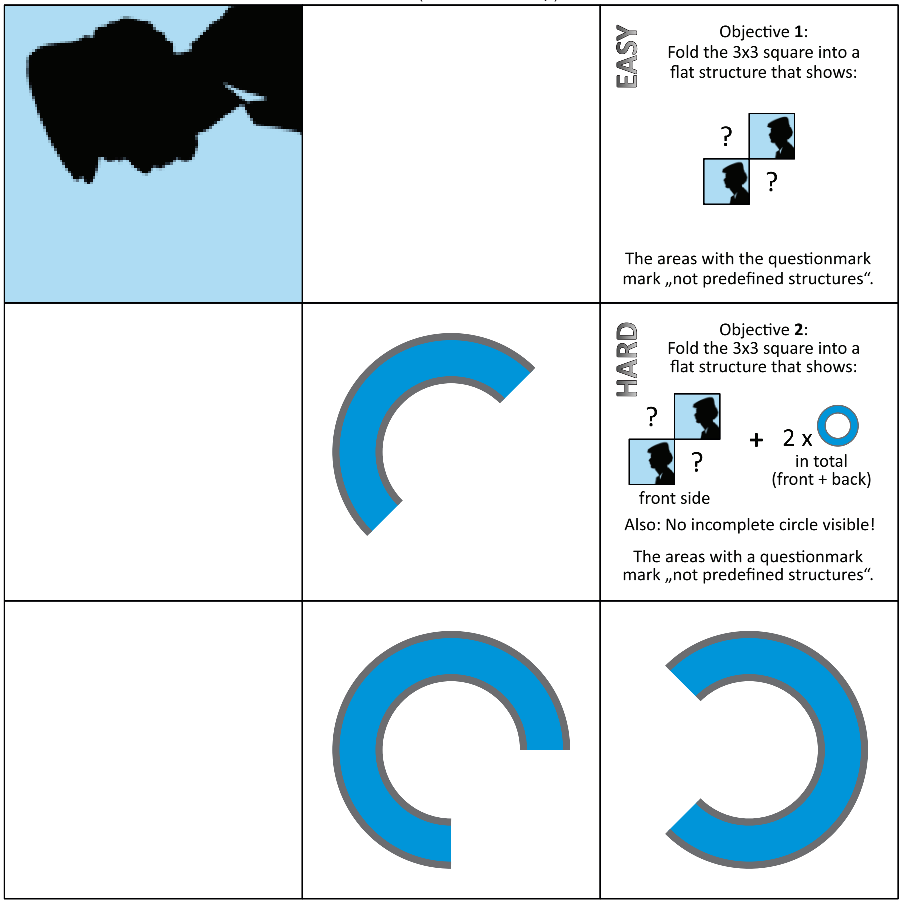
Level 6/7 (front side, bottom)

Level 7/7 (front side, top)



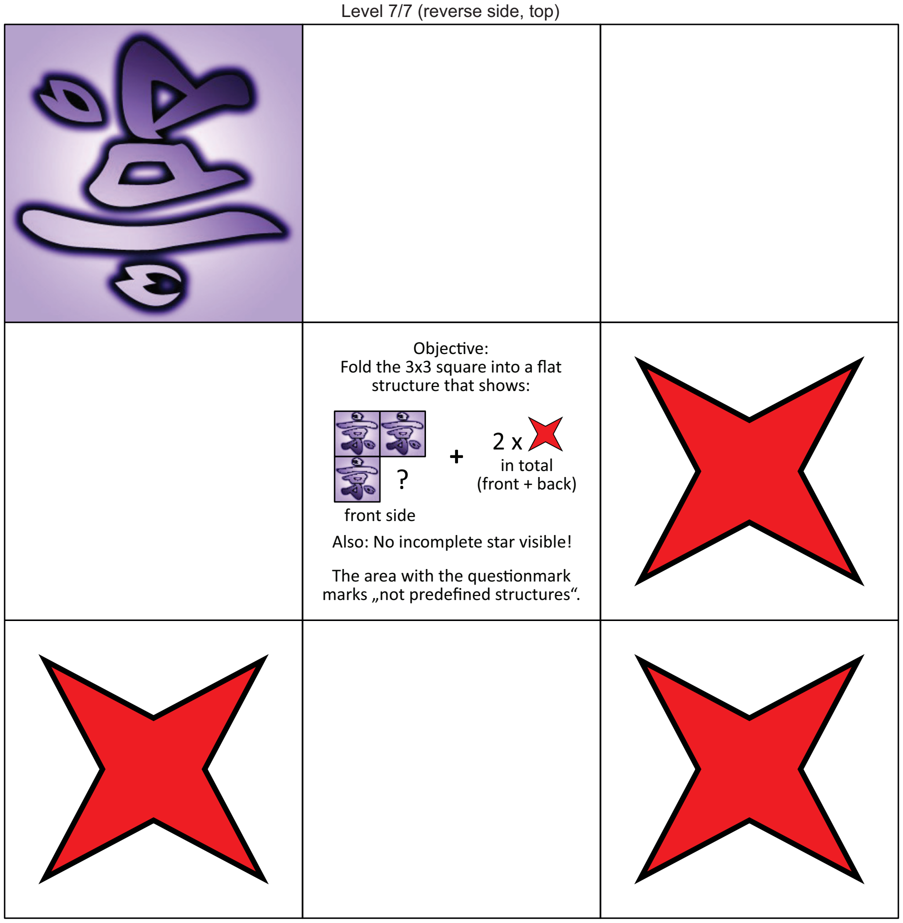
Level 7/7 (front side, bottom)

Level 6/7 (reverse side, top)



Level 6/7 (reverse side, bottom)

Level 7/7 (reverse side, top)



Level 7/7 (reverse side, bottom)

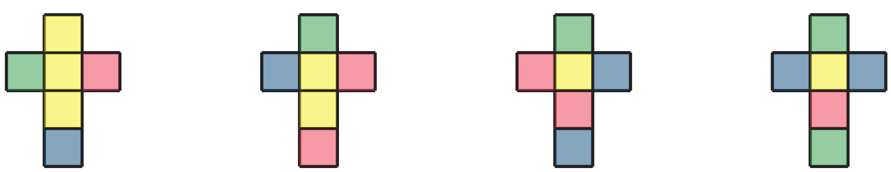
The Mutando of Insanity

by Érika. B. Roldán Roa

Puzzles based on coloured cubes and other coloured geometrical figures have a long history in the recreational mathematical literature. Martin Gardner wrote about them in: Chapter 16 of *New Mathematical Diversions*, Chapter 16 of Mathematical Magic Show , and Chapter 6 of *Fractal Music, Hypercards and More Mathematical Recreations from Scientific American Magazine* ([7, 5, 6]). One of the most commercially famous of these puzzles is the Instant Insanity that consists of four cubes. Their faces are coloured with four different colours in such a way that each colour is present in each one of the four cubes. To solve the puzzle, one needs to stack the cubes in a tower in such a way that each one of the colours appears exactly once in the four long faces of the tower. The main purpose of this paper is to study the combinatorial richness of a mathematical model of this puzzle by analysing all possible ways of colouring cubes to form a puzzle analogous to the Instant Insanity. We have done this analysis for n cubes and n colours for $n = 4, 5, 6$. This combinatorial analysis allowed us to design the Mutando of Insanity, a puzzle that we presented in a talk at the G4G 12.

Introduction

In the book *A Lifetime of Puzzles*, Rik van Grol [4] wrote a paper about the Insanity kind of puzzles. In this paper, he gave a wonderful account of the history of the "Instant Insanity" puzzle and related puzzles. The majority of the Insanity puzzles consist of n cubes, with a fixed $n \in \{4, 5, 6\}$, and the six faces of each cube coloured with one of n colours in such a way that each one of the four colours appears in each cube. To solve the puzzle one has to find a way of stacking the n cubes in a tower (a $n \times 1 \times 1$ prism) in which the n colours appear in each one of the long faces of the tower. As an example, for $n = 4$ the way in which the cubes of The Instant Insanity puzzle are coloured is depicted in the next figure (they are not the original colours of the Instant Insanity but the structure is exactly the same):



For the rest of the paper, when we refer to an Insanity puzzle we mean a collection of n coloured cubes with n colours as we described it in the last paragraph. For each fixed $n \in \{4, 5, 6\}$, there are a lot of ways of colouring one cube with n colours and even more ways to select n of these coloured cubes to have one Insanity puzzle. The Instant Insanity is just one example constructed by taking $n = 4$. There are also Insanity puzzles with $n = 5$ that have been in the market as the Hanayama, the Trench Tantalizer, and, the Allies Flag Puzzle (the configurations of these puzzles can be find in [4]).

Despite of the huge amount of possibilities to construct an Insanity puzzle, the majority of Insanity puzzles that have been commercialised have the same colouring structure (some of them use four symbols instead of four colours). I share Rik Van Grol's (and O'Bernie's [12]) amazement that people or firms bringing out and commercialising a new Insanity puzzle, do not make an effort to design their own version, but simply replace the original pictures or colours with other figures or colours. This paper is inspired by this amazement.

We have found all possible Insanity puzzles with four, five and six cubes and four five and six colours respectively. We have also classified them by their number of solutions. In doing so we have been able to answer questions like:

What is the maximum number of solutions that an Insanity puzzle can have?

Given a number s between this maximum and zero, how many Insanity puzzles have s solutions?

Once we have fixed one of these Insanity puzzles, is it possible to form a $2 \times 2 \times 1$ prism with monochromatic faces?

We have used combinatorial game theory and graph theory tools to model the Insanity puzzles. The model that we use to analyse Insanity puzzles has been inspired by an algorithm that T.A. Brow presented in [1]. He has used this algorithm to solve by hand the Instant Insanity puzzle.

For those interested in creating and designing new Insanity puzzles, we hope that with the results that we present in this paper, it will be possible to construct Insanity puzzles without using the same combinatorial structure of puzzles that are already on the market. Also, these results open the possibility of creating a multilevel Insanity puzzle that can have different puzzles varying with the number of possible solutions.

Other kind of Insanity Puzzles

There exists a whole family of variations of Insanity puzzles. Rik Van Grol presents in [4] other kind of Insanity puzzles such as On the spot Insanity (that he has designed and presented in the 23rd International Puzzle Collectors Party).

In the same paper, Rik Van Grol also mentions the Mutando puzzle that consists of four cubes with faces that have four colours but does not fulfill the requirement that all the cubes must have at least one face with each one of the colours. We present in the last section of this paper a detailed description of the Mutando puzzle. We also present a new Insanity puzzle that we called the Mutando of Insanity that is a puzzle analogous to the Mutando but that fulfils the properties of an Insanity puzzle.

In [3], Demaine et al. defined a whole different family of Insanity puzzles. These puzzles do not necessarily use cubes, they are constructed with regular prisms in such a way that all the coloured faces must apear in the solution of the puzzle. There exist other great puzzles that consists of coloured cubes and figures that are not considered as Insanity puzzles. We refer the interested reader to a book by P.A. Macmahon [11] were he has written about several coloured figure puzzles. There exists also a generalisation of the Instant Insanity to other Platonic Solids other than the cube [10].

These other kind of Insanity puzzles are very interesting and some of them are still in need of a mathematical modelling to analyse them. In this paper we are not going to analyse this other type of Insanity puzzles.

History of mathematical solutions to the Instant Insanity

In a personal conversation at G4G 12, David Singmaster gave us references about the first paper that give and analyse a mathematical model of The Tantalizer puzzle (commercialised before the Instant Insanity but with the same colouring structure) [2]. In this paper, some (at most four) undergraduate students at Cambridge University, who used to write under the pseudonym F. De Carteblanche, used graph theory to model the Tantalizer and to find the unique solution to the puzzle.

One year after the Instant Insanity was commercialised (in 1967 by Parker Brothers Division), T.A. Brown [1] used combinatorial number theory to solve the Instant Insanity.

We have found of mathematical interest that these two mathematical models are equivalent. It is possible to answer questions about possible graph structures with the combinatorial number theory structures that Brown has used and vice versa. In this paper we are not going to talk about this relationship. The mathematical model that we are going to define in the next section has a natural graph theoretical structure and the results that we have found can be settled in the language of graph theory.

As far as we know, the only two mathematical models for the Insanity puzzles that are analysing in this paper are the ones presented in [2] and [1]. Although these are the first papers in which these models were presented, there are other articles and books that talk about Insanity puzzles ([9, 8, 14, 15]). Some of these papers give a refinement or a different way of explaining what Brown and De Carteblanche have done. The results that we present in this paper were computed in less than 24 hours for $n = 4, 5, 6$ but it was proved in [13] by Eduard Robertson and Ian Munro that the generalisation of Brown's algorithm to all $n \in \mathbb{N}$ is $NP - complete$.

Mathematical Modelling of Insanity puzzles

The definitions that follow were inspired by the algorithm that Brown [1] used to solve the Instant Insanity. We do not know if the definitions and structures that we are defining in this section have already been studied, but they are a natural way of generalising the matrix structures that Brown has used in [1]. We believe that by stating the definitions in such a general framework we allow this structures to be useful not only to answer graph theoretical questions (the Insanity puzzles can be transformed in coloured and labeled graphs and be solved with graph theory) but also to be used in more general settings as combinatorial number theory, or algebraic topological structures as simplicial complexes.

From a finite set of prime numbers $\mathcal{P} = \{p_1, ..., p_n\}$, we form the set of k products

$$\mathcal{P}_k = \{p_{i_1} \cdot p_{i_2} \cdot ... \cdot p_{i_k} \mid p_{i_l} \in \mathcal{P} \text{ for } l = 1, ..., k\}.$$

For each $m_1, m_2 \in \mathbb{N}$ we define a set containing all matrices with m_1 rows and m_2 columns such that each entry of the matrix is an element of $\mathcal{P}_k$. We will denote it by $[\mathcal{P}_k]^{m_1 \times m_2}$.

The next definitions will lead us to model the specific structure of the Insanity puzzles with a set of prime numbers and the set of matrices with prime product entries that we defined above. The set of prime numbers $\mathcal{P} = \{p_1, ..., p_n\}$ will represent the set of n colours or labels that we are going to use to colour or mark the faces of the cubes. In the Insanity puzzles, as we have defined them, the number n of labels is the same as the number of cubes which determine the number $m_1 = n$ of rows in the matrices $[\mathcal{P}_k]^{m_1 \times m_2}$. The labeled structure of each cube will be represented in one row of the matrix in the following way: each entry of a matrix will represent the product of the labels of two opposite parallel faces in a cube and the three opposed faces of each cube will be represented in the three elements in one and only one row. Then, we want to take $k = 2$ and $m_2 = 3$. Because our Insanity puzzles have the property that each label must appear in at least one face of each cube, the product of the three elements of each row of a matrix representing an Insanity puzzle must be divisible by $\Pi_{i=1}^{n} p_n$. Any matrix of $[\mathcal{P}_2]^{n\times 3}$ that has this property will be called a proper matrix. It is easy to show that each possible Insanity puzzle is represented by at least one proper matrix contained in $[\mathcal{P}_2]^{n\times 3}$. We are going to denote the subset of all proper matrices by $[\mathcal{P}_2]_*^{n\times 3}$

It is clear that we can represent an Insanity puzzle with more that one proper matrix but we can define an equivalence relation to be able to have a one to one correspondence between all possible Insanity puzzles and the elements of $[\mathcal{P}_2]_*^{n\times 3}$. We are going to consider two elements in $[\mathcal{P}_2]_*^{n\times 3}$ as equivalent if we can get from one to another by permuting the rows or if we can get from one to another by permuting the entries of a row (within the same row).

Example 1 (Instant Insanity) *For the Instant Insanity we have $n = 4$, we can take $\mathcal{P} = \{2, 3, 5, 7\}$, and then $\mathcal{P}_2 = \{4, 6, 10, 14, 9, 15, 21, 25, 35, 49\}$. By labelling the faces of the four cubes of the Instant Insanity with the prime numbers in $\mathcal{P}$ instead of colours we can have the following configuration for this puzzle:*

Although we could relabel these four cubes in a different way but still have the configuration of the Instant Insanity by permuting the labels, we are going to consider as different the matrices corresponding to two different ways of labelling the Instant Insanity and we are going to take the one given above as the canonic label of the Instant Insanity.

The next three elements of $[\mathcal{P}_2]_*^{4\times 3}$ represent the Instant Insanity puzzle and are equivalent because we can get from one to the other by permuting its rows or by permuting the elements of the first and the third rows within the same rows

$$\begin{pmatrix} 14 & 25 & 15 \\ 6 & 35 & 10 \\ 6 & 14 & 15 \\ 9 & 14 & 35 \end{pmatrix}, \qquad \begin{pmatrix} 14 & 25 & 15 \\ 6 & 14 & 15 \\ 6 & 35 & 10 \\ 9 & 14 & 35 \end{pmatrix}, \qquad \begin{pmatrix} 14 & 15 & 25 \\ 6 & 14 & 15 \\ 6 & 10 & 35 \\ 9 & 14 & 35 \end{pmatrix}.$$

The next definitions will allow us to model the solutions of the Insanity puzzles. For an element $A = [a_{(i,j)}] \in [\mathcal{P}_2]^{n\times 3}$ and a *magic number* M, we define the set of M *partial solutions* for A as $V_M^A := \{\{(1, s_1), (2, s_2), ..., (n, s_n)\} \mid \Pi_{i=1}^{n} a_{(i,s_i)} = M\}$, where $1 \le s_i \le 3$ for all $i \in \{1, ..., n\}$. We define two elements of V_M^A to be *independent* if they have empty intersection. Finally, we define the set of l_M*-solutions* of A as the set $[S_M^A]_l$ containing the sets of l pairwise independent elements of V_M^A. A solution to an Insanity puzzle consists of four long faces of a tower of n cubes in such a way that each long face of the tower has each one of the n labels. Then, to find all solutions of an Insanity puzzle represented by a matrix A, we need two find the set $[S_M^A]_2$ with $M = \Pi_{i=1}^n p_i^2$. We select $l = 2$ because there are two opposite long faces in a tower built with n cubes. We choose $M = \Pi_{i=1}^n p_i^2$ because want each label to appear once in each one of the faces of the $n \times 1 \times 1$ tower, which means appearing twice when we consider two opposite faces.

Example 2 (Instant Insanity) *Taking A as the first matrix in Example 1 and $M = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2$ it is easy to prove that the only elements of V_M^A are $\{(1,2),(2,1),(3,2),(4,3)\}$, $\{(1,3),(2,3),(3,2),(4,3)\}$, and $\{(1,3),(2,2),(3,2),(4,2)\}$. Then, the set of 2_M-solutions of A with $M = 44,100$ is given by*

$$[S_M^A]_2 = \{\{(1,2),(2,1),(3,2),(4,3)\},\{(1,3),(2,2),(3,2),(4,2)\}\}.$$

We can conclude that the Instant Insanity has only one solution and we can form the solution with the cubes by taking, for example, $\{(1,2),(2,1),(3,2),(4,3)\}$ as the front and back faces of the tower and {(1,3), (2,2), (3,2), (4,2)} as the right and left faces of the $4 \times 1 \times$1 tower.

In general, for an Insanity puzzle with n labels represented by a proper matrix $A \in [\mathcal{P}_2]_*^{n\times 3}$ each element of $[S_M^A]_2$ we can form a $n \times 1 \times 1$ tower showing a solution. For this tower if we select its front, back, left, and right faces then the group S_n of permutations of four elements and the group D_4 of the symmetries of the square is acting on the tower and we have $\mid S_n \mid \times \mid D_4 \mid = n! \cdot 8$ different ways of rearranging the cubes in the tower but still have the same solution of the puzzle.

All possible coloured cubes

Before finding the number of different Insanity puzzles and the number of solutions that each one has, we need first to know all possible ways for labelling a cube with n labels. With the mathematical model that we have proposed for Insanity puzzles, what matters is which label is in front of which label in the three different opposite pair of faces of each cube. Because of this, we are going to enumerate bellow all possible sets of three elements that we can form with $\mathcal{P}_2$ for $\mathcal{P} = \{2,3,4,5\}$ when $n = 4$, $\mathcal{P} = \{2,3,4,5,7\}$ when $n = 5$ and $\mathcal{P} = \{2,3,4,5,7,11\}$ when $n = 6$. Also we need to guarantee that the product of the three elements is divisible by $\Pi_{i=1}^n p_i$ because we need each label to be present in each one of the cubes. We are going to denote this set by $[\mathcal{P}_2]_n^{1\times 3}$. It is an easy task to compute by hand and check with a computer the elements of $[\mathcal{P}_2]_n^{1\times 3}$ for $n = 4,5,6$.

For $\mathcal{P} = \{2,3,4,5\}$ the set $[\mathcal{P}_2]_4^{1\times 3}$ has 52 elements. These elements are:

1) $\{4,9,35\}$	6) $\{25,49,6\}$	11) $\{9,15,14\}$	16) $\{49,14,15\}$
2) $\{4,25,21\}$	7) $\{4,6,10\}$	12) $\{9,21,10\}$	17) $\{49,21,10\}$
3) $\{4,49,15\}$	8) $\{4,10,21\}$	13) $\{25,10,21\}$	18) $\{49,35,6\}$
4) $\{9,25,14\}$	9) $\{4,14,15\}$	14) $\{25,15,14\}$	19) $\{4,15,21\}$
5) $\{9,49,10\}$	10) $\{9,6,35\}$	15) $\{25,35,6\}$	20) $\{4,15,35\}$

21) $\{4,21,35\}$
22) $\{9,10,14\}$
23) $\{9,10,35\}$
24) $\{9,14,35\}$
25) $\{25,6,14\}$
26) $\{25,6,21\}$
27) $\{25,14,21\}$
28) $\{49,6,10\}$
29) $\{49,6,15\}$
30) $\{49,10,15\}$
31) $\{6,10,14\}$

32) $\{6,15,21\}$
33) $\{10,15,35\}$
34) $\{14,21,35\}$
35) $\{6,6,35\}$
36) $\{10,10,21\}$
37) $\{14,14,15\}$
38) $\{15,15,14\}$
39) $\{21,21,10\}$
40) $\{35,35,6\}$
29) $\{49,6,15\}$
30) $\{49,10,15\}$

31) $\{6,10,14\}$
32) $\{6,15,21\}$
33) $\{10,15,35\}$
34) $\{14,21,35\}$
35) $\{6,6,35\}$
36) $\{10,10,21\}$
37) $\{14,14,15\}$
38) $\{15,15,14\}$
39) $\{21,21,10\}$
40) $\{35,35,6\}$
41) $\{6,10,21\}$

42) $\{6,10,35\}$
43) $\{6,14,15\}$
44) $\{6,14,35\}$
45) $\{6,15,35\}$
46) $\{6,21,35\}$
47) $\{10,14,15\}$
48) $\{10,14,21\}$
49) $\{10,15,21\}$
50) $\{10,35,21\}$
51) $\{15,35,14\}$
52) $\{14,21,15\}$

For $\mathcal{P} = \{2,3,4,5,7\}$ the set $[\mathcal{P}_2]_5^{1\times 3}$ has 45 elements. These elements are:

1) $\{4,15,77\}$
2) $\{4,21,55\}$
3) $\{4,33,35\}$
4) $\{9,10,77\}$
5) $\{9,14,55\}$
6) $\{9,22,35\}$
7) $\{25,6,77\}$
8) $\{25,14,15\}$
9) $\{25,22,21\}$
10) $\{49,6,55\}$
11) $\{49,10,33\}$
12) $\{49,22,15\}$
13) $\{121,6,35\}$
14) $\{121,10,21\}$
15) $\{121,14,15\}$
16) $\{6,10,77\}$
17) $\{6,14,55\}$
18) $\{6,22,35\}$
19) $\{10,14,33\}$
20) $\{10,22,21\}$
21) $\{14,22,15\}$
22) $\{6,15,77\}$
23) $\{6,21,55\}$
24) $\{6,33,35\}$
25) $\{15,21,22\}$
26) $\{15,33,14\}$
27) $\{21,33,10\}$
28) $\{10,15,77\}$
29) $\{10,35,33\}$
30) $\{10,55,21\}$
31) $\{15,35,22\}$
32) $\{15,55,14\}$
33) $\{35,55,10\}$
34) $\{14,21,35\}$
35) $\{14,35,33\}$
36) $\{14,77,15\}$
37) $\{21,35,22\}$
38) $\{21,77,10\}$
39) $\{35,77,6\}$
40) $\{22,33,35\}$
41) $\{22,55,21\}$
42) $\{22,77,15\}$
43) $\{33,55,14\}$
44) $\{33,77,10\}$
45) $\{55,77,6\}$

For $\mathcal{P} = \{2,3,4,5,11,13\}$ the set $[\mathcal{P}_2]_6^{1\times 3}$ has 15 elements. These elements are:

1) $\{6,35,143\}$
2) $\{6,55,91\}$
3) $\{6,65,77\}$
4) $\{10,21,143\}$
5) $\{10,33,91\}$
6) $\{10,39,77\}$
7) $\{14,15,143\}$
8) $\{14,33,65\}$

9) $\{14, 39, 55\}$ 11) $\{22, 21, 65\}$ 13) $\{26, 15, 77\}$ 15) $\{26, 33, 35\}$

10) $\{22, 15, 91\}$ 12) $\{22, 39, 35\}$ 14) $\{26, 35, 55\}$

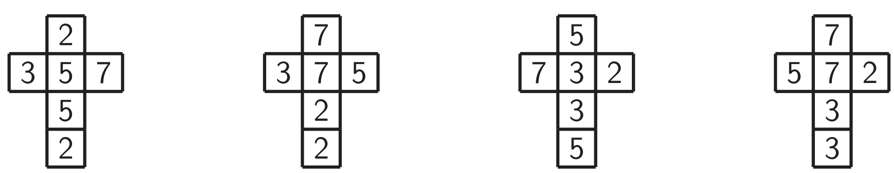
Solutions to all Insanity puzzles

The purpose of this section is to analyse all possible Insanity puzzles for $n \in \{4, 5, 6\}$. We were able to find the number of solutions to all Insanity puzzles. In particular, for $n = 4$ we can know how many Insanity puzzles can be designed that have a different abstract structure with only one solution. This gives us a whole set of different Insanity puzzles with unique solution that is different from the Instant Insanity puzzle. To form a matrix A representing an Insanity puzzle, we take four distinct elements of $[\mathcal{P}_2]_n^{1\times 3}$ as rows to form an $n \times 3$ matrix. Then we calculate the set of solutions $[S_M^A]_2$ and its cardinality will tell us how many different solutions has the Insanity puzzle represented by A. Based on the mathematical model that we have constructed for the Insanity puzzles, we have implemented an algorithm to calculate all Insanity puzzles. We have found the next results.

For $n = 4$ there exists an Insanity puzzle with 72 solutions and there are no Insanity puzzles with more than 72 solutions. It is not true that for all $0 \leq m \leq 72$ there exists an Insanity puzzle with m solutions. For $m = 13, 15, 17, 19, 22, 23, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40$ $41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69,$ 70, or $m = 71$, there is no Insanity puzzle with m distinct solutions.

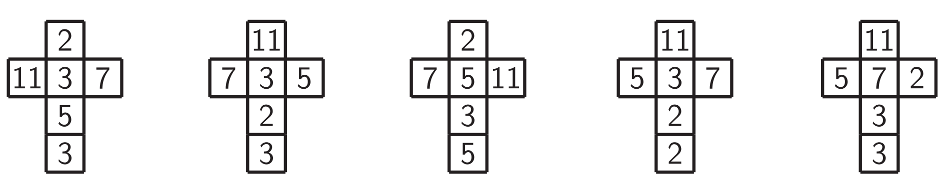
The next Insanity puzzle with four cubes has only one solution and its configuration is different from the Instant Insanity.

The next Insanity puzzle with four cubes has 72 solutions.

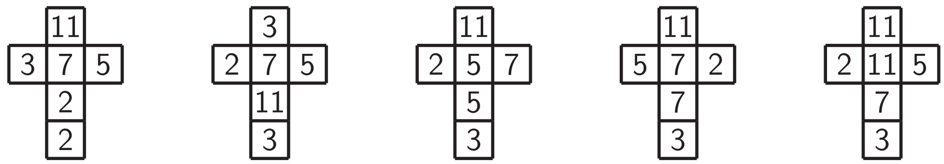


For $n = 5$ there exists an Insanity puzzle with 18 solutions and there are no Insanity puzzles with more than 18 solutions. It is not true that for all $0 \leq m \leq 18$ there exists a Insanity puzzle with m solutions. For $m = 14$ and $m = 15$ there is no Insanity puzzle with 5 cubes and m distinct solutions.

The next Insanity puzzle with five cubes has only one solution.

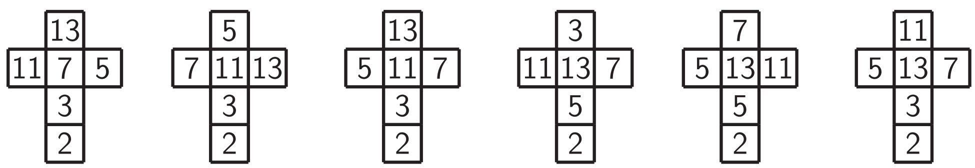


The next Insanity puzzle with five cubes has 18 solutions.

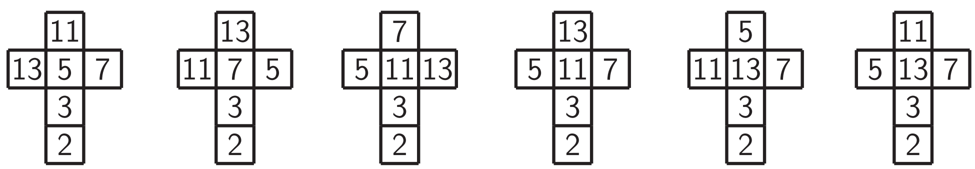


For $n = 6$ there exists an Insanity puzzle with 18 solutions and there are no Insanity puzzles with more than 18 solutions. It is not true that for all $0 \leq m \leq 18$ there exists an Insanity puzzle with m solutions. For $m = 5, 8, 10, 12, 14, 15, 16$, and $m = 17$ there is no Insanity puzzle with 5 cubes and m distinct solutions.

The next Insanity puzzle with six cubes has only one solution.

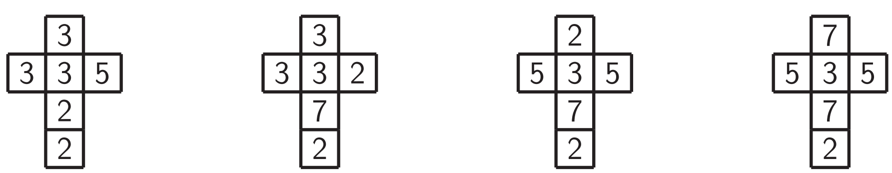


The next Insanity puzzle with five cubes has 18 solutions.



The Mutando of Insanity

The Mutando is a puzzle designed by E. Kunzell in 1997. It was commercialise in 2000 by Ingo Uhl in 2000. This puzzle consists of four cubes coloured with four colours with a structure analogous to the next four cubes.



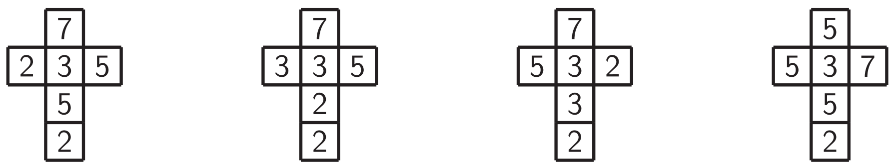
The Mutando is not an Insanity puzzle because it does not have the four labels present in the four cubes. The two puzzles that the Mutando asks to solve are:

- Puzzle 1: With the four cubes, form a $4 \times 1 \times 1$ prism in such a way that in each long face of the prism the four labels are present.
- Puzzle 2: With the four cubes form a $2 \times 2 \times 1$ prism in such a way that in each face of the prism has all square faces with the same label.

Observe that Puzzle 1 is what we ask from an Insanity puzzle to be solved. It is a natural question to ask if there exists an Insanity puzzle (all labels present in each one of the cubes) for $n = 4$ such that it is possible to solve Puzzle1 and Puzzle 2 as defined above.

Based on the same mathematical model of prime product matrices we were able to find an Insanity puzzle that has a solution to Puzzle 1 and Puzzle 2. We have already described in previous section the mathematical model that we have used to solve Puzzle 1. With the same mathematical structure, but representing with a product of two primes the labeled faces of the cube that share an edge (the dimensions of the matrices were not the same because there are 12 pair of faces of a cube sharing and edge instead of three pairs of opposite faces), we were also able to solve Puzzle 2.

This Insanity puzzle was presented in the G4G 12 as a Gift Exchange and the configuration of the four cubes is:



Acknowledgments

I would like to thank Martin Gardner for the legacy that he had let us in very different ways. Part of his legacy is G4G. I was able to assist to the G4G 12 thanks to Colm Mulcahy invitation and the generous financial support that the G4G offers to students. At the event I had the opportunity to talked about this puzzles with Rik Van Grol and David Singmaster. I thank both for the interesting conversations and the information that they generously gave me about Insanity puzzles.

References

[1] Thomas Andrew Brown. A Note on Instant Insanity. *Mathematics Magazine*, 41(4):167–169, 1968.

[2] F De Carteblanche. The coloured cubes problem. *Eureka*, 9:9–11, 1947.

[3] Erik D Demaine, Martin L Demaine, Sarah Eisenstat, Thomas D Morgan, and Ryuhei Uehara. Variations on Instant Insanity. *Space-Efficient Data Structures, Streams, and Algorithms*, pages 33–47, 2013.

[4] Erik D Demaine, Martin L Demaine, and Tom Rodgers. *A lifetime of puzzles*. CRC Press, 2008.

[5] Martin Gardner. *Mathematical Magic Show from Scientific American*. Random House Inc., 1977.

[6] Martin Gardner. *Fractal Music, Hypercards and More Mathematical Recreations from Scientific American Magazine*. W.H. Freeman,, 1992.

[7] Martin Gardner. *New Mathematical Diversions*. JSTOR, 1995.

[8] AP Grecos and RW Gibberd. A diagrammatic solution to Instant Insanity problem. *Mathematics Magazine*, 44(3):119–124, 1971.

[9] Frank Harary. On The Tantalizer and Instant Insanity. *Historia Mathematica*, 4(2):205–206, 1977.

[10] Andrews Jebasingh and Andrew Simoson. Platonic Solid Insanity. *Congressus Numerantium*, pages 101–112, 2002.

[11] Percy Alexander MacMahon. *New Mathematical Pastimes*. The University Press, 1921.

[12] Thomas Hay O'Beirne. *Puzzles and Paradoxes*. Dover, 1984.

[13] Edward Robertson and Ian Munro. Np-completeness, puzzles and games. *Utilitas Math*, 13:99–116, 1978.

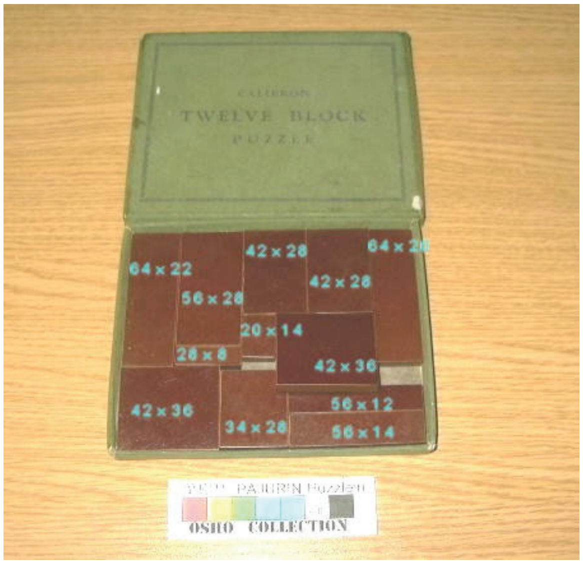
[14] BL Schwartz. An Improved Solution to Instant Insanity. *Mathematics Magazine*, 43(1):20–23, 1970.

[15] Joan Van Deventer. Graph theory and Instant Insanity. *The Many Facets of Graph Theory*, pages 283–286, 1969.

The Mystery of
The Calibron Twelve Block Puzzle

by George Miller and Nick Baxter
G4G12 Atlanta, 2016

Background



The original *Calibron Twelve Block Puzzle*, from the Osho Collection

This story starts in 1933 when the manufacturing company Calibron, run by Theodore Edison, youngest son of Thomas Edison, published a Bakelite puzzle named *Calibron Twelve Block Puzzle*, also known as the *Calibron 12*. As we will find out, this was a subtle and well thought out puzzle used in a marketing effort to spread the name of Calibron. It was presented in a rectangular box with 12 red pieces and one black spacer piece. The challenge was to arrange the 12 red pieces into a solid rectangle of unspecified dimensions. There is only one way to do this and it is very difficult.

Fast forward sixty or seventy years, the puzzle has become somewhat rare, reportedly having sold less than 200 units. One collector lucky enough to own a copy is Osho (Naoyuki Iwase), a well known collector from Japan who publishes photos of his puzzle collection online. In the case of the *Calibron 12*, he also included hand-measured dimensions of the pieces (in millimeters), so that others could reproduce the design for their personal enjoyment. But as is typical for puzzles of that age, pieces go missing, instructions get misplaced, etc., and thus some of the original subtlety of the *Calibron 12* puzzle was unknowingly lost in this presentation.

Recent Reproductions

Calibron 12 from Pavel's Puzzles

In 2010, Pavel Curtis was commissioned to reproduce the *Calibron 12* in laser-cut Acrylic, using Osho's dimensions (found on Rob Stegmann's Puzzle Page) as a guide. Soon Pavel started advertising and selling the puzzle commercially and generously etched the dimensions on each piece, taunting the solver to think that such information might actually be useful!

Not to be outdone, in 2012, Creative Crafthouse, also started publishing the same puzzle with the same dimensions using laser-cut hardwoods with a side slot for storing one of the larger pieces (see below). In this case, they knowingly provide a spoiler by giving away an important part of the challenge: showing the correct rectangle for the solution.

Calibron 12 from Creative Crafthouse

JCC/Strijbos Variation – with reversible edge

In 2014, Jean-Claude Constantin also reproduced the design under the name *Werkzeugbrett* (or *Tool Board*). This design was enhanced in 2015 by Wil Strijbos, adding a reversible edge to the tray and an extra challenge, to figure out which one of the 12 pieces to remove and then pack the now smaller rectangle with the remaining 11 pieces.

In an online puzzle forum post, Dominik Münch describes the JCC version of *Calibron 12*. He includes the dimensions of the pieces, but having given the puzzle away prior to the post, he could only measure the pieces in pixel units from a scanned image. An anonymous reader, known only as Bobson, scaled and rounded Münch's pixel measurements to get a simplified and compelling new set of measurements.

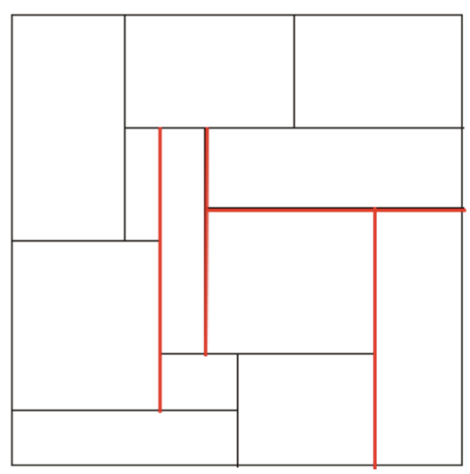
In the meantime in the Gathering for Gardner community, Jerry Slocum's exchange paper from G4G3 (1998) included a transcription of the original instructions as well as measurements of the pieces, this time in inches with two digits of accuracy.

As it turns out, these four sets of measurements are very close, proportionately, and they all appear to give the same unique solution, roughly. But it turns out they are all different. Who's right?

Rediscovery and Analysis

Last year, a Spanish puzzle collector, Primitivo Familiar Ramos, acquired three copies of the *Calibron 12*, all in virtually new condition with original instructions and spacer pieces. He discovered two astounding facts.

Armed with the original version, the Creative Crafthouse version, and a digital caliper, Ramos discovered minor discrepancies between the two versions, and then reached out to various people in the puzzle community looking for an explanation. What we eventually confirmed is that none of the previously documented measurements were correct. Ironically, the most indirect measurement was the most accurate: Bobson missed just one length by 2% rounding error!

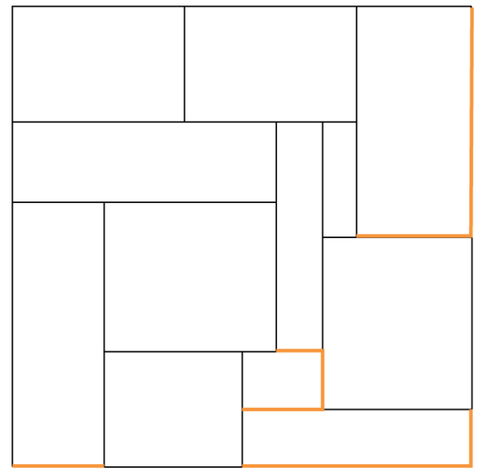


Osho – Moved Edges

But only Osho's dimensions actually work to precisely assemble a proper rectangle with no holes, meaning that all of the equivalent edge combinations are properly aligned (both sides of the 10 internal compound seams must be equal with no gaps).

To illustrate the subtle differences, above is the solution using Osho's dimensions, indicating in red the edges that are placed ever so slightly differently from the original solution. It's clear that this is just one of an infinite number of possible lookalike puzzles, where compound edges can be moved at will.

The errors inherent in the other measurements are more profound, causing the puzzle to be insoluble (in the absolute precise sense). For example, the Slocum measurements, which are the most consistently accurate, produce a "solution" with small holes, both internally, and making gaps around the outer edge, as indicated in the diagram in orange. The dimensions, despite their apparent precision, simply don't add up.



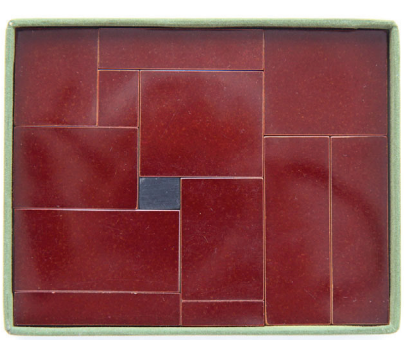
Slocum – Alignment Holes

The following table shows the accurate millimeter measurements from Ramos, the base integer units they represent, and then the other four sets of measurements with relative error (scaled as appropriate).

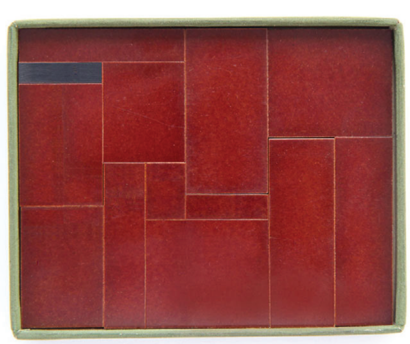
	Units		Ramos (mm)		Iwase (mm)				Dominik Münch (pixels)				Bobson				Slocum (in)			
1	7	5	19.88	14.20	20	14	**2.0%**	0.0%	234	164	**2.1%**	0.2%	1.4	1.0	0.0%	0.0%	0.8	0.6	**-1.6%**	**-1.6%**
2	10	3	28.40	8.52	28	8	0.0%	**-4.8%**	327	94	-0.1%	**-4.3%**	2.0	0.6	0.0%	0.0%	1.1	0.3	0.2%	**-1.6%**
3	12	10	34.08	28.40	34	28	1.2%	0.0%	397	327	1.1%	-0.1%	2.4	2.0	0.0%	0.0%	1.3	1.1	-0.1%	0.2%
4	15	10	42.60	28.40	42	28	0.0%	0.0%	491	327	0.0%	-0.1%	3.0	2.0	0.0%	0.0%	1.7	1.1	-0.4%	0.2%
5	15	10	42.60	28.40	42	28	0.0%	0.0%	491	327	0.0%	-0.1%	3.0	2.0	0.0%	0.0%	1.7	1.1	-0.4%	0.2%
6	15	13	42.60	36.92	42	36	0.0%	-1.1%	491	422	0.0%	-0.8%	3.0	2.6	0.0%	0.0%	1.7	1.5	-0.4%	-0.2%
7	15	13	42.60	36.92	42	36	0.0%	-1.1%	491	422	0.0%	-0.8%	3.0	2.6	0.0%	0.0%	1.7	1.5	-0.4%	-0.2%
8	20	4	56.80	11.36	56	12	0.0%	**7.1%**	655	140	0.1%	**6.9%**	4.0	0.8	0.0%	0.0%	2.2	0.5	-0.7%	0.6%
9	20	5	56.80	14.20	56	14	0.0%	0.0%	655	164	0.1%	0.2%	4.0	1.0	0.0%	0.0%	2.2	0.6	-0.7%	**-1.6%**
10	20	10	56.80	28.40	56	28	0.0%	0.0%	655	327	0.1%	-0.1%	4.0	2.0	0.0%	0.0%	2.2	1.1	-0.7%	-0.7%
11	23	7	65.32	19.88	64	20	-0.6%	**2.0%**	750	234	-0.4%	**2.1%**	4.5	1.4	**-2.2%**	0.0%	2.6	0.8	-0.5%	-0.3%
12	23	8	65.32	22.72	64	22	-0.6%	**-1.8%**	750	258	-0.4%	**-1.5%**	4.5	1.6	**-2.2%**	0.0%	2.6	0.9	-0.5%	-0.5%

Table 1. Original base units, and relative error for various measurements

Ramos' second discovery is actually much more exciting than uncovering some minor measurement errors, something that no one in recent documented history had known: Calibron actually produced *three* different versions of the puzzle! The 12 red puzzles pieces are always the same, but each type came with a differently sized black spacer piece, either 5x4, 10x2, or 20x1 units. Furthermore, the box was sized to perfectly fit the 12 puzzle pieces and any one of the spacers in a 45x36 (or 1620) unit rectangle (where the puzzle solution formed a 40x40 square).



5 x 4 Spacer



10 x 2 Spacer

20 x 1 Spacer

Thus in any of the three cases, the puzzle could be presented and stored flat and firm, as shown above, and never giving away the dimensions of the solution. In each case, the storage configuration was slightly easier than the puzzle itself, allowing for multiple assembles: 32 assemblies using the 5x4 spacer, 72 assemblies using the 10x2 spacer, and 104 assemblies using the 20x1 spacer.

So now we know that the *Calibron Twelve Block Puzzle* is not just a haphazard assembly of 12 rectangular blocks that just fit together uniquely. Rather, it is actually a very clever collection of pieces that intentionally assembles in four different but related ways, and was designed at a time long before BurrTools! It turns out that Theodore Edison was perhaps just as great an inventor, or at least as great a puzzle designer as his father!

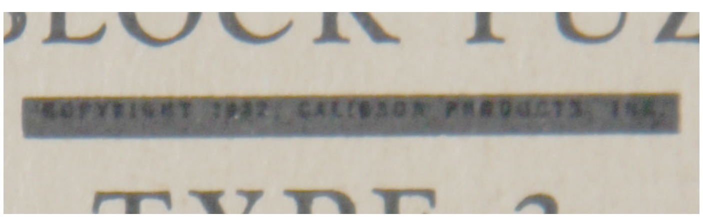
The Last Mystery Solved?

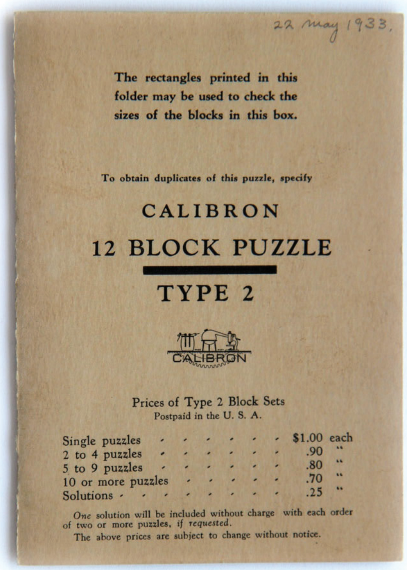
The source of the sizing confusion is still something of a mystery—why are the precise dimensions of the pieces so irregular? If they were in whole millimeters, then Osho's measurements would likely have been accurate; and if they were reasonable fractional inches, then Jerry Slocum could have nailed it. Why instead are the "correct" measurements in hundredth of a millimeter?!

One intriguing hypothesis is that the puzzle was originally designed using the base units from Table 1, and then scaled so that the solution would be a very normal 4-inch square. But for some reason, perhaps the Bakelite fabricators worked in metric, the dimensions had to be converted to millimeters. And instead of applying the correct conversion of 2.54 cm/inch, a careless typo or bad handwriting resulted in 2.84 cm/inch being used instead. If this had happened, the resulting piece dimensions would be *exactly* the irregular dimensions that Ramos discovered!

Acknowledgements

The authors are grateful to the hero of this story, Primitivo Familiar Ramos, for questioning the status quo and rediscovering the original features of the *Calibron 12*, and for allowing us to further research and tell the full story. And thanks go to all the others players and reviewers of this paper, including Jean-Claude Constantin, Pavel Curtis, David Janelle (Creative Crafthouse), Josh Jordan, Osho, Jerry Slocum, Rob Stegmann, and Wil Strijbos.





Web References

1. http://robspuzzlepage.com/assembly.htm#calibron
2. http://www.pavelspuzzles.com/2010/08/the_calibron_12block_puzzle.html
3. http://www.creativecrafthouse.com/index.php?main_page=product_info&cPath=107&products_id=844
4. http://puzzling.stackexchange.com/questions/16642/fitting-rectangles-into-square-optimal-perfect-rectangle-packing
5. https://www.puzzlemaster.ca/browse/inventors/constantin/6977-werkzeugbrett
6. http://www.mrpuzzle.com.au/toolkit-12-piece-packing-puzzle.html

AN EXCHANGE FOR
G4G12
Atlanta, March 2016

A New 12-Puzzle

By Todd Estroff
and
Jeremiah Farrell

This puzzle is a continuation of the tribute to the magician Paul Swinford. The following 18 two-letter words use each of the 12 letters of PAUL SWINFORD exactly three times each. The words are to be placed on the nodes of the grid so that each hexagon and each of the three diagonals contain the 12 letters of our honoree's name.

It is also possible to play a tic-tac-toe game with the tiles where the winner is first to select three tiles with a common letter. Two players alternately choose tiles (either at random or by choice).

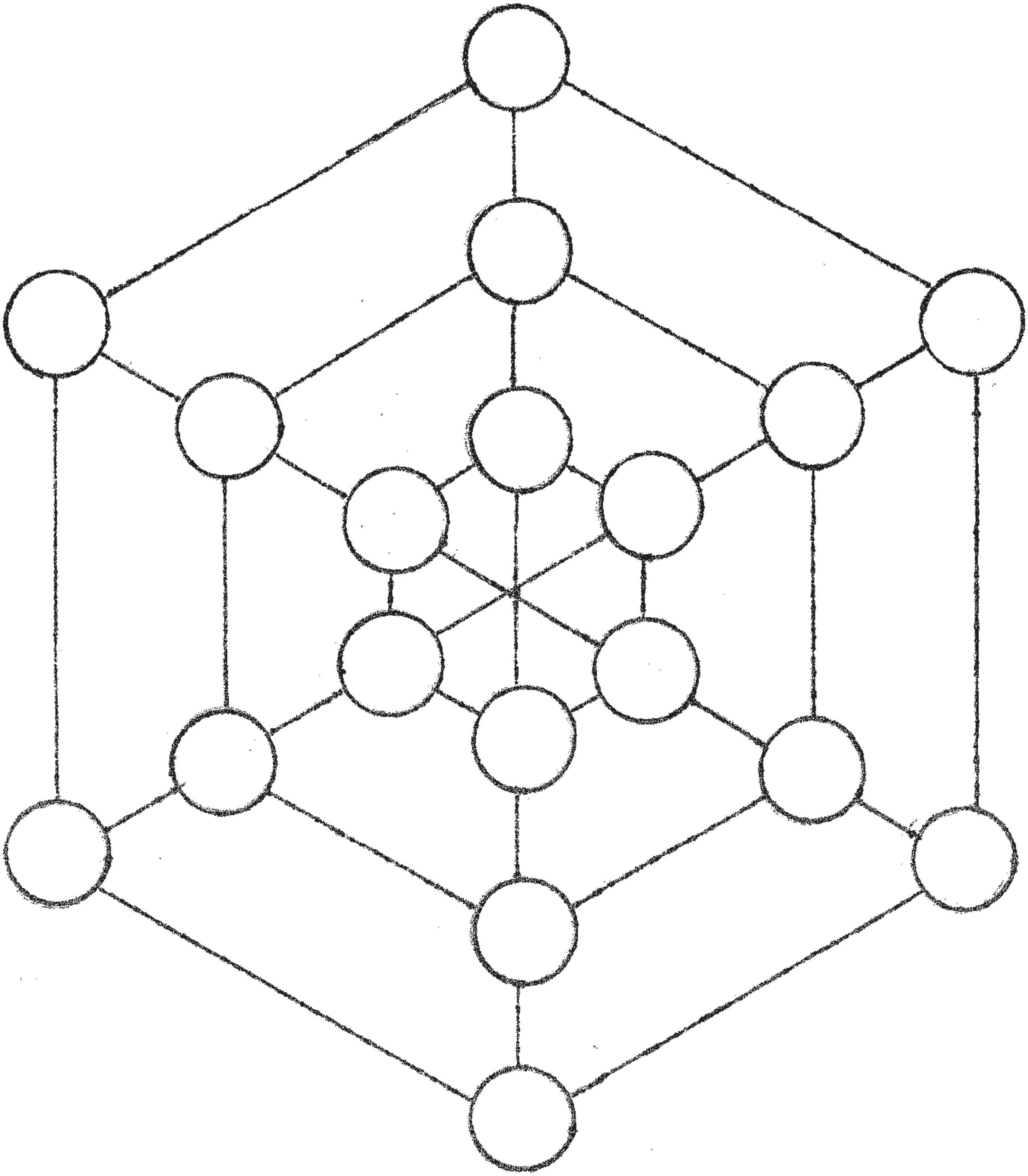
Some definitions of the more uncommon words:

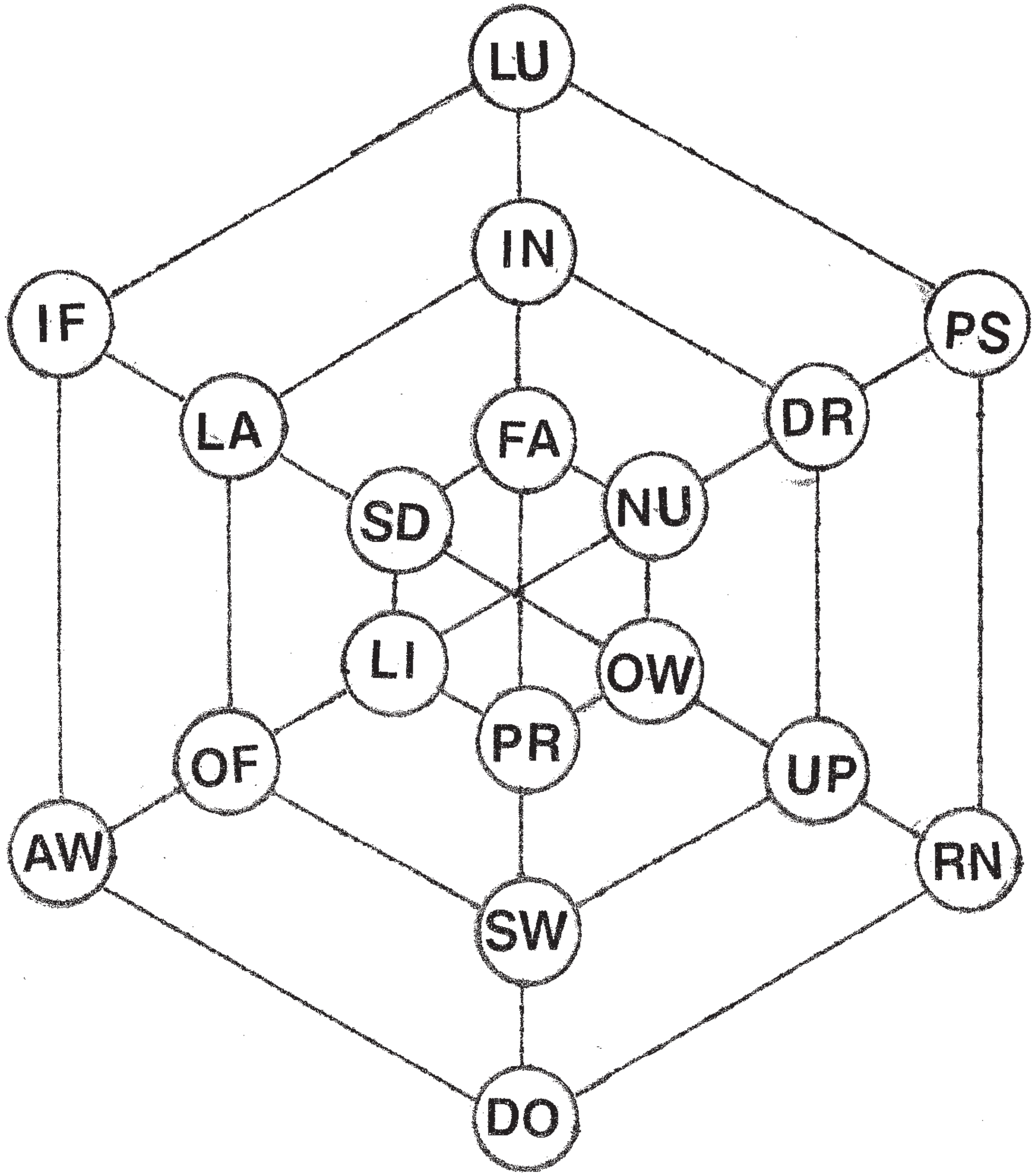
LU (chem) Lutetium	LI (chem) Lithium)	PR (chem) Praseodymium
SD South Dakota	SW South West	

AW DO DR FA IF IN

LA LI LU NU OF OW

PR PS RN SD SW UP




LU
IN
IF
PS
LA
FA
DR
SD
NU
LI
OW
OF
PR
UP
AW
RN
SW
DO

Perplexing Puzzle Poem in Pentameter

Daniel Levy
daniel.levy98@gmail.com

The message is found in a special book,
The name of which I'll have to overlook.
The author is a man we all know well;
His name is Martin Gardner, I must tell.

The page numbers come from a special draw.
The Marxists would call it most bourgeois.
They had to do the draw some 20 times
Before some guys would go and claim the prize.

For numbers two stanzas ahead, I plea:
Let us refer to one of them as "z."
I bid thee, go "z" marks and char'cters forward.
Your letter will appear; it's quite straightforward!

Please use each of these numbers as the "z."
You must go in the order that you see.
Go to the start of the page, for each set.
Don't go back to the start; finish the rest.

The first set of numbers is nine, one, seven.
The second set is ten, four, twenty; even.
The third set will be thirteen and nineteen.
The fourth is ten, twenty-four, and sixteen.

The fifth set is quite long, as you can see:
Forty-two, forty-six, three, twenty-three,
Nineteen, two, eleven, one, eight, and last:
Thirty-three. Phew, I'm glad the big set's passed.

The sixth set now, we're about to contrive:
Sixty-seven, twelve, eight, thirty-nine, five,
Sixteen, twenty-six, five, seventy-nine.
Those are the rules for you that I assign.

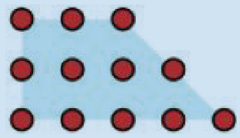
Book 1

Julia Robinson Mathematics Festival®

A Sample of Mathematical Puzzles

Compiled by Nancy Blachman, Founder, Julia Robinson Mathematics Festival

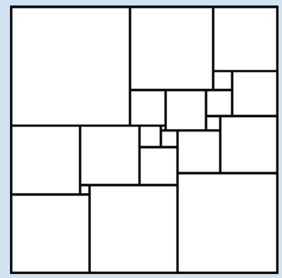
Hugs & Kisses



Trapezoidal Numbers

thesmartkitchenblog.com

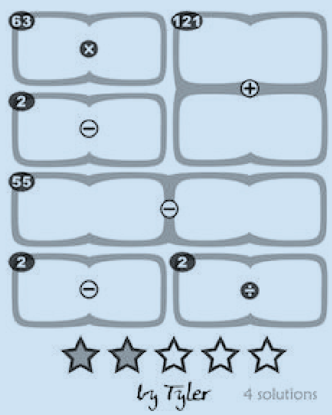
Squareable Numbers



www.MathPickle.com

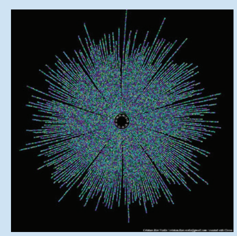
Squaring Puzzles

Switching Light Bulbs



www.MathPickle.com

Cartouche Puzzles



www.itsokaytobesmart.com

Digit Sums

"The JRMF really gets it right. Usually the best parts of mathematics are kept away from the public, as if you needed to be a mathematician to get to the fun stuff! It's refreshing to see a festival that brings this stuff to light, and in such a relaxed atmosphere. If you're lucky enough to have a JRMF near you, don't miss it! It's the best math party around."

– Vi Hart, Recreational Mathemusician, youtube.com/user/ViHart

The Festival activities are designed to open doors to higher mathematics for K–12 students, doors that are not at the top of the staircase, but right at street level.

If you are interested in volunteering, organizing or hosting a Festival, email us at info@jrmf.org

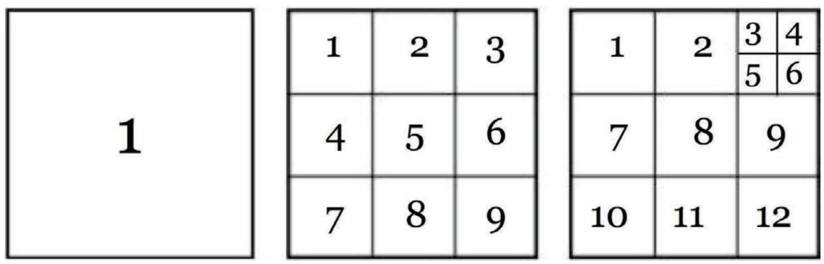
Your mom has three containers of candy for your summer treats! One container has hugs, one has kisses, and one has both. But, your little brother changed all the labels—he's even told you that every single label is WRONG. He'll let you pick one bag, and pick out one candy (blindfolded). Then, you have to figure out which candy is in which container. This chart may help you figure it out.

		Label SAYS:		
		"Kisses"	"Hugs and Kisses"	"Hugs"
In the containers				
				
				

Squareable Numbers

by Daniel Finkel and Katherine Cook, Math for Love

The number n is “squareable” if it is possible to build a square out of *n* smaller squares (of any size) with no leftover space. The squares need not be the same size. For example, 1, 9, and 12 are all squareable, since those numbers of squares can fit together to form another square.



Is there a simple way to tell if a number is squareable or not?

Which numbers from 1 to 30 are squareable? Experiment. Every time you come up with a way to break a square into some number of squares, circle that number.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Is there a pattern? Can you predict squareability in general?

Here’s why Dr. Finkel proposed this problem to Gary Antonick, who published it in the New York Time Numberplay online blog, wordplay.blogs.nytimes.com/2013/04/08/squareable.

> *I think this puzzle is amazing because it’s compelling right away, and you can work on it without worrying too much about wrong answers. If you’re trying to show 19 is squareable and can’t, maybe you’ll accidentally show 10 is squarable on the way. (Of course, neither of those numbers is necessarily squareable. No spoilers here.) It’s great to be able to experiment with a puzzle in an environment where virtually everything you do gives you some positive gains.*
>
> *I also like it because the willy-nilly approach most people start with eventually leads to a more strategic approach, and it takes a combination of deeper strategies to solve the problem. I also like it because just about anyone can get started on it, and make some serious headway —you don’t need a sophisticated math background.*

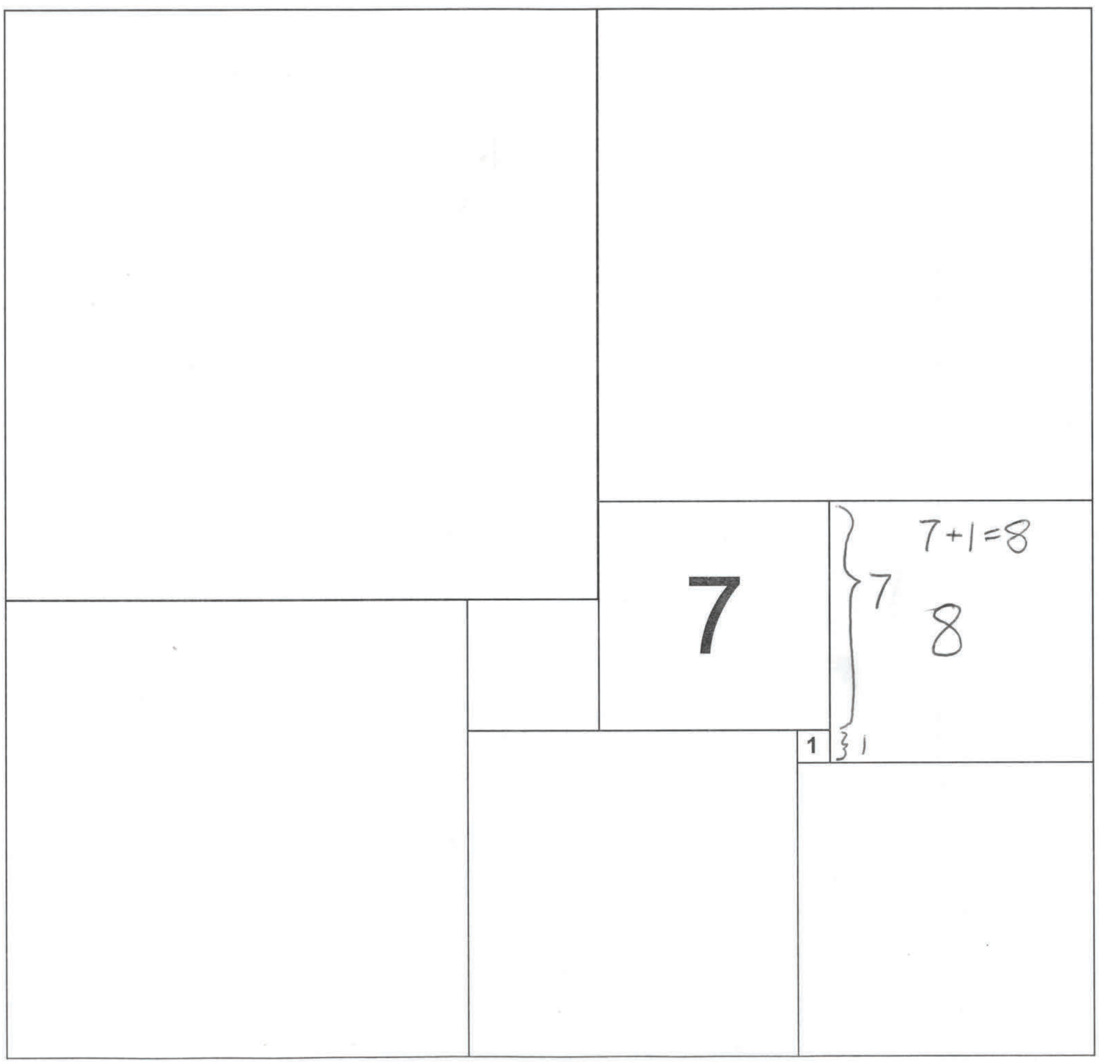
Find this and other Math for Love puzzles online at mathforlove.com/lesson-plan/.

Squaring Puzzles

by Gord Hamilton, Math Pickle

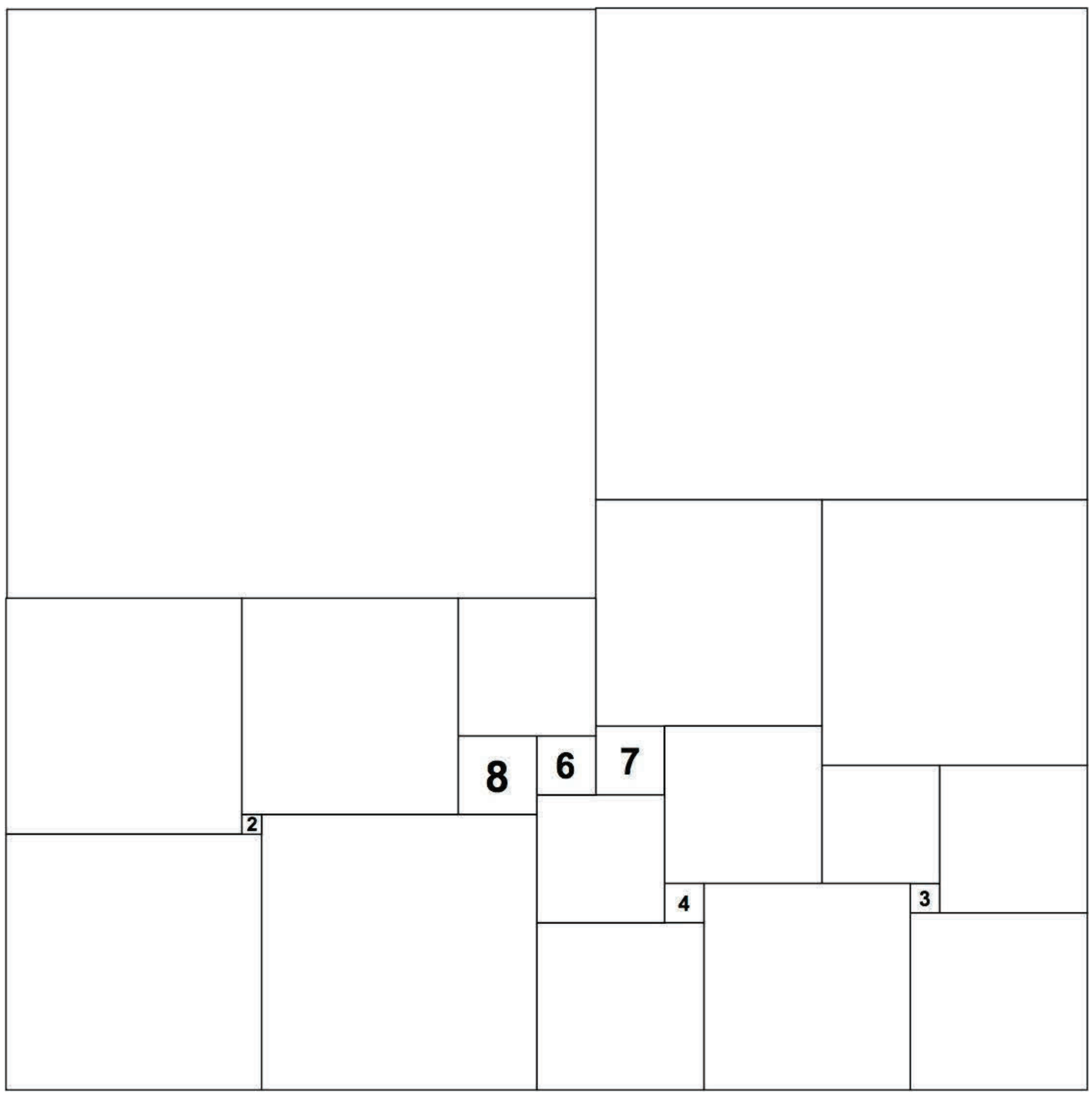
These abstract squaring puzzles give students addition and subtraction practice with numbers usually below 100. They also link these numerical activities to geometry. What a beautiful way to practice subtraction! —*Gord Hamilton, Founder of Math Pickle.*

The number in each square represents the length of a side of that square. Determine the length of a side of all the squares in this rectangle and the lengths of the sides of the rectangle.



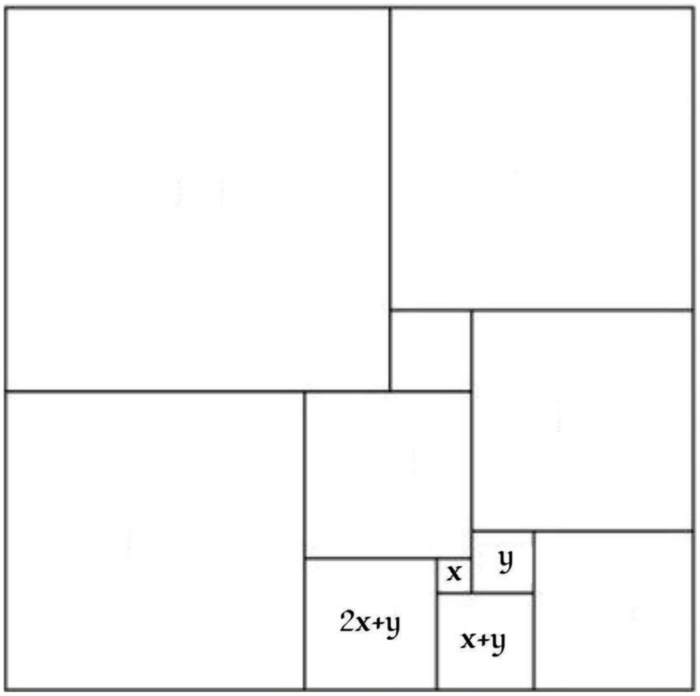
Find more square and subtracting puzzles here:
mathpickle.com/project/squaring-the-square/.

Here's a more challenging puzzle. As in the previous puzzle, the number in each square represents the length of the sides of that square. Determine the dimensions of all the squares in this rectangle and the lengths of the sides of the rectangle.



Algebra on Squares

by Gord Hamilton, Math Pickle
mathpickle.com/project/algebra-on-rectangles



Imagine all the interior rectangles are squares. The letter in each square represents the length of a side of that square.

Determine the length of a side of each square in this rectangle and write it inside the square.

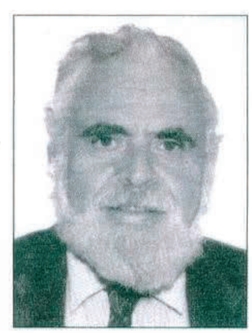
Also determine the lengths of the sides of the rectangle.

Find more of these algebra puzzles on the MathPickle link above.

If you want even more of a challenge, try the following puzzle.

Golomb's Puzzle Column™ Number 35: Rectangles With Consecutive-Integer Sides

Solomon W. Golomb



The sides (lengths and widths) of five rectangles measure each of the values 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 (units), in an unspecified order. As one of many possibilities, the five rectangles could be 1×6, 2×3, 4×10, 5×8, and 7×9, in which case their total area would be $6 + 6 + 40 + 40 + 63 = 155$ (square units).

1. How many different sets of five rectangles are possible? (The sequential order of the five rectangles does not matter, and we do not distinguish between an $a \times b$ and a $b \times a$ rectangle.)

2. What are the maximum and minimum values (A_{max} and A_{min}) for the total areas of the five rectangles?

3. Between A_{min} and A_{max}, which integer values of total areas are possible, and which are impossible?

4. There are a few sets of five rectangles (of the type we are considering) which can be assembled (without gaps or overlaps) to form a square.
   a.) Can you show, by a simple argument, that the total number of such sets of rectangles must be *even*?
   b.) Can you show that the side of any square so formed must have an *odd* length?

5. Can you exhibit any or all of the sets of rectangles, and the squares they form, as described in Problem 4?

September 1996 *IEEE Information Theory Society Newsletter*

Trapezoidal Numbers

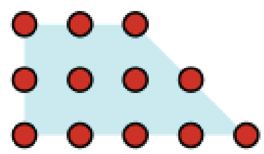
Compute

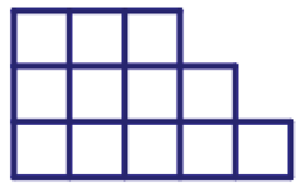
1. What is the sum 3 + 4 + 5?

2. What is the sum 4 + 5 + 6 + 7 + 8?

3. What is the sum 5 + 6 + ... + 80 + 81?

All of the results of these computations are called *trapezoidal* numbers, because you can draw a trapezoid that illustrates the answer to problem 1 with dots or blocks like this:





where each row has one more dot than the row before. So for instance 13 is trapezoidal because it is equal to 6 + 7. A trapezoidal number has to have at least two rows.

Patterns

4. What numbers can be written as 2-row trapezoidal numbers, like 13?

5. What numbers can be written as 3-row trapezoidal numbers, like 3 + 4 + 5?

6. What numbers can be written as 4-row trapezoidal numbers?

7. What about 5-row, 6-row, and so on? Can you explain a general rule, so that we can tell whether 192 is a 12-row trapezoidal number?

8. Can you name a large number that is not trapezoidal, no matter what number of rows you try? How do you know it can't be trapezoidal?

9. Can you name a large number that is trapezoidal in only one way? How do you know?

10. How many trapezoidal representations does 100 have? Why? How about 1000?

11. How many trapezoidal representations does 221 have? Why?

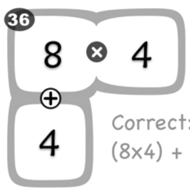
12. How can you determine how many trapezoidal representations a number has?

13. What if we allow negative numbers, like –2 + –1 + 0 + 1 + 2 + 3 + 4 + 5, in a trapezoidal representation? What if we allow "staircases" like 3 + 7 + 11?

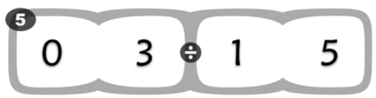
Find more Julia Robinson Mathematics Festival problem sets at jrmf.org/problems.php.

Cartouche *puzzles*

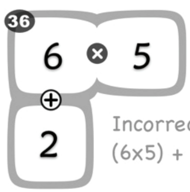
In each of these puzzles choose four digits from 0 through 9. Place one of these in each row and column. Use the operators in the white regions first, then between white regions. If there is no operator, the digits form a single number. The operator ⊖ takes the difference of the two numbers. The operator ÷ divides the bigger number by the smaller number.



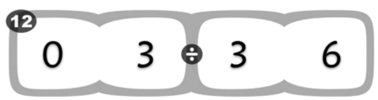
Correct:
(8x4) + 4 = 36



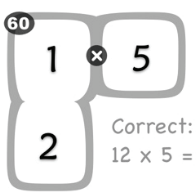
Correct:
15 ÷ 3 = 5



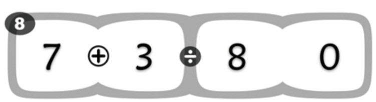
Incorrect:
(6x5) + 2 ≠ 36



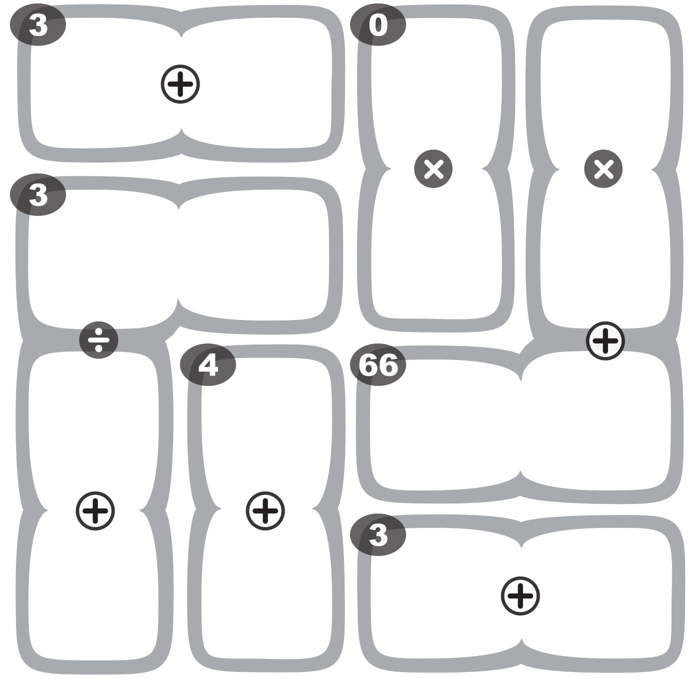
Incorrect:
36 ÷ 3 = 12
but the row contains duplicate 3s.



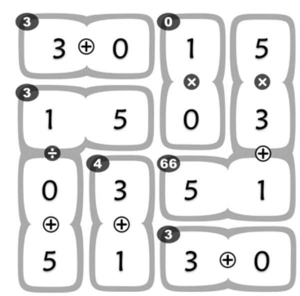
Correct:
12 x 5 = 60



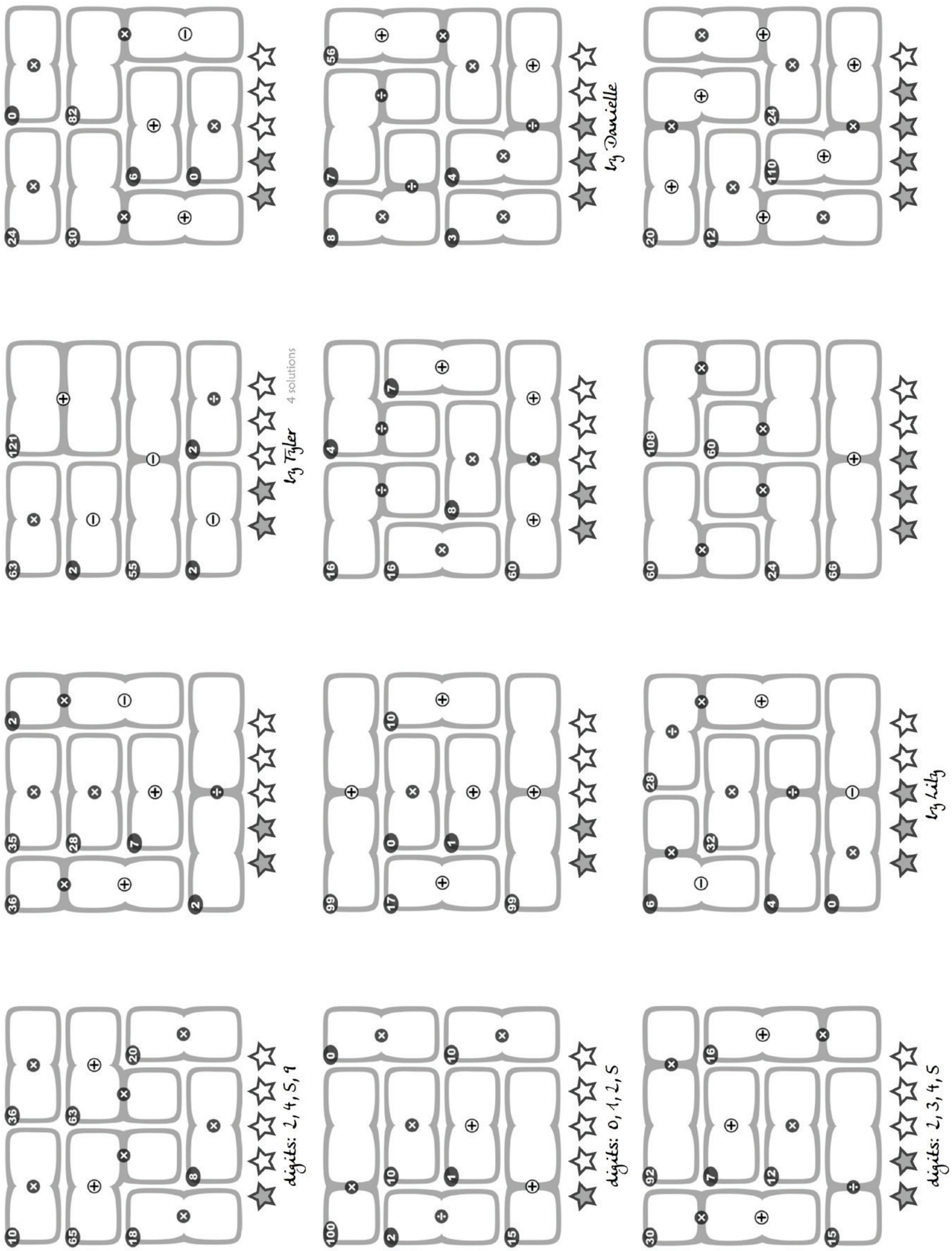
Correct:
80 ÷ (7+3) = 8



Sample Puzzle



Solution

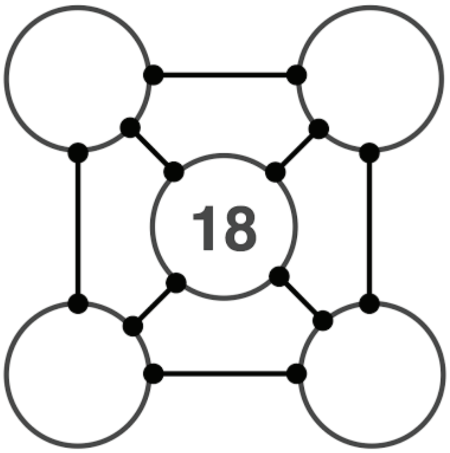


Find more MathPickle Cartouche puzzles online at mathpickle.com/project/cartouche/.

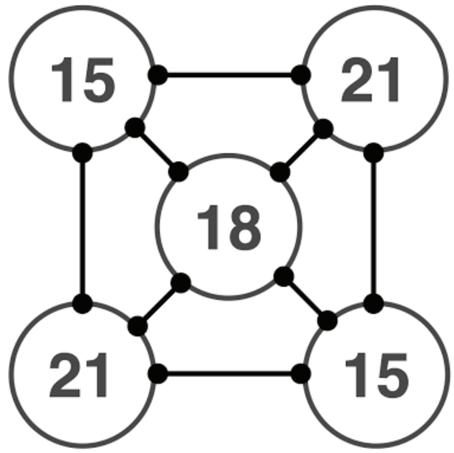
Digit Sums and Graphs

In each diagram, fill in the circle with positive whole numbers in such a way that each circle's number is the sum of the digits of all the numbers connected to it. Thanks to Erich Friedman for this idea!

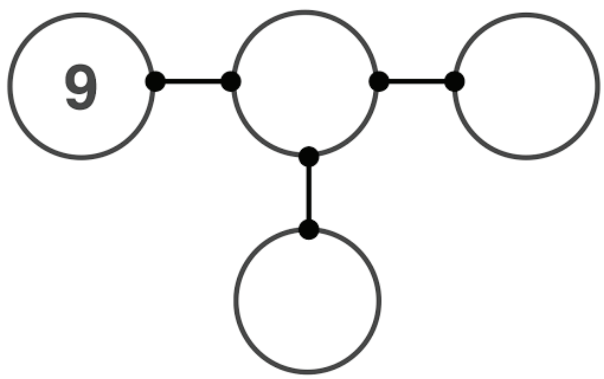
EXAMPLE

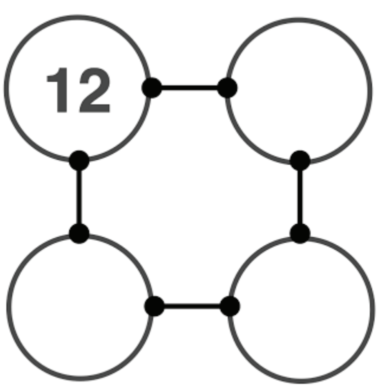


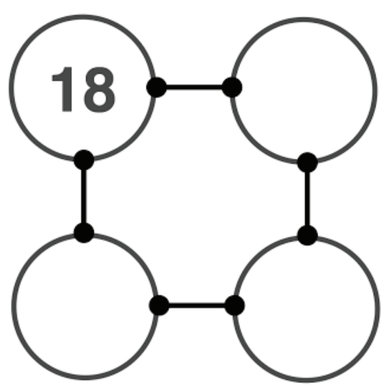
SOLUTION



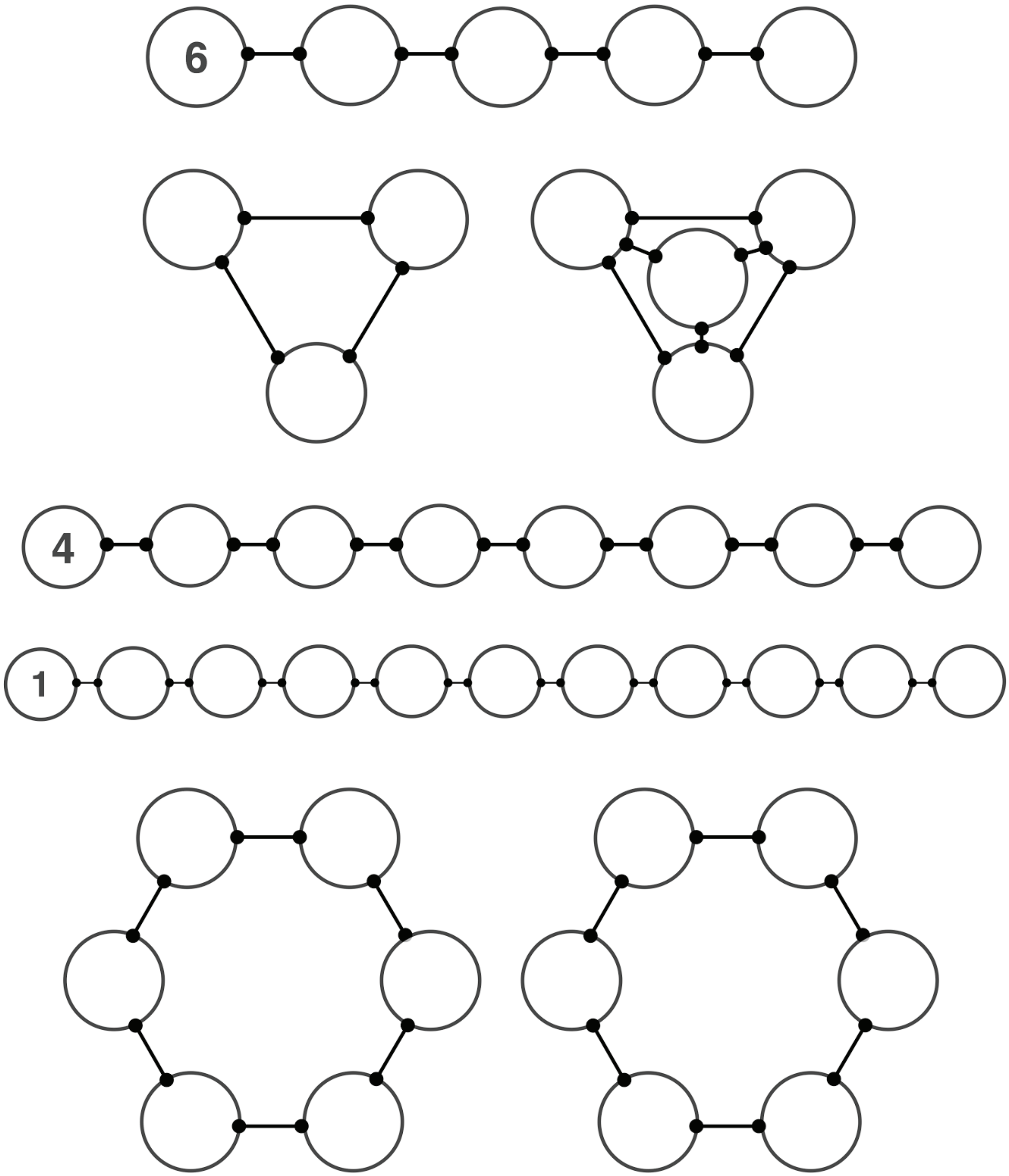
The solution works because
15 = (2+1) + (1+8) + (2+1) for the two corners
21 = (1+5) + (1+8) + (1+5) for the other two corners
18 = (1+5) + (2+1) + (1+5) + (2+1) in the center







Some of these may have more than one solution



Find more Julia Robinson Mathematics Festival problem sets at jrmf.org/problems.php.

Switching Light Bulbs

A long hallway has 1000 light bulbs with pull strings, numbered 1 through 1000. If the light bulb is on, then pulling the string will turn it off. If the light bulb is off, then pulling the string will turn it on. Initially, all the bulbs are off.

At one end of the hallway, 1000 people numbered 1 through 1000 wait. Each person, when they walk down the hallway, will pull the string of every light bulb whose number is a multiple of theirs. So, for example, person 1 will pull every string; person 2 will pull the strings of bulb number 2, 4, 6, 8, 10, …, and person 17 will pull the strings of bulb number 17, 34, 51, 68, … .

For each situation below, which light bulbs are on after all the indicated people are done walking?

1. Everyone
2. The evens, or in other words, all the people whose numbers are even.
3. The odds
4. The primes
5. The perfect squares
6. The multiples of 3
7. The perfect cubes
8. The people 1 more than a multiple of 4.
9. The people 2 more than a multiple of 4 (that is, the evens not divisible by 4).
10. Any other interesting sets you'd like to consider?
11. Given the set of people who walked, what is a general strategy for figuring out which light bulbs are turned on?

For each situation below, which people should walk in order for the indicated sets of light bulbs to end up being the only ones turned on?

12. All the bulbs.

13. The odds, or in other words, all the light bulbs whose numbers are odd.

14. The evens

15. The primes

16. The perfect squares

17. The perfect cubes

18. The multiples of 3

19. The multiples of 4

20. The multiples of 6

21. Any other interesting sets you'd like to consider?

22. Given the set of light bulbs that are turned on, what is a general strategy for figuring out which people walked?

23. For any set of light bulbs, does there necessarily exist a set of people who can walk such that the given set of light bulbs ends up being the only set turned on? If so, prove it. If not, describe the sets of light bulbs that are impossible.

24. Suppose that there are still 1000 people, but there are more than 1000 light bulbs. Not knowing which people walked, but only knowing which of the first 1000 light bulbs are turned on, what can you predict about which of the bulbs beyond #1000 are turned on?

Thanks to Stan Wagon's Macalester problem of the week for the idea behind this extension of the famous locker problem. Thanks to Glenn Trewitt and Car Talk for the idea of using light bulbs instead of lockers.

Find more Julia Robinson Mathematics Festival problem sets at jrmf.org/problems.php.

Casting Out Nines

The “digital root” of a number is the result you get if you add up its digits, and then add up the digits of that result, and so on, until you end up with a single digit. For instance, the digital root of 44689 is computed by finding that $4 + 4 + 6 + 8 + 9 = 31$, and then $3 + 1 = 4$ gives you a single-digit answer.

1. Let's look at two numbers that add up to 44689, such as 31847 and 12842. What relationship can you find among the digital roots of these numbers?

2. What about two numbers that subtract to make 44689, like 83491 and 38802? Is there a relationship among their digital roots? What can you do with 100000 and 55311?

3. What about two numbers that multiply to make 44689, like 67 and 667? Or two other numbers that multiply to make 44689, like 23 and 1943?

4. The process of taking the digital root is called “Casting out nines” for a reason: what you're actually doing in computing the digital root is another way of determining the remainder when you divide by 9. In other words, you keep throwing away multiples of 9 until you're eventually left with a number smaller than 9. Well, that's not quite true: why not?

5. In the original example of 44689, we obtained 31 after the first step. Let's see the 9s disappearing as we go from 31 to 3 + 1: 31 means $3 \times 10 + 1$ which is the same as $3 \times 9 + 3 \times 1 + 1$, so after throwing away the 9s we have $3 \times 1 + 1$, which finally is $3 + 1$. Can you give a similar explanation for how 44689 turns into $4 + 4 + 6 + 8 + 9$ after throwing away a lot of 9s?

6. One of the major uses of casting out nines is to check arithmetic quickly. If your calculation (like in the first few problems here) doesn't match up, then you know there was an arithmetic mistake. Which of the following can be proved wrong by casting out nines? Are the other ones actually correct?
   a) $1234 + 5678 = 6812$
   b) $12345 - 9876 = 2469$
   c) $10101 - 2468 = 7623$
   d) $1234 \times 5678 = 7006652$
   e) $4321 \times 8765 = 37783565$
   f) $345 \times 543 = 196335$
   g) $2^{17} = 130072$ (warning! How should you handle exponents? Think about this very carefully!)

7. On the other hand, certain kinds of mistakes will never be found by casting out nines. Can you give some examples of these? Examples that might be common?

8. Why is this process a bad idea for division when it works so well for addition, subtraction, and multiplication? Give an example where casting out nines seems to be "wrong" even though the answer is correct.

9. On the other hand, you can use casting out nines to check division problems by rewriting them as multiplication and addition. How would you rewrite "23894 divided by 82 is 291 with a remainder of 32" using only multiplication and addition, so you could then check it by casting out nines?

10. Another way to think about casting out nines is that as you add 9 to a number, you increase the tens digit by 1, and decrease the ones digit by 1, so adding 9 won't change the digital root. What is the flaw in this logic? Can you repair it?

11. Casting out nines has some other interesting applications as well. What is the digital root of 3726125? Can you use that information to explain why 3726125 is not a perfect square?

12. You can also cast out elevens instead of nines. Start with the rightmost digit, and alternately add and subtract. So with 44689 you'd take $9 - 8 + 6 - 4 + 4 = 7$. If you end up with a negative number, remember you're casting out elevens, so just add 11 as many times as you'd like. Can you explain why this process works?

13. There are some common mistakes that you wouldn't be able to catch with casting out nines, but you can catch by using casting out elevens. Give at least one example.

14. There's a magic trick that is most often done using a calculator. Pass the calculator around the room, and each person types in one digit and presses the multiplication key. After a while, the calculator screen is full of digits. The person holding the calculator at that point eliminates any one digit 1 through 9 (not 0), and then takes the remaining digits and writes them in any order. For example, they might write 3004129. Then, a mathematician almost instantly says what the missing digit is. Which digit is missing? How could the mathematician know? But sometimes the mathematician is wrong. Why?

15. What is the digital root of 4444^{4444}? Can you determine how many times you will have to sum the digits before obtaining a single digit answer?

For more mathematical puzzles, visit ...

NRICH promotes the learning of mathematics through problem solving. NRICH provides engaging problems, linked to the curriculum, with support for teachers. (Grades K-12)
nrich.maths.org

Julia Robinson Mathematics Festival

Explore the richness and beauty of mathematics through puzzles and problems that encourage collaborative and creative problem-solving. (Grades K-12)
jrmf.org

Empowering minds through play. (Grades K -12)
thinkfun.com

Alex Bellos' Monday Puzzle. (Grades 5-Adult)
www.theguardian.com/science/series/alex-bellos-monday-puzzle

Wild Maths is mathematics without bounds. Visitors are free to roam and develop as mathematicians. (Grades K-12) wild.maths.org

Cool math problems that are beautiful and thought provoking. Favorite lessons and complex problems. (Grades K-6)
mathforlove.com/lesson-plan/

While a standard textbook cannot adapt to each individual learner, expii.com was created to do just that. (Grades 5-12) expii.com and expii.com/solve

Galileo.org strives to inspire a passion for learning. (Grades K-12)
galileo.org/classroom-examples/math/math-fair-problems

Gord Hamilton has a passion for getting students to realize that mathematics is beautiful. (Grades K-12)
MathPickle.com

On the NY Times website, Numberplay generally presents mathematical and/or logical puzzles and problems. (Grades 5-Adult)
wordplay.blogs.nytimes.com/category/Numberplay

Brilliant's problems are created by people all over the world. Members learn how to solve problems by engaging in a vibrant community. (Grades 2-Adult) brilliant.org

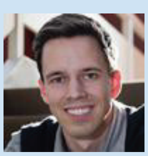
Interactive mathematics miscellany and puzzles. (Grades 1-Adult)
cut-the-knot.org

MathsChallenge.net is a website dedicated to the puzzling world of mathematics. (Grades 4-Adult)
MathsChallenge.net

Project Euler offers for free engaging computation problems that will require more than just mathematical insights to solve. (Grades 5-Adult) projecteuler.net

Math Central is an award-winning website with investigations for teachers and students. (Grades 7-12)
mathcentral.uregina.ca/mp

G4G features puzzles, games, magic tricks, and crafts. (Grades K-Adult)
celebrationofmind.org/puzzles_games



Dan Meyer has created problems and videos to inspire students to solve problems. (Grades 4-12)
blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story

Youcubed's main goal is to inspire, educate, and empower teachers of mathematics, by providing accessible and practical materials. (Grades K-12) youcubed.stanford.edu/tasks

A resource for educators passionate about improving students' mathematics learning and performance. (Grades K-12)
insidemathematics.org

The Grabarchuk family produces puzzles for websites, mobile devices, and books. (Grades 4-12) GrabarchukPuzzles.com

Secret Messages in Juggling and Card Shuffling

Erik D. Demaine* Martin L. Demaine*

Abstract

We present several puzzles around the idea of hiding messages within two types of mathematical performance: juggling and card shuffling.

1 Introduction

Over the past dozen years, we have developed several different typefaces/fonts that express text through mathematical theorems or open problems in broadly accessible forms, often through the use of puzzles. The fonts are all free to play with on the web.[1]

Two of our most recent such fonts are based on ball-juggling patterns and perfect (Faro) shuffling of cards. These fonts were originally prepared for the 80th birthday party of Ron Graham [DD], who Martin Gardner wrote about on several occasions and to whom Martin dedicated his book *Wheels, Life and Other Mathematical Amusements*. Here we explore how to develop standalone puzzles based on variations of these fonts.

2 Juggling Font Puzzles

Our juggling font[2] consists of one three-ball juggling pattern for each letter of the alphabet. The idea is that the trajectories of the balls trace out the desired letter. But seeing just the pattern animated, it is a puzzle to figure out the letter:

Puzzle 1 *The juggling patterns in Figure 1 spell what word?*

Puzzle 2 *The juggling patterns in Figure 2 spell what word?*

3 Card Shuffling Font Puzzles

In our card-shuffling font,[3] the magician starts with a sorted deck of 26 cards labeled A through Z, and given a word (e.g., from the audience), the magician can bring the word's letters to the top of the deck, one at a time in order, by repeatedly shuffling the deck. The trick is to use *perfect shuffles* (often called *Faro shuffles*), which are riffle shuffles where the cards exactly alternate between the two exact halves of the deck [Gar89]. The magician carefully chooses between making *outside* and

*MIT Computer Science and Artificial Intelligence Laboratory, 32 Vassar St., Cambridge, MA 02139, USA, {edemaine,mdemaine}@mit.edu

[1] http://erikdemaine.org/fonts/

[2] http://erikdemaine.org/fonts/juggling/

[3] http://erikdemaine.org/fonts/shuffle/

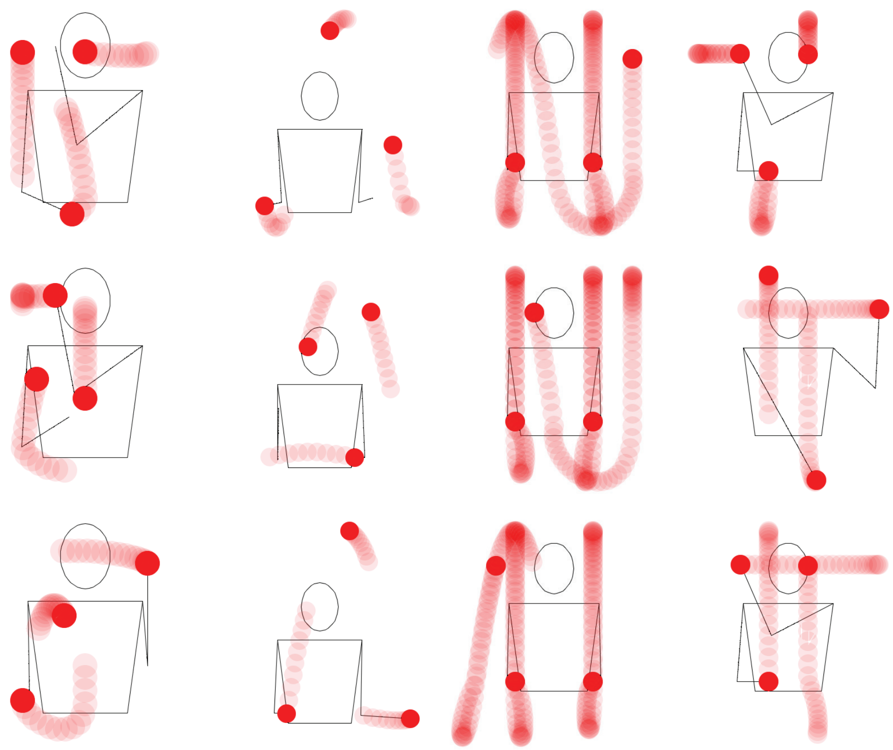


Figure 1: Four juggling patterns representing four letters, with time looping vertically. Underlying animations produced with Juggling Lab [B+14].

inside perfect shuffles, that is, choosing which half of the deck to start the shuffle with in order to keep the outside (top and bottom) cards on the outside or move them one card inside, respectively. Using an algorithm of Diaconis and Graham [DG07], any card can be brought to the top within $\lceil \log_2 26 \rceil = 5$ perfect shuffles. Figure 3 shows how this works to produce the initials of Martin Gardner.

Puzzle 3 *Starting from a sorted deck of 26 letters (A on top, Z on bottom), what card is on top after one inside perfect shuffle?*

Puzzle 4 *Starting from a sorted deck of 26 letters (A on top, Z on bottom), what card is on top after two inside perfect shuffles?*

A new and different type of trick involves stacking (re-arranging) the deck so that few operations produce large effects.

Puzzle 5 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are M, A, T, H, with M on top.*

Puzzle 6 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are M, A, R, T, I, N, with M on top.*

Puzzle 7 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are G, A, T, H, E, R, with G on top.*

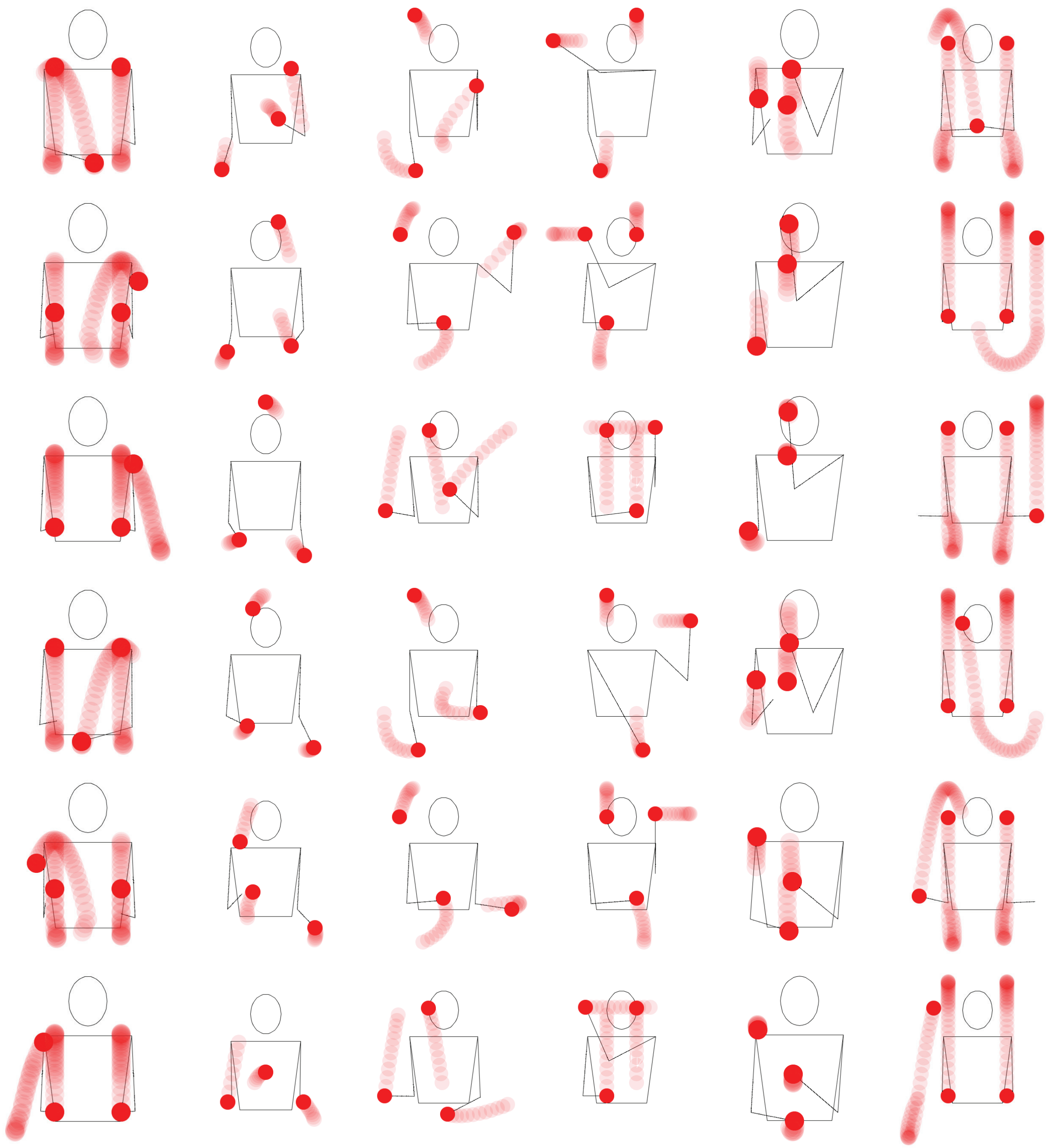


Figure 2: Six juggling patterns representing six letters, with time looping vertically. Underlying animations produced with Juggling Lab [B+14].

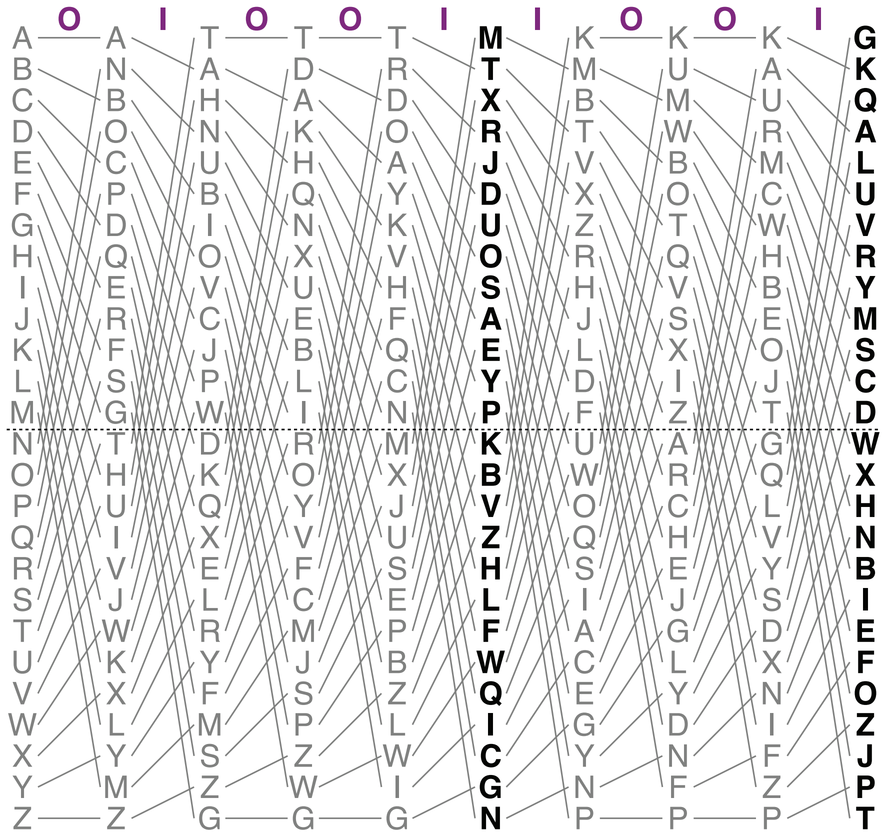


Figure 3: Starting from the sorted alphabet deck on the left, five perfect shuffles bring the letter M to the top, and four subsequent shuffles bring the letter G to the top. I and O denote inside and outside perfect shuffles, respectively.

References

[B+14] Jack Boyce et al. Juggling lab. http://jugglinglab.sourceforge.net/, 2014.

[DD] Erik D. Demaine and Martin L. Demaine. Juggling and card shuffling meet mathematical fonts. In *Connection in Discrete Mathematics: A celebration of the work of Ron Graham*. Cambridge University Press. To appear.

[DG07] Persi Diaconis and Ron Graham. The solutions to Elmsley's problem. *Math Horizons*, 14:22–27, February 2007.

[Gar89] Martin Gardner. Card shuffles. In *Mathematical Carnival*, chapter 10. Mathematical Association of America, 1989.

Puzzle Solutions

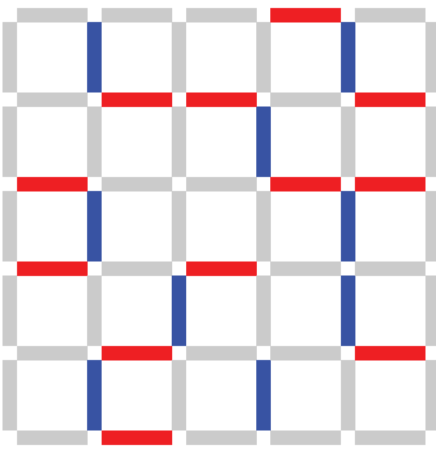
1. FONT • 2. MARTIN • 3. N • 4. G • 5. AHCDEFGBIJKLNMTPQRSOUVWXYZ
6. ATNDEFGHPIJKLCMRIQOSBUVWXYZ • 7. AHRDPFNBIJKLMGTEQCSOUVWXYZ

Andrea Gilbert

A collection of six "step-over" logic mazes.
Exchange gift for G4G12 Atlanta. March 2016.

This paper offers a collection of simple "step-over" logic mazes, where each maze is based on a different logical rule. The mazes in this collection illustrate not only how adaptable the "step-over" rule can be, but also how complex such logic mazes can be even on small (4x4 or 5x5) grids. These mazes work in small spaces and can be easily presented as walk-round floor mazes, for inside or outside parties and gatherings. For instance ...

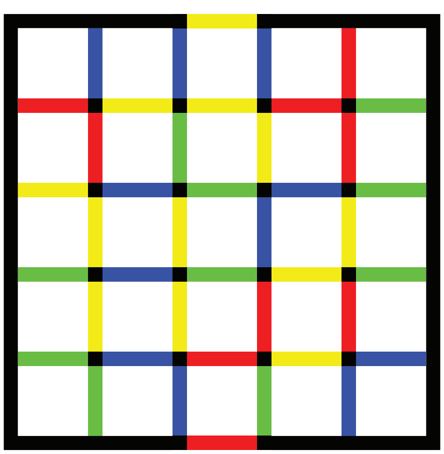
* Draw on tarmac or concrete using coloured chalk
* or ... use strips of bright-coloured sticky insulating tape
* or ... masking tape on carpet, coloured with highlighter pen
* or ... rivet/sew together strips of webbing and pin down on grass



Red-Blue Stop-dead Maze (new for 2016)

START : Step in at the bottom over red.
GOAL : Step out at the top over red.

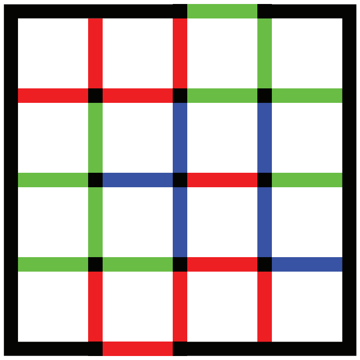
RULE : When you step over a red or blue line stop dead immediately. Then turn either left or right and walk in a straight line until you step over another red or blue line (you will alternate red/blue). Never step over the same colour twice and only walk in straight lines. Remember to stop-dead each time you step over a red or blue line.



Stick-with-it Maze (new for 2016)

START : Step in at the bottom over red.
GOAL : Step out at the top over yellow.

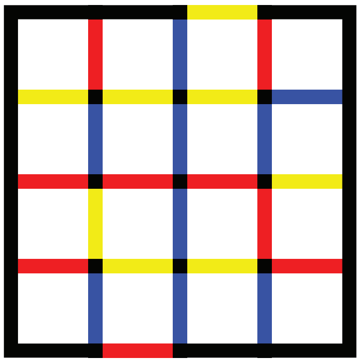
RULE : The colour you select on your first step you must stick with until it runs out. So start with red and keep steppng over red until there is no new red line to step over. Then choose a new colour and stick with that one until it also runs out. Repeat as needed.



Three-step Maze (new for 2016)

START : Step in at the bottom over red (first step of three).
GOAL : Step out at the top over green (third step of three).

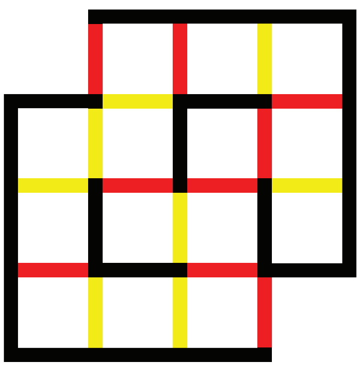
RULE : Take three steps over red, then change to a new colour and repeat. You must change colour every third step. Keep moving forward (don't step back). If you can't take full three steps in one direction, try a different direction. You must exit on a third step.



Red-Blue-Yellow Sequence Maze

START : Step in at the bottom over red.
GOAL : Step out at the top over yellow.

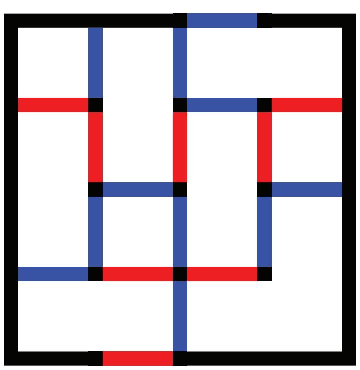
RULE : Step over red then blue then yellow in strict sequence. Repeat as needed; red, blue, yellow, red, blue yellow ...



Pair-wise Maze (for two people)

START : Enter in opposite corners.
GOAL : Meet in any square.

RULE : Each player must step over the same colour, at the same time, into an adjacent square. Step over either red or yellow (black is a wall). Agree your move then step together, repeat until you can meet in the same square. Neither player can forfeit their step but backtracking is allowed.



Red-Blue Alternating Maze

START : Step in at the bottom over red.
GOAL : Step out at the top over blue.

RULE : Step over red then blue in strict alternating series. You can move freely within each open area.

Doug McKenna

The Sleeping Beauty Paradox Resolved

Abstract

The Sleeping Beauty problem is a famous open paradox about probabilities that has divided and polarized communities of mathematicians – probability theorists, decision theorists and even philosophers – for over fifteen years. This simply stated problem in self-locating belief, involves a rational agent undergoing amnesia (https://en.wikipedia.org/wiki/Sleeping_Beauty_problem). It appears like a seemingly simple puzzle in subjective probability. It has two possible solutions, one-half and one-third, both of which have staunch adherents, the “halfers” and the “thirders.” Using frequentist methods, the answer clearly seems to be one-third (Elga 2000); yet, this equally clearly violates a basic natural Bayesian assumption that belief should not change in the absence of new evidence (Lewis, 2001). The passion generated in arguments between these two entrenched camps, which I recently experienced firsthand in a column for Quanta magazine (https://www.quantamagazine.org/20160114-sleeping-beautys-necker-cube-dilemma/) puts political ideological debates to shame. This has led several commenters to examine and question the validity of subjective probability or “credence.” Yet, interestingly, competent halfers and thirders reach identical correct solutions when challenged with well-specified variations of the original problem, regardless of whether the solutions are consonant with their positions or not.

I believe I have pinpointed the reason for this passionate polarization. There are two alternative interpretations of this puzzle, hopelessly entangled, that are based on two different construals of the problem statement. These construals are based on two different, perfectly natural, ways that past events can be imagined. Each of the alternate ways, which I call the “action interpretation” and the “property interpretation” respectively, has strong intuitive appeal, and guides the way that different individuals think about and solve the problem. My resolution of the paradox shows that both Bayesian and frequentist methods give the same answers and demonstrate that there is no quarrel between the definitions of the notion of credence of the halfers and the thirders: It’s just that the two camps apply their understanding to two completely different propositions. This is the reason why both camps are constantly talking past each other. This resolution of the paradox is consistent with that proposed by Berry Groisman (2008), giving two separate probabilistic experiments. My treatment pinpoints in a novel way, the exact place where ambiguity creeps in to cause the paradox, and shows how the naturalness of the two alternative interpretations completely explains the intuitive reasons for the passion and polarization that this paradox continues to generate.

Elga, A. (2000). "Self-locating Belief and the Sleeping Beauty Problem". Analysis 60 (2): 143–147. doi:10.1093/analys/60.2.143. JSTOR 3329167.

Lewis, D. (2001). "Sleeping Beauty: reply to Elga". Analysis 61 (3): 171–76. doi:10.1093/analys/61.3.171. JSTOR 3329230.

Groisman, B. (2008) “The end of Sleeping Beauty's nightmare” arXiv:0806.1316

The Sleeping Beauty Problem, Resolved!
Episode 2: Lost in Time

The Sleeping Beauty problem, described below, is a famous open problem about probabilities that has divided and polarized communities of mathematicians – probability theorists, decision theorists and even philosophers – for over fifteen years. The passion generated in arguments between the two entrenched camps, the "halfers" and the "thirders" puts political ideological debates to shame. In a recent Quanta Insights column, I compared the problem to a Necker cube, a famous visual illusion which can be perceived in two mutually exclusive ways. Most people can flip quite easily between the two views of the Necker cube, however, while in case of the Sleeping Beauty problem halfers and thirders remain firmly entrenched in their view, stubbornly experiencing the other view as completely wrong. Yet, curiously, as we saw in a previous column, competent halfers and thirders reach identical correct solutions when challenged with well-specified variations of the original problem, regardless of whether the solutions are consonant with their positions or not. Both camps can certainly do math, so what makes them butt heads so hard in vain? Is the problem underspecified? Is it ambiguous?

Here is the problem:

> The famous fairy-tale princess Sleeping Beauty participates in an experiment that starts on Sunday. She is told that she will be put to sleep, and while she is asleep a fair coin will be tossed that will determine how the experiment will proceed. If the coin comes up heads, she will be awakened on Monday, interviewed, and put back to sleep, but she won't remember this awakening. If the coin comes up tails, she will be awakened and interviewed on Monday and Tuesday, again without remembering either awakening. In either case, the experiment ends when she is awakened on Wednesday without being interviewed.
> Whenever Sleeping Beauty is awakened and interviewed, she won't know which day it is or whether she has been awakened before. During each awakening, she is asked: "What is your degree of certainty* that the coin landed heads?" What should her answer be?

*The phrase "degree of certainty" has been variously expressed as "belief", "degree of belief", "subjective certainty", "subjective probability" or "credence."

It seems very hard to believe that this simply stated problem should have remained open for over fifteen years. It is possible that the problem is underspecified, but it doesn't feel that way – both camps feel confident that they have solved it. This fact hints at some deep ambiguity in the problem's statement about which smart people vehemently disagree. In our previous column, we saw some dichotomies in the two camps: Halfers count experiments, thirders count awakenings; halfers calculate from the experimenter's point of view, thirders from Sleeping Beauty's. But these are mathematical techniques that both camps know how and when to use. If they do reach different conclusions, it is probably not a matter of mistaken calculation: they must, in effect, be solving two entirely different problems.

This point – that the problem is ambiguous, and that both sides are correct – has been made by several commenters in web discussion groups. A paper by Berry Groisman showed that there are two interpretations of the problem that are both consistent under standard probability theory. I agree with this viewpoint, but it still remains to be explained why both halfers and thirders are so entrenched in their view, and are strongly convinced that they are right. This strong feeling arises, I suggest, because this quarrel is not about mathematics, but rather, it is actually a hidden, subconscious fight about *two different ways of understanding the problem statement.* Specifically, there are two equally valid interpretations or construals of the phrase "landed heads" which refers to something that happened in the past.

To reveal these two meanings, let me add a small hitherto unknown detail to the famous Sleeping Beauty story.

> Imagine that when the coin was tossed on Sunday, it was mounted on a brass plaque in the position it landed, so that the result of the coin toss can be checked at any time. This plaque is kept in a locked safe in Sleeping Beauty's room.

That's all we need to add to prime our intuitions. Certainly this act cannot, in any way, make any difference to the logic of the actual problem. But it enables us to see that the original question can be interpreted in two different ways:

> Interpretation 1: The Action Focus: "What is your degree of belief that the coin landed heads" = What is your degree of belief that the coin landed heads *in the act of tossing*?
> (Image to prime your intuition: Imagine the coin being tossed)
>
> Note that the belief, though current, is about a previous event: The verb "landed" is in the past tense. Whenever a past event is evoked in speech or writing, the listener or reader has to decide how much of the event's background is relevant. Sometimes, a phrase referring to the past requires the listener to "import" the event's background without the speaker explicitly saying so – a "frozen past tense."[1] It's like a photo taken when you were ten years old, unchanging forever,

[1] Note that, as a couple of language experts including Professor Steven Pinker have told me, this is not a phenomenon specific to a particular language. How much of the past background one needs to

which shows your old house in the background, even though you've changed a lot and the house is gone. Here's a reference to the past that requires this kind of implicit background importing: "*What is your belief that my friend the rock star spent a full year's pay on his first guitar?*" This question refers to your belief about the money my friend was making when he bought the guitar, not what he makes today. After all, he is a rock star now, twenty years later. In a similar way, the first meaning of the Sleeping Beauty proposition imports its background act: It can be intuitively accessed by invoking the image of the coin being tossed. It refers to the probability that the coin landed heads when it was tossed: obviously one-half.

Interpretation 2: Property Focus: "What is your degree of belief that the coin landed heads" = What is your degree of belief that the preserved coin in the safe is *showing heads now*?
(Image to prime your intuition: Imagine what face is showing on the coin in the safe.)

Here the past tense predicate "landed heads" is used as a way to describe a *property* previously gained by the coin you are referring to today. You are no longer concerned with how likely it was to have landed heads or tails when originally tossed, but are now concerned with the likelihood that the coin being referred to today, the preserved coin, shows heads and therefore "landed heads" at some time in the past (as opposed to landed tails). The same past tense verb construction is used but now it does not import the background action.

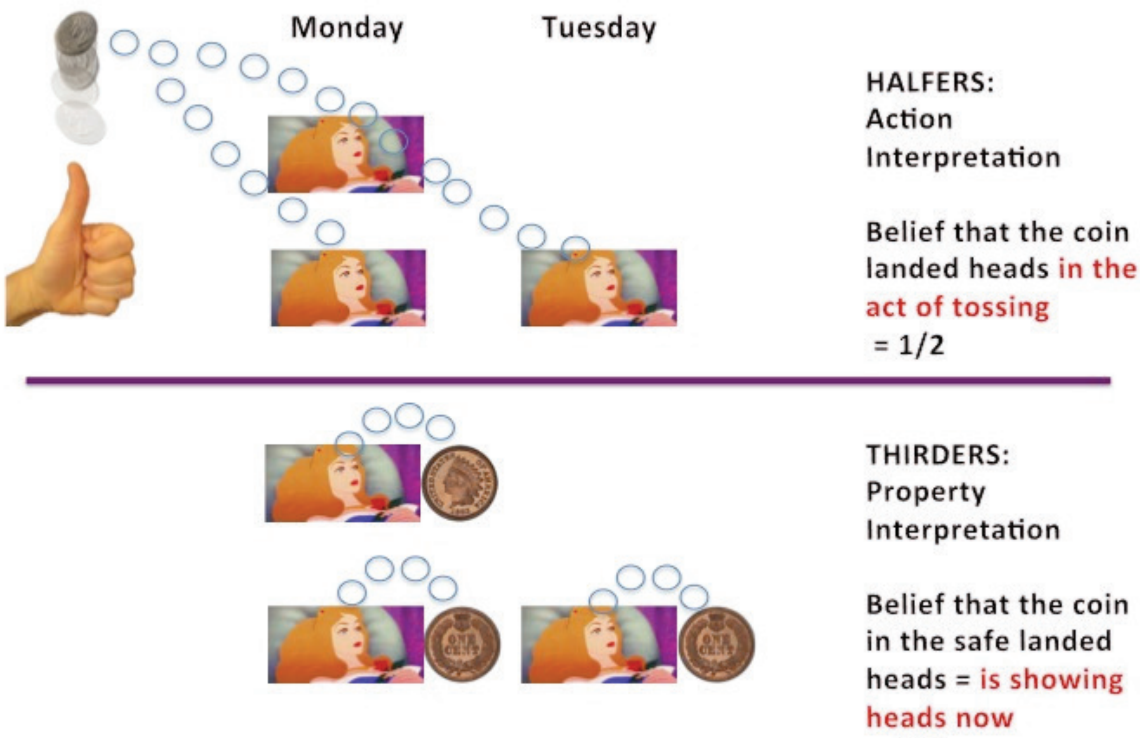
As an example, remember that in the year 2015[*], 6 of the 9 US Supreme Court justices attended Harvard Law School and 3 attended Yale Law School. If I had asked you then, *"What is your degree of belief that a random US Supreme Court Justice attended Yale Law?"* you would probably have answered 1/3. Here, the question is not intended to import the background of the event: I was not asking you about the odds that the person chose Yale Law School of all the law school choices they had back then, you are using "attended Yale" as a property of the person within the current group of Justices, as opposed to the opposite property, "attended Harvard." In a similar way, the second meaning of the Sleeping Beauty proposition does not import the past action, but merely looks at heads as a present property, previously acquired. It can be intuitively accessed by invoking the image of comparing the coin encountered with the plaque in the safe. It refers to the probability that the coin is showing heads now if you had to bet on it, or carried out this comparison on a number of similar occasions: this probability could be anything from 0 to 1. In this case it is 1/3.

---

assume for a given statement, is a universal problem when referring to the past, usually resolved by context.

[*] The numbers in this example have been rendered obsolete by the passing of Justice Scalia. I have kept the original numbers because of the memorable, and relevant ratio they generate.

In the real life example sentences I gave, it is quite clear from the context what is expected: Whether we should invoke the background of the event (the friend's previous salary) or merely use the acquired property (the justice's law school affiliation). However in case of a coin toss, the context does not force one interpretation or the other. Both are up for grabs – and boy, are they grabbed tightly by the two different camps!



Two meanings of "landed heads"

Halfers, I suggest, consciously or subconsciously find the first interpretation, image or intuition to be more salient, use it in their modeling, and come up with a value of one-half; while thirders subconsciously prefer the second interpretation, image or intuition, base their calculations upon it, and come up with a value of one-third.

How can this be? Isn't the coin in the safe the same coin that landed heads or tails on Sunday?

Let's ask the sophisticated and intelligent princess Sleeping Beauty, who is well versed in the natural arts and sciences such as linguistics, math and science, besides of course, fauna and flora. Let's catch her at the time of her interview with the experimenter's assistant at one of her awakenings.

EA: 'What is your degree of belief for the proposition that the coin landed heads?'
SB: 'That question is ambiguous: It can be interpreted in two different ways. Do you mean my belief about the likelihood of heads in the act of tossing the coin on Sunday or do you mean my belief about the likelihood of heads being shown on the preserved coin in the safe?'
EA: 'But the coin in the safe is the same coin that was tossed on Sunday, and shows the same result.'
SB: 'Yes, but you can have a different credence about the probability of heads in the act of tossing a fair coin (which is always one-half) and the probability of the heads in the same coin some time later.'

'Let's suppose that while walking on the seashore, I see 15 coins, 10 of which show tails and 5 heads. Perhaps, a boy who was on the beach before me, removed half of the coins that came up heads because he liked that side. No matter how many coins I gather, I always find two showing tails to every one showing heads. Half of the coins are, to me, **lost in space**. Now my credence that a new coin I encounter shows heads and therefore "landed heads" at some time in the past is only 5 out of 15 or one in three. I can only base my credence on the clear-cut and reliable statistics of the coins I encounter. Maybe one day, I'll find the boy's stash of coins that landed heads, and if I do, my expectation that there are equal numbers of coins that landed heads will return to one-half—or maybe I never will. Notice that we use the verbs "landed" or "came up" in two senses: in the act of tossing the coin, as in "the coin just landed heads," and in the act of finding it later as in "here's a coin that shows heads, and therefore landed heads sometime in the past." '

'Here's a different situation. Imagine I have a specific kind of double vision: the only objects it affects are coins showing tails. When a coin shows heads I see it as one; when it shows tails, a strange optical effect makes me see double. I actually saw only 10 coins, 5 showing heads and 5 showing tails. But my strange affliction, unknown to me, causes me to see heads and tails in the ratio 1:2 and hence my expectation that any new coin I find will show heads, and therefore landed heads, is one in three. Later if I find out that I have this condition, I can correct my erroneous, but at that time fully valid, belief. Of course, knowing that these are all fair coins, I never waver in my belief that they originally landed heads one in two times in the act of tossing.'

'Now, imagine that I find the same 15 coins, but I entered a time warp, unbeknownst to me, and five of the tails I saw were the same ones I had seen before. All the coins showing heads somehow escaped the time warp. Now half of the heads are **lost in time,** or alternatively, the tails are doubled in time. Again, my credence for heads is, validly, one-third. If and when I come out of the time warp, and realize it, I change my credence back to one-half.'

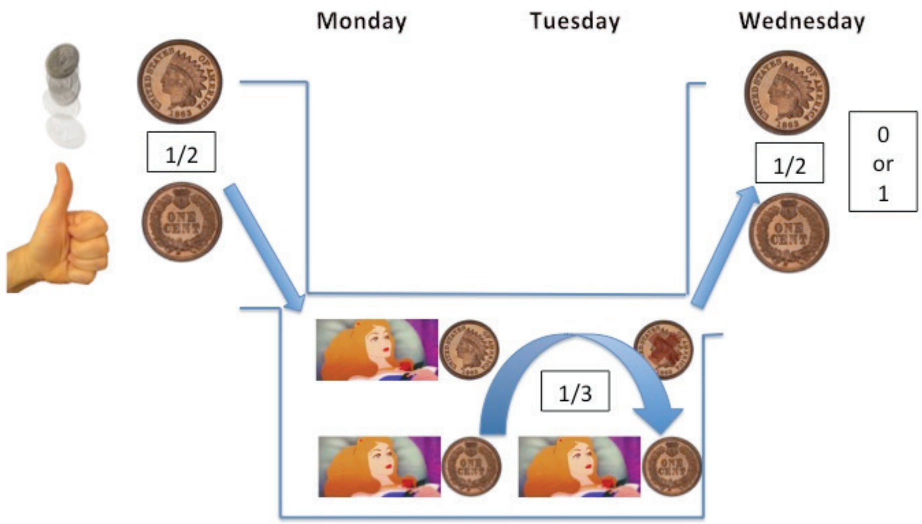
‘I don’t want you to think that such distortions necessarily reflect some kinds of errors of which I am unaware. It may be that half of all coins that landed heads self-destructed on landing so the reliable ratio of coins that I find actually reflects the existing ratio in the world. As long as I am unaware of any systematic errors, I therefore have to trust the ratio that I find, actually or by calculation, as the basis for my belief in the actual probability of the coins I am likely to find.’

‘Thus distortions in time, space and perception that I am unaware of, or differential longevities of the two kinds of coins, or *any systematic process that alters the frequencies* of the two coin toss results differentially, can alter the relative frequencies I systematically find. All these processes influence my valid belief regarding the proportion of the coins I am likely to encounter now that landed heads.’

‘Let’s return to your question, which is actually two separate questions.’

‘The first question is: What is my degree of belief that the coin landed heads *in the act of tossing*? This value, of course, was one-half on Sunday, and will remain one-half until I actually find out what happened. I am a true ***halfer*** about this.’

‘What is my degree of belief that the preserved coin is *showing heads now* which is the same as saying **“**what is my credence that the coin in the safe landed heads sometime in the past?” On Monday and Tuesday, it is one-third, because I am in a time warp with half the heads being lost in time. I am definitely a **thirder** on these two days.’



Heads lost in time:
Sleeping Beauty's experimental time warp

> 'When I will emerge from the experiment's time warp on Wednesday, the value of my credence for heads will once again return to one-half, *because the two different interpretations will coincide*. Then I expect that your boss, the Professor, will tell me how the coin actually landed. At that time, my credence that the coin landed heads will settle on exactly 0 or exactly 1.'
>
> EA: 'Wow, you are one formidable princess. All that sleeping must be good for the brain! Ouch, my head hurts. I think I'll take some of the amnesia drug I will soon be giving you…'

That's all there is to it, halfers and thirders. The rest, as they say, is just plain… math.

Speed Cubing

Solving the *Rubik's Cube*

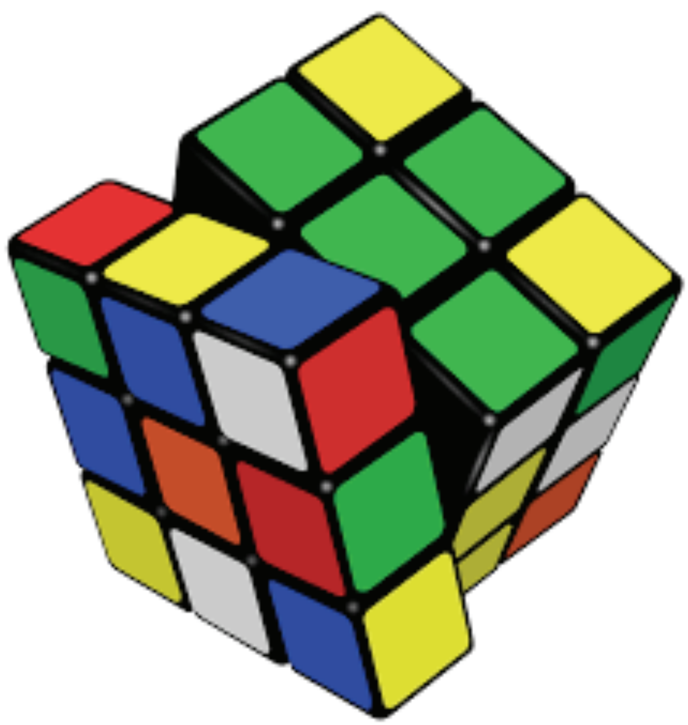
By humans and robots
What are the limits?

Gift from Rik van Grol

Article also submitted for the souvenir book

For G4G12

March 30 - April 3, 2016



Speed solving the *Rubik's Cube* What are the limits?

by Rik van Grol, NL
Rvgrol@hotmail.com

Introduction

The *Rubik's Cube* [1] is probably one of the world's biggest and longest lasting puzzle crazes (another one was the 15 puzzle). Everybody knows the *Rubik's Cube*. The *Rubik's Cube* was invented in the late seventies of the 20th century by Ernõ Rubik, a Hungarian teacher. It was an enormous craze during the early eighties. Then during the nineties it simmered in the background, but in the twenty-first century the craze took off again. The craze is about the magical puzzle itself, the perceived complexity, the universe of Rubik's Cube variants, the analysis of its solutions, the solution methods themselves, pretty patterns made with the cube, and above all, speed solving contests. In this article I will focus on speed solving, both by hand and by robot[1].

For a long time, humans could solve the *Rubik's Cube* more quickly than robots, but for a while now, robots have been quicker. But is the comparison fair? Both humans and robots are still getting quicker. What are the limits? This article will address these issues.

My personal history with Rubik's Cube solving

The *Rubik's Cube* has many attractions. When I see an unsolved *Rubik's Cube*, anywhere, in a house, in a shop, I feel the urge to pick it up and solve it. Most of the time I can resist the urge, but even now, more than 35 years after the introduction of the *Rubik's Cube*, I still feel this urge. When I first got a hold of the *Rubik's Cube* in 1980, I was in fourth grade, and was surprised that I could easily solve the first layer, while our math teacher could not even solve a single side. Intrigued as I was, I spent a bundle (25 Dutch guilder was a lot for a 15 year old; 72 Dutch guilders in 2015) to buy my first genuine *Rubik's Cube*. At the time, solutions books were not available yet, and I solved the *Rubik's Cube* in my own way. It took me one week to reliably solve the Cube up to two remaining twisted edge pieces. It took me another two weeks to reliably solve the complete *Rubik's Cube*. Then it was interesting to compete with fellow students to solve the cube more quickly. I remember I was able to solve the cube in 1 minute 30 seconds, but I never got further – there was no competition at school and I had no contact with other cubers at the time. It was not until the end of the 80's that I got in contact with the Dutch Cube Club where there would be a traditional speed-solving contest at the yearly Dutch Cube Day. I never competed there, as I was not nearly fast enough. During the nineties the interest for speed solving in the club faded away, and it was not until 2004 that we got in contact with the rising community of speed solvers that were seeking out venues to hold their speed solving contest. Since then, they have kept one of their contests in conjunction with our yearly Dutch Cube Day. Some of the quickest human speed solvers have attended the Dutch Cube Day. I have never competed in speed solving, and my shortest solving time is still 1.5 minutes, but I am increasingly intrigued by the way the Cube is solved these days. But that is not the subject of this article.

Human or manual cube solving - speed cubing

The *Rubik's Cube* is a puzzle, meant to be used by humans. When the Cube was born, the first obstacle was how to solve it in the first place. Once that was mastered, some people became interested in producing pretty patterns, others in solving the cube as quickly as possible. When cubers came together, there would always be a friendly competition. Cubers would also exchange ideas for how to solve the Cube with the shortest number of moves. They did all of this in living room environments.

Professionalization of speed solving

[1] In Cubism For Fun (http://CFF.helm.lu), you find information on the other aspects of the *Rubik's Cube* craze.

So how did speed cubing develop since the eighties? To discuss this I will address a number of aspects. Speed solving started as a relaxed game, but as it became more official it needed professionalization for many aspects: timing the solving, creating the cube to solve, etc.

Registering the solving time

In the beginning speed solving would simply be done by starting at same time and noting who would be ready sooner. Then, a simple watch or stopwatch would suffice to time contestants. Soon this was not accurate enough, so within the community a special device has been developed to time cube solving. It consists of a mat and a timer (see Figure 1). The contest nowadays works as follows. The contestant gets the cube (which was covered up). As soon as the cover is lifted, the contestant is allowed to look at it for a maximum of 15 seconds[2]. Then the contestant puts his hands on the mat. As soon as he lifts his hands off the mat, the timer starts. As soon as he has finished solving the cube, he puts both hands back on the mat, which stops the timer.

Figure 1. StackMat timer, a special timer used for speed cubing

Configuration

The *Rubik's Cube* knows 43,252,003,274,489,856,000 states, but any *Rubik's Cube* can be solved within 20 moves [2]. Many configurations can be solved in less moves (if you know which moves). So in order to keep the timing fair, it is necessary to realize that not all starting configurations are equally difficult/easy to solve quickly. In a contest, each contestant is presented with the same configuration to solved. The official world record for a single solve currently is 4.9 seconds. Because there is some luck involved in a particular cube, there is also an official world record for an average solve. Here the time is averaged over five attempts. The shortest and longest solving times are eliminated and the average of the other three is the result.

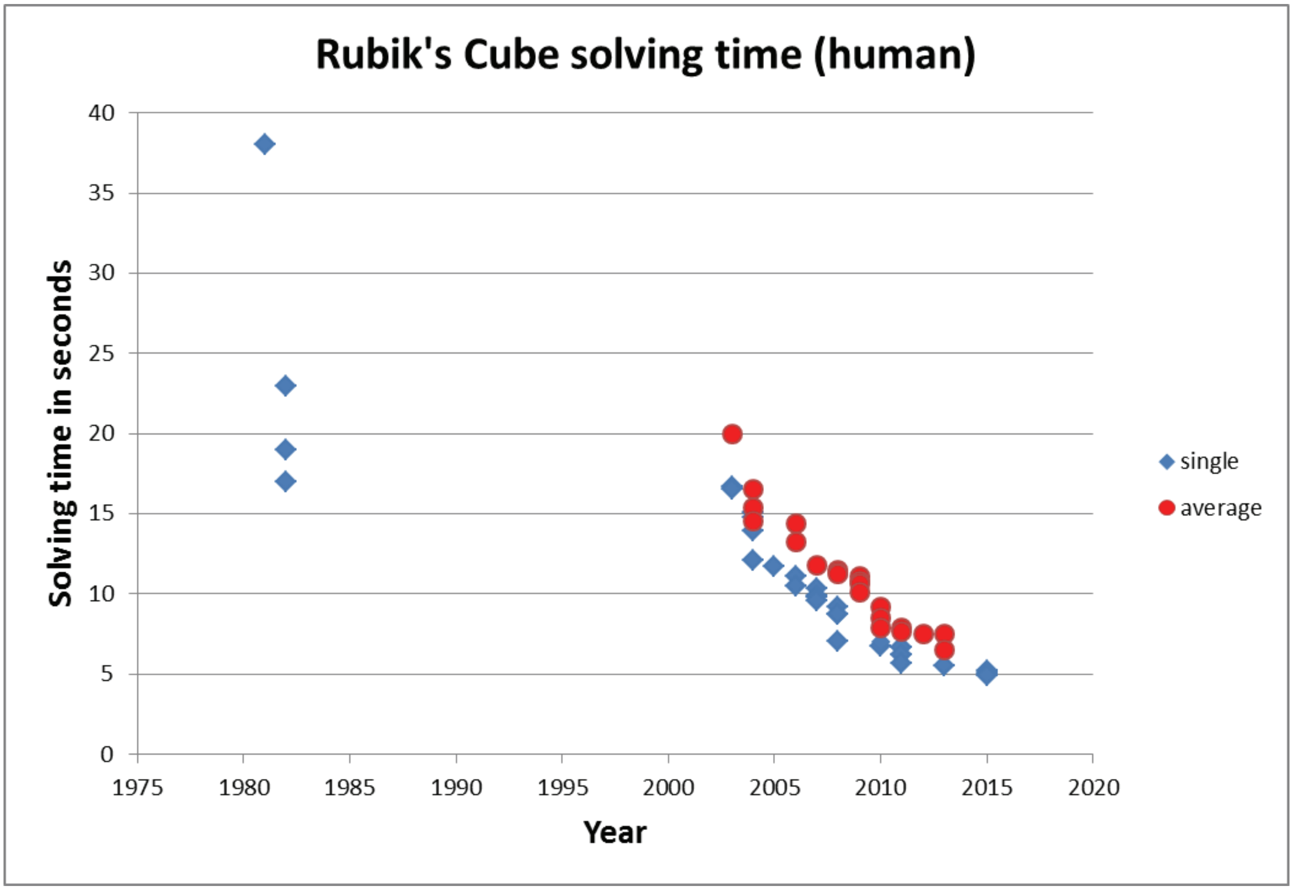
In principle, this should be fair and provide a level playing field for all contestants. The playing field might not be so level as expected. Let us assume that each configuration presented to the contestants is at maximum distance from a solved cube (20 moves)[3]. We can safely assume that there is no (quick) solution method (known yet) that always provides the shortest solution of 20 moves. This means that the solution method uses some strategy, which generally results in a detour before it heads for the solution. Different solution strategies are not very likely to have exactly the same length detours, which means that for each starting configuration the favourite solution strategies may be ranked. Unless all strategies are known in advance and the provided starting configurations are distributed in such a way that they favour certain solving strategies in equal frequencies, the level playing field is not really level. In reality, not all starting configurations are at max distance, and from a starting position with say 18 moves, some strategies may lead to moving away from the solution (to 19 or 20 moves) before heading for the solution, whereas others may move towards the solution more directly. It is unlikely that all of this is taken into account[4].

Official speed records

As mentioned earlier, official records are recorded for single solves and for average solves. There are several records to be found on the Internet[5,6]. Most records have been achieved since 2003, only a few official records from 1982. The graph below shows the solving times over the years. Currently the records are:

[2] See the rules on speed cubing: https://www.worldcubeassociation.org/regulations/#article-4-scrambling.
[3] Based on the rules on speed cubing the scrambling is just done randomly, which does not guaranty a maximum distance configuration.
[4] Based on the rules for speed cubing it is even 100% sure.
[5] https://en.wikipedia.org/wiki/Rubik%27s_Cube
[6] https://www.worldcubeassociation.org

- **Single** in 4.904 seconds by **Lucas Etter** (USA) at River Hill Fall in 2015.
- **Average** in 6.54 seconds by **Feliks Zemdegs** (Australia) at the Melbourne Cube Day 2013.



Mechanical or Robot cube solving

Can a *Rubik's Cube* be solved by a robot? How does solving by a robot compare to human cube solving? Who is currently better at it, and who will be the winner in the end? Those are amongst the questions I was asking myself when I came across new world records.

I have no idea when the first mechanical cube solver was constructed. For the moment I can only go by what is to be found on the Internet[7]. What is clear, however, is that mechanical (automated) cube solving required three developments:

1. a computer program capable of solving the *Rubik's cube*
2. a camera with pattern recognition software, and
3. some sort of mechanical device capable of manipulating a *Rubik's Cube*.

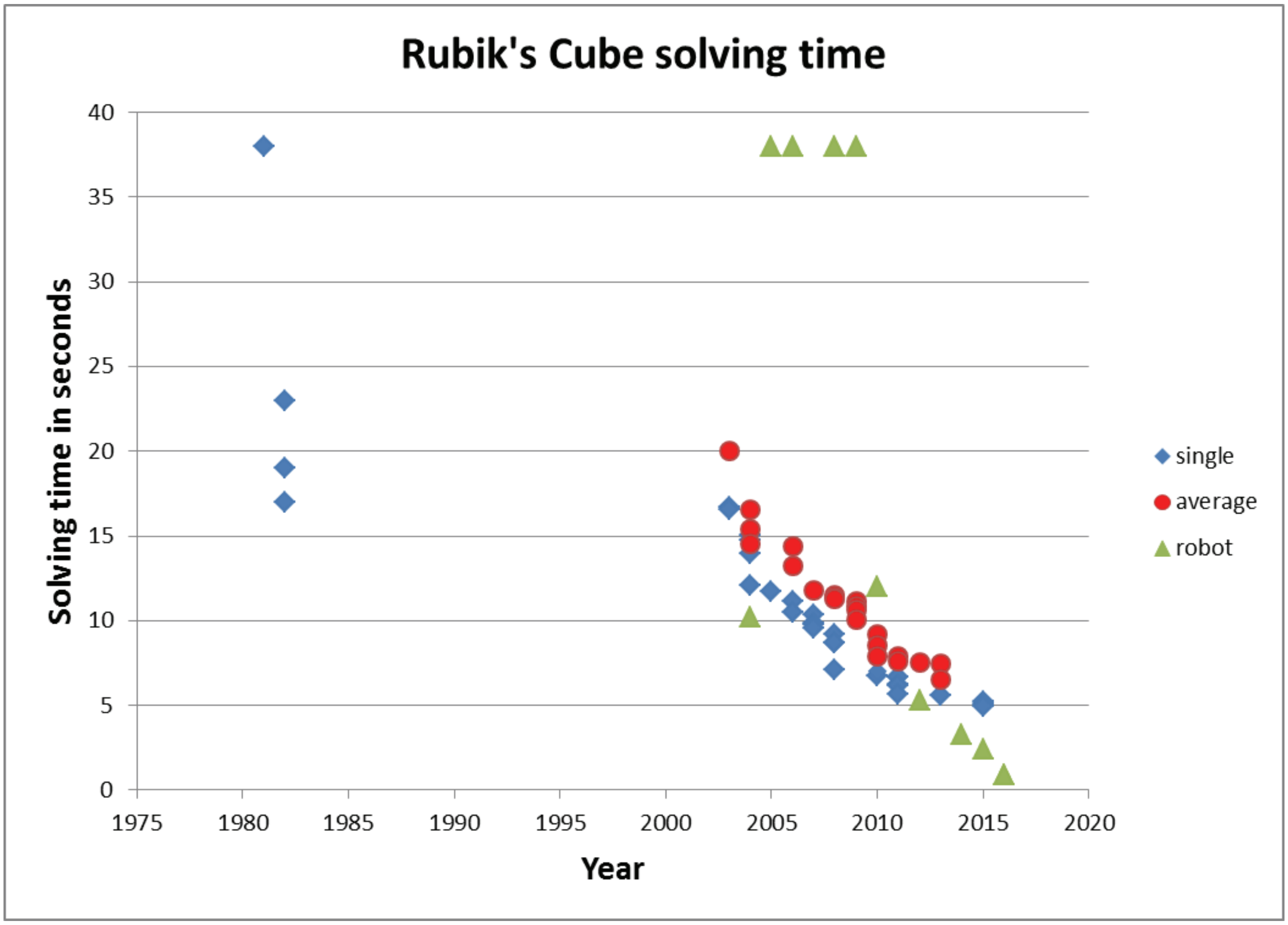
It is my guess that the last problem has been the easiest to solve, but was in fact the last to be realised; what is the use of a machine capable of manipulating a cube if nothing can tell it what to do? For that, you require a solution program and a way to tell in which state the cube is. Programs to solve the *Rubik's cube* were probably around quite soon after the *Rubik's Cube* became popular, but I have not been able to find any record claiming to be the first program. The first I found is from 1988, but it cannot believe it was the first. Of course next to a program, a computer must also be able to compute the solution fast enough. The enormous increase in computing power over the past decades has certainly contributed to solving that problem. The early programs required the state of the *Rubik's Cube* to be entered manually. To resemble a human solver, the robot solver must be able to "read" the *Rubik's Cube* independently. As far as I can tell, it was before 2006 that the three required components came together. Technically it could have been done sooner, but probably the new Rubik's speed cubing craze in 2003 reinitiated the interest in making robot solvers. It took a few years to get all components ready but in 2006, a first serious attempt was presented. In fact this robot, called Rubot II, looks like a human being and may

[7] If a reader knows more, please let me know!

actually be the only version that can be compared with a human solver. This 2006 robot would pick up the cube from a table, look at it and solve it. In a video it wins against a human being: its maker. Its official speed record is 64 seconds, but this includes both the time to pick up the cube and the scanning and “thinking time”.

With 64 seconds Rubot II was in fact fairly slow, but since then robot solving has made some serious developments. It is interesting to mention that in a parallel development *Rubik’s Cube* solving robots have been developed using LEGOs. The first versions were seriously slow, and it took a bit of patience to watch. Later versions are actually amongst the quickest.

Official speed records
The speed records for Robots are less organised than those for humans, so it took quite a while to gather all of the information. In the Figure below, the Robot solving times have been added. Note that the four highest times are actually higher (60+ seconds). The time of 10 seconds in 2004 is “invalid” because this robot had no “eyes” and the solution was pre-set. This is unlike the record of 0.887 seconds, which includes scanning, thinking and solving.



1. Deep Cube
2004, 10.2 seconds (measured from the video), six arms, no camera, PC, Evan Gates.
Not a real solver, because the solution-sequence was predetermined (20 moves). It is actually quite slow considering the number of moves.
YouTube: *Rubix cube deep cube.*

2. *Rubik’s Cube* Solver Robot
2005, 64 seconds (13 scanning + 14 computing + 37 solving; from video), six arms, PC.
University of Michigan - Ann Arbor - Design of Microprocessor Based Systems - Doug Li, Jeff Lovell, Mike Zajac.
YouTube: *Rubik's Cube Solver Robot.*

3. Rubot - prototype
2006, up to 15 minutes, two arms, video camera, PC, Pete Redmond (Ireland).
Time is determined by the computing time, looking for the solution.
YouTube: *RuBot - Rubik's Cube Soving Robot Prototype*

4. Lego Rubik Utopy
2008, 61 seconds max, turntable, two holding hands, Danielle Benedettelli.
LEGO Rubik Utopia project.
YouTube: *Danny's Rubik's Cube Solver - faster than ever!*

5. Rubot II
2009, 64 seconds officially (8 pick-up, 17 scan, 39 solve), average 35 seconds, max 43 seconds
Two arms, video camera, PC, picks up the cube by itself (!).
In 2009 at the BT Young scientist and technology exhibition in Dublin, Peter Redmond developed the fastest robot to solve a *Rubik's Cube* called the Rubout 2 and it's in the Guinness book of world records. Nicknamed, "The Cubinator", this amazing robot set the Guinness World Record for a *Rubik's Cube* solving robot and appears in the 2010 Guinness Book of World Records.
YouTube: *RuBot II, The Cubinator - A Rubik's Cube Solving Robot.*

6. Lego Mindstorms
2010, minutes, Autonomous Cube solver – not a speed cuber, see Figure 2.
YouTube: *Lego Mindstorms Rubik's Cube Solving Robot.*

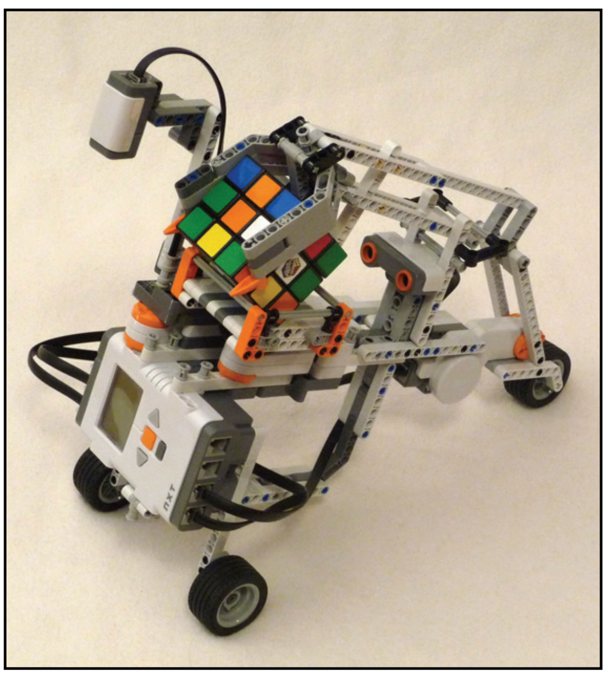


Figure 2. Tilted Twister LEGO MINDSTORMS *Rubik's Cube* Solver

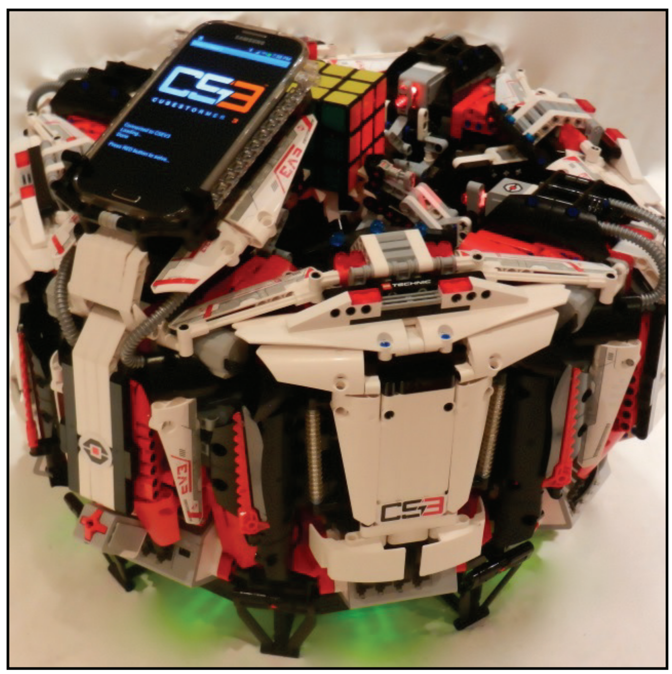


Figure 3. Cubestormer 3

7. Lego Mindstorms EV3 – *Rubik's Cube* Solver
2013, 95 seconds, turntable and flipper, spot-scanner, autonomous.
YouTube: *Lego Mindstorms EV3 – Rubik's Cube Solve*r.

8. CubeStormer
2010, Less than 12 seconds, four arms, regular cube, Mike Dobson.
YouTube: *CubeStormer.*

9. Arduino controlled Rubik's Solver
September 2012, 16 minutes, turntable and flipper, regular cube.
This is not a speed solver but the result of a school project.

YouTube: *Arduino controlled Rubik's Cube Solver.*

10. CubeStormer 2
June 2012, **5.270 seconds**, four arms, regular cube, David Gilday and Mike Dobson, see Figure 3.
Youtube: *The CubeStormer 2 - World Record Rubik's Cube Solver made from LEGO NXT Mindstorms.*
YouTube: *This is how you solve a Rubik's cube in 5 seconds.*

11. CubeStormer 3
March 2014, **3.253 seconds**, four arms, regular cube, David Gilday and Mike Dobson
YouTube: *CUBESTORMER 3 Smashes Rubik's Cube Speed Record.*

12. Fastest robot to solve a *Rubik's Cube*
November 2015, **2.39 seconds**, six arms, regular cube (?), Zackary Gromko (USA).
YouTube: *Fastest robot to solve a Rubik's Cube - Guinness World Records.*

13. World's Fastest *Rubik's Cube* Solving Robot - Now Official Record is 0.900
January 2016, **0.900 seconds**, six arms, predrilled Rubik’s Cube (!), Jay Flatland.
YouTube: *World's Fastest Rubik's Cube Solving Robot - Now Official Record is 0.900 Seconds.*

14. Fastest robot to solve a *Rubik's Cube*
February 2016, 0.887 seconds, six arms, regular cube, Albert Beer (Germany).
YouTube: *Fastest robot to solve a Rubik's Cube - Guinness World Records.*
Website: http://bit.ly/GWR-RubikRobot

Human versus or Robot cube solving

So, with 0.887 seconds versus 4.9 seconds, clearly the robot wins against the human! Or does it? I do not think it is so clear. It is like comparing apples and oranges. Let us investigate this.

Let us consider the human first (quickest and slowest alike):

- A human gets 15 seconds maximum to inspect the cube before the timing starts. The “scanning” and thinking time is not taken into account.
- The human puts the cube down and gets into the starting position (hands on the stackmat).
- The timer starts when the first hand leaves the mat and stops when the last hand is back on the mat.
- The human uses any regular cube.
- The human uses two hands (although there is also a one-handed contest).

And now for the quickest robot solver, see Figure 4:

- The *Rubik’s Cube* is placed in the six arms of the Robot.
- The robot is timed from the moment that the view of the cameras is unblocked.
- The timing includes scanning, thinking and solving.
- The robot uses any regular cube (the fastest robot needs a specially prepared cube).

So is this a fair comparison?

- Two hands versus six arms.
- Only solving versus scanning, thinking and solving.
- Regular cube versus (sometimes) a specially prepared cube.
- *Rubik’s cube* needs to be picked up versus *Rubik’s Cube* already in solving position.

There are so many differences that it is hard to compare this in a fair way. In my view, the closest contest between human and robot has been Rubot II versus his maker (see Figure 5).

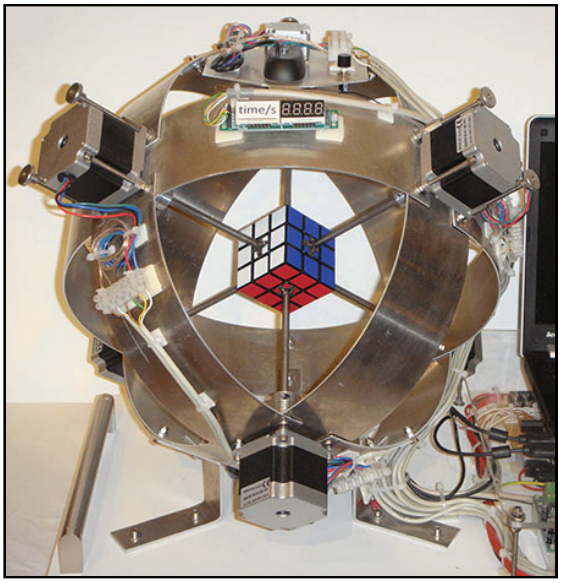


Figure 4. The fastest *Rubik's Cube* solving robot from Albert Beer

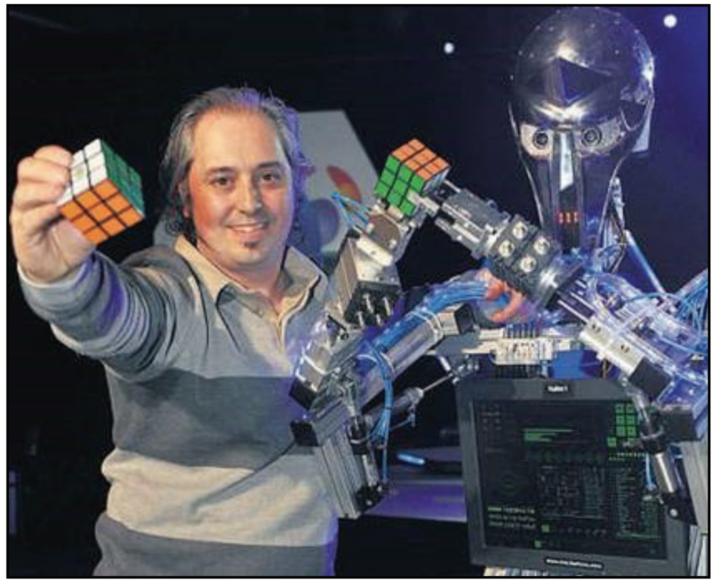


Figure 5. Rubot II battling its maker Peter Redmond

Rubot II has two arms and hands, picks up the cube, looks at it, thinks, solves the puzzle and puts it down again. This is much like a human. Strangely enough the recorded time for Rubot II includes scanning and thinking, whereas the human gets 15 seconds to do that. Even though this is the fairest comparison, there are still differences, such as the number of fingers on a hand. Still, for a fair comparison between humans and robots, Rubot II seems to be the best starting point. I do not really think this will be pursued further. I would love the see a Rubot III.

Will humans get faster? Only with better algorithms, I believe. A quick algorithm to find the solution for God's number would help. Without better algorithms, only by some luck the single solving time may decrease. This is why the single solving time should be abandoned. Only the average solving time is fair, although a larger number than five would be preferable.

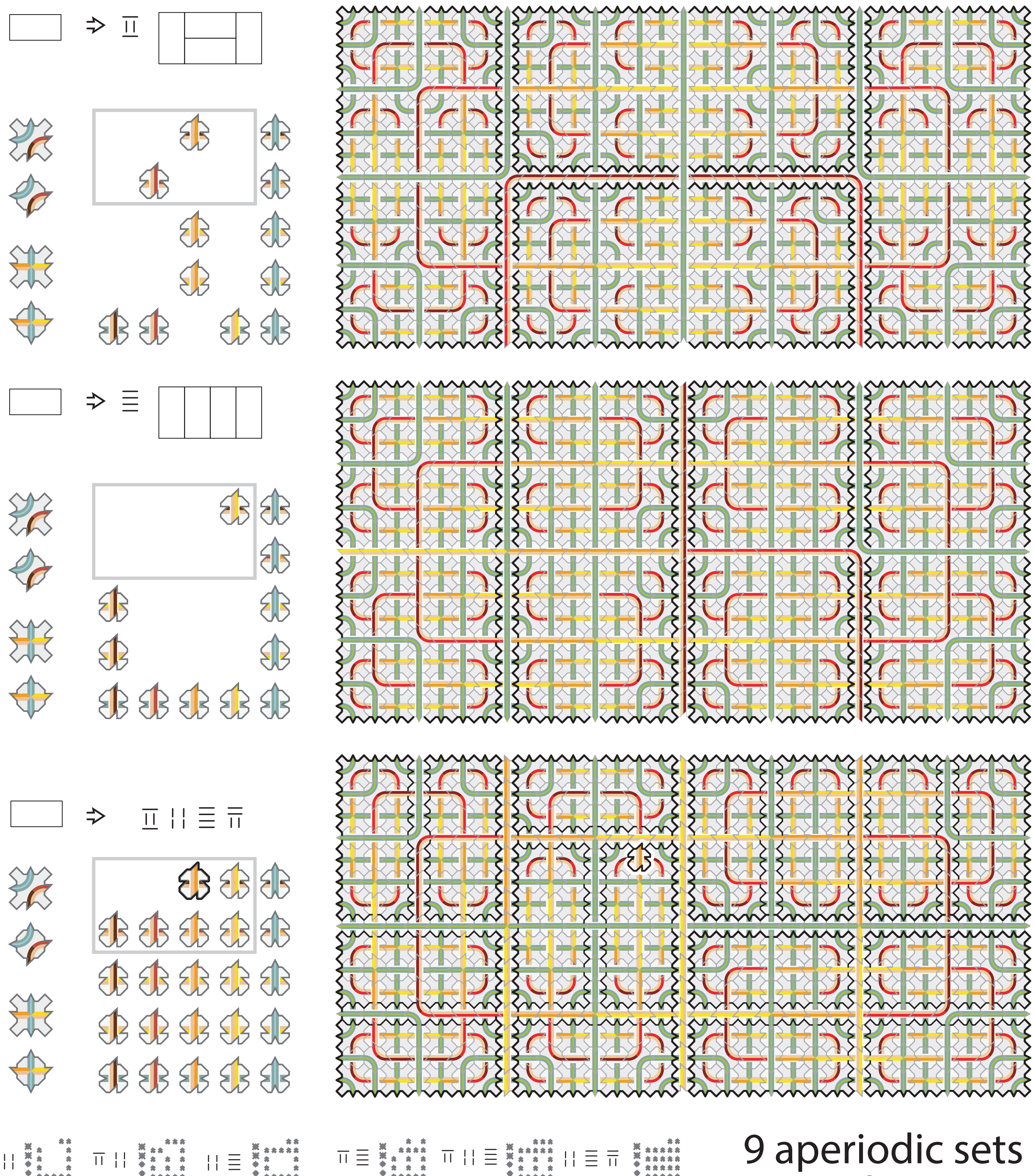
Will robots get faster? With 2.39 seconds I thought the limits were nearly reached. And then weeks ago the time was more than halved! However, now mechanically the limit has probably been reached. Or has it? Obviously a better algorithm will also help here in reducing the solution sequence and thus the solving time.

I will be waiting to be surprised both by humans and robots.

References

[1] Jerry Slocum et al., *The Cube – The Ultimate Guide to the World's Bestselling Puzzle*, 2009, ISBN 978-1-57912-805-0.

[2] Rik van Grol, *The Quest for God's Number*, in The Best Writing on Mathematics 2011, Princeton University Press, ISBN 978-0-691-15315-5.

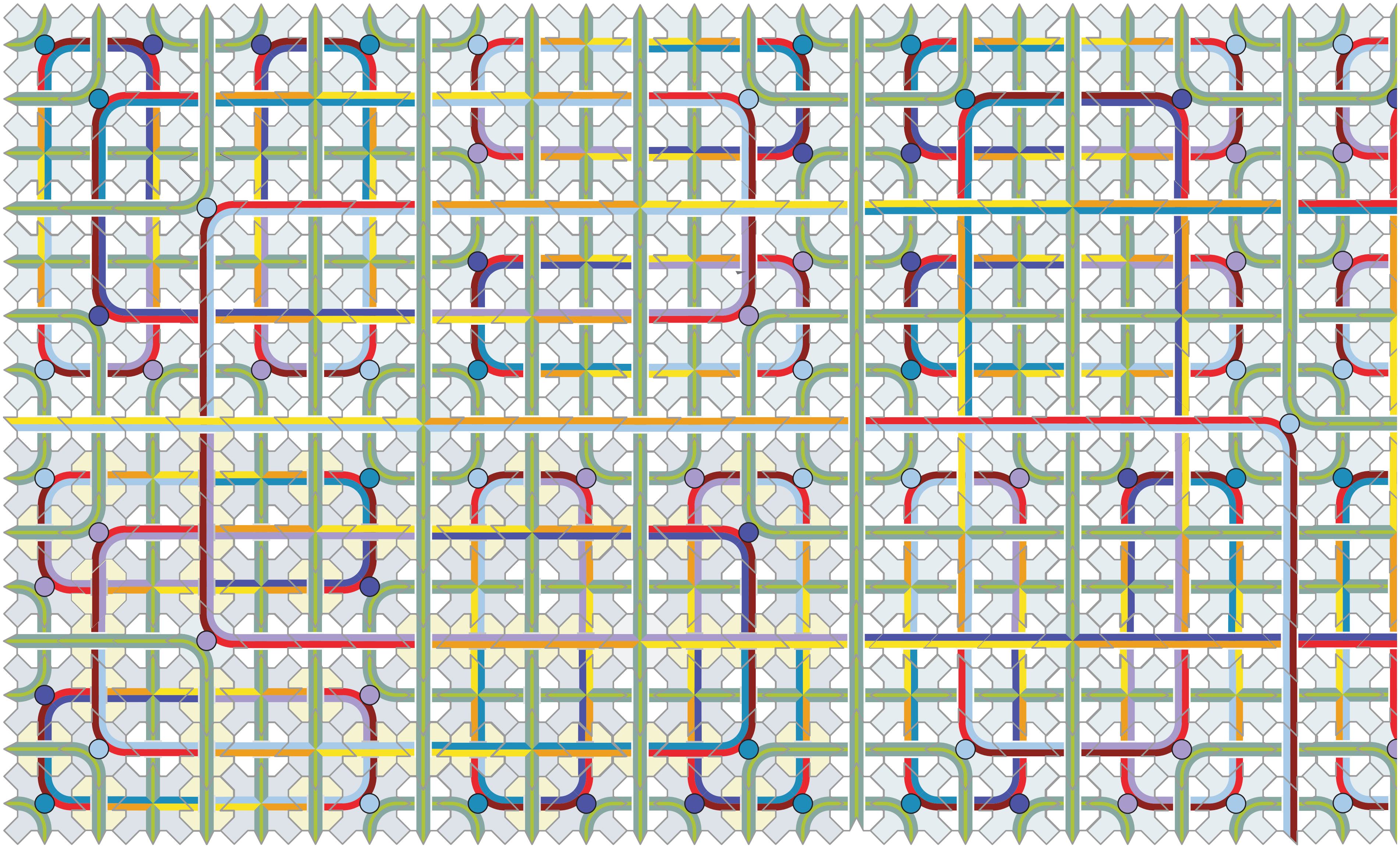


9 aperiodic sets in 27 tiles.

Table Tiling

Control which hierarchies are allowed by including or not including various tiles

Lots of Aperiodic Sets of Tiles
Chaim Goodman-Strauss,
forthcoming, 2016.

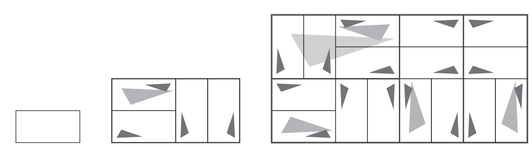


Lots of Aperiodic Sets of Tiles

Easy generation of new aperiodic sets

50625 aperiodic sets in 205 tiles,
enforcing the rich family of dimer substitutions $\overline{\Pi}$,
increasing the number of known examples thousand-fold.

Lots of Aperiodic Sets of Tiles
Chaim Goodman-Strauss,
forthcoming, 2016.



the tiles shown here must
exactly form this hierarchy

A Trio of Coin-Jumping Puzzles

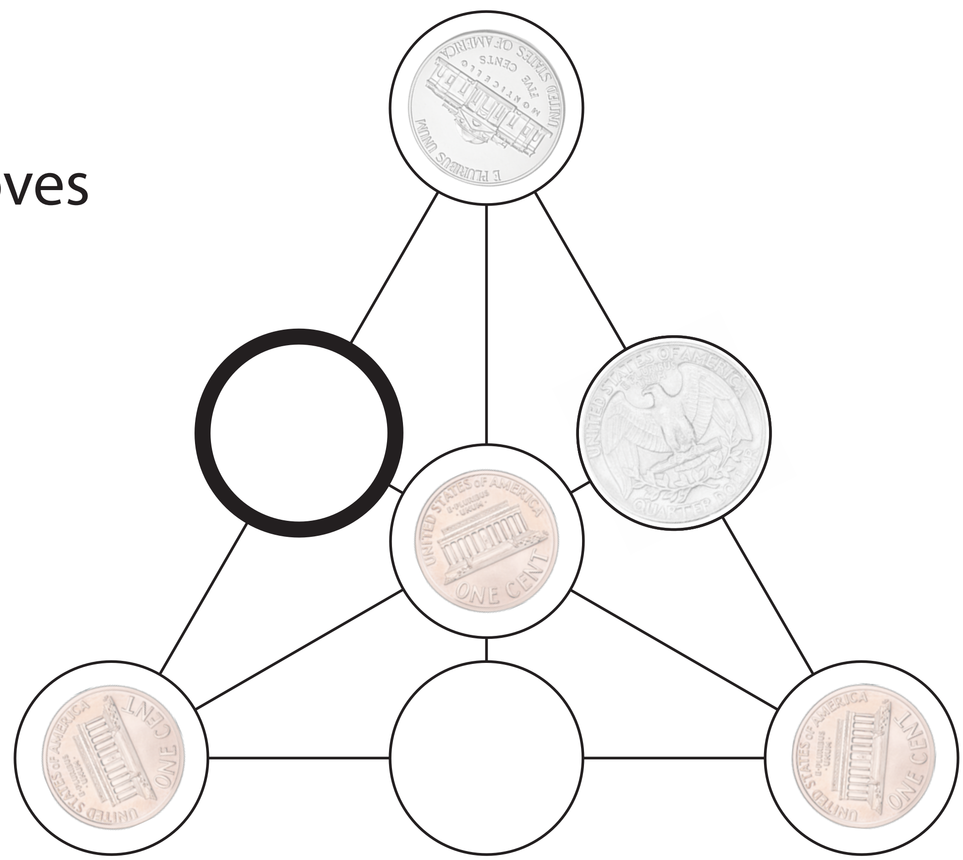
Bob Hearn
Gathering 4 Gardner 12
March 30 - April 3, 2016
Atlanta, GA

Martin Gardner wrote about both peg solitaire and coin-sliding puzzles. These puzzles, composed for G4G12, have elements of both. Like peg solitaire, a move is made by jumping one coin over an adjacent one, in a straight line, into an adjacent empty space. Unlike peg solitaire, the jumped coin is NOT removed.

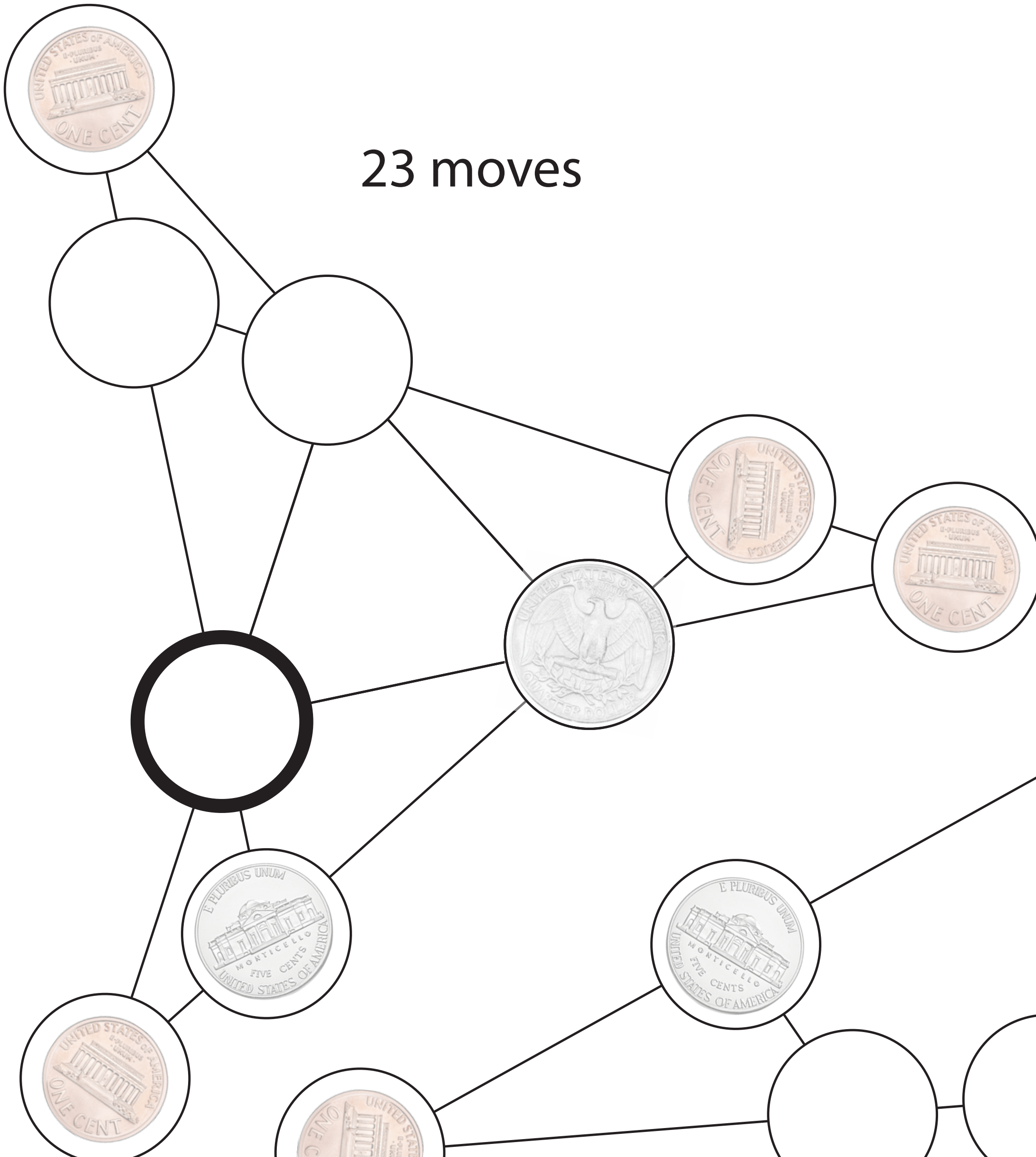
Your goal is to get a quarter into the thick-bordered circle. Sound easy? Oh, one more thing: smaller coins cannot jump over larger coins. So pennies can only jump pennies, nickels can jump pennies or nickels, and quarters can jump anything. You'll find this makes the problems a bit challenging.

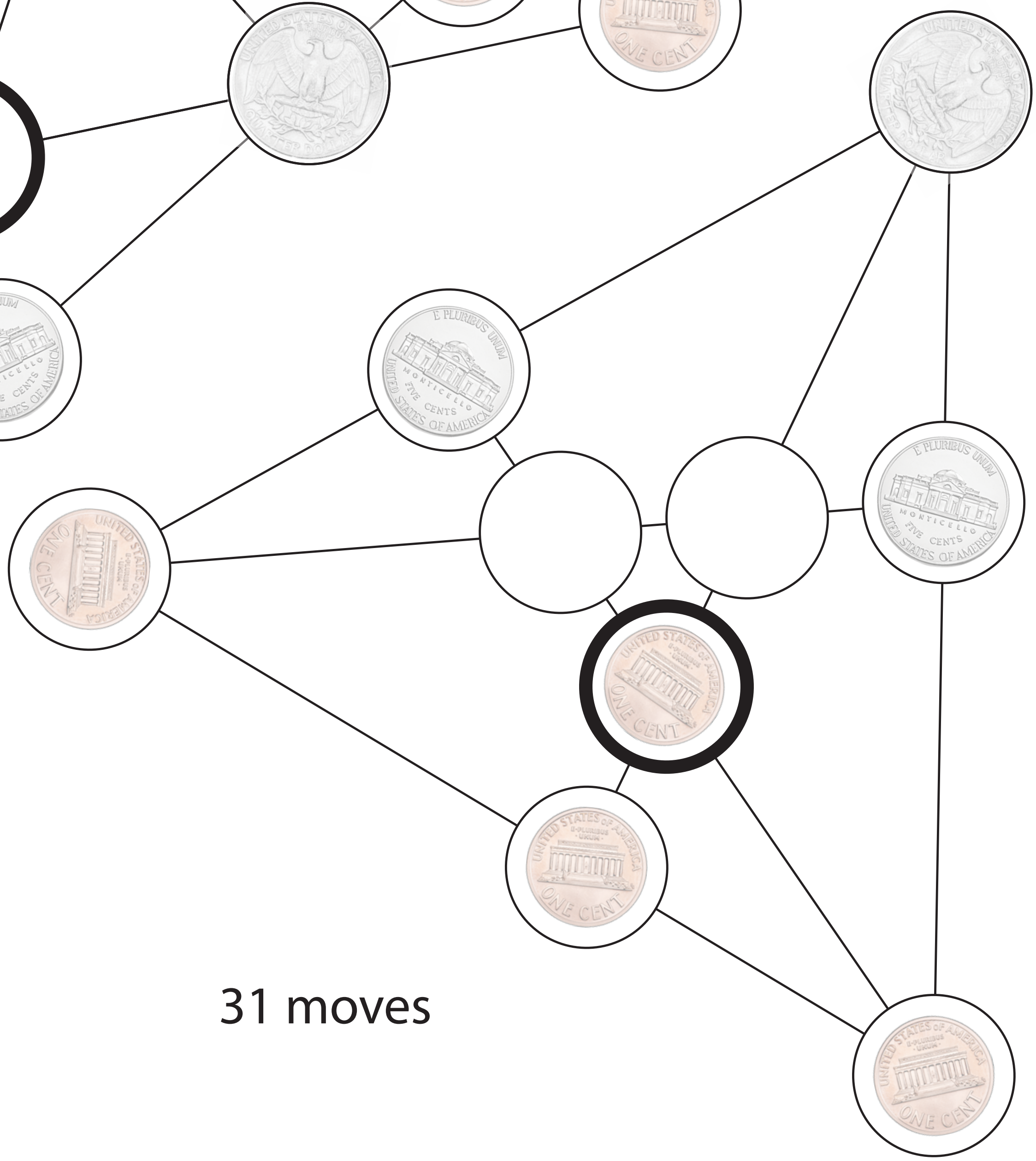
Good luck!

12 moves



23 moves



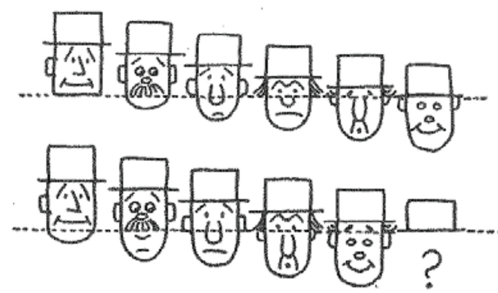


31 moves

G4G12 EXCHANGE
from Stuart Moskowitz
Humboldt State University
Arcata, CA 95521
stuart@humboldt.edu

WHICH MAN DISAPPEARS?

In Mathematics Magic and Mystery, Martin Gardner includes "The Vanishing Face" puzzle (pictured at right) and notes that "it is useless to ask which face vanishes because after the shift is made, four of the faces have been broken into two parts and the parts redistributed so that each face gains a small amount…"



In the middle of the night, right after sharing Martin Gardner's Vanishing Face puzzle during my workshop titled "Make Puzzles Less Puzzling with Math" at Western Michigan University in 2001, the following was sent to me via email. It provides one of the best means for explaining why the question "which man disappears?" is a misleading question.

Thu, Feb 22, 2001 at 5:43 AM
To: stuart@humboldt.edu

Stuart
I enjoyed your talk yesterday.
Last night I had an amusing thought about which of the six men in top hats disappears. If you print their names between the heads, then we can talk about exactly which guy goes.

```
J                          J
O   R   D   R   A          O   R   D   R   A
E   O   O   O   L          E   O   O   O   L
--------------------       --------------------
    Y   N   N   B   L      Y   N   N   B   L
        A   N   E   E          A   N   E   E
        L   I   R   N          L   I   R   N
        D   E   T              D   E   T
```

See what happens?
After the shift we have Joey, Ronald, Donnie, Robert, and Allen.

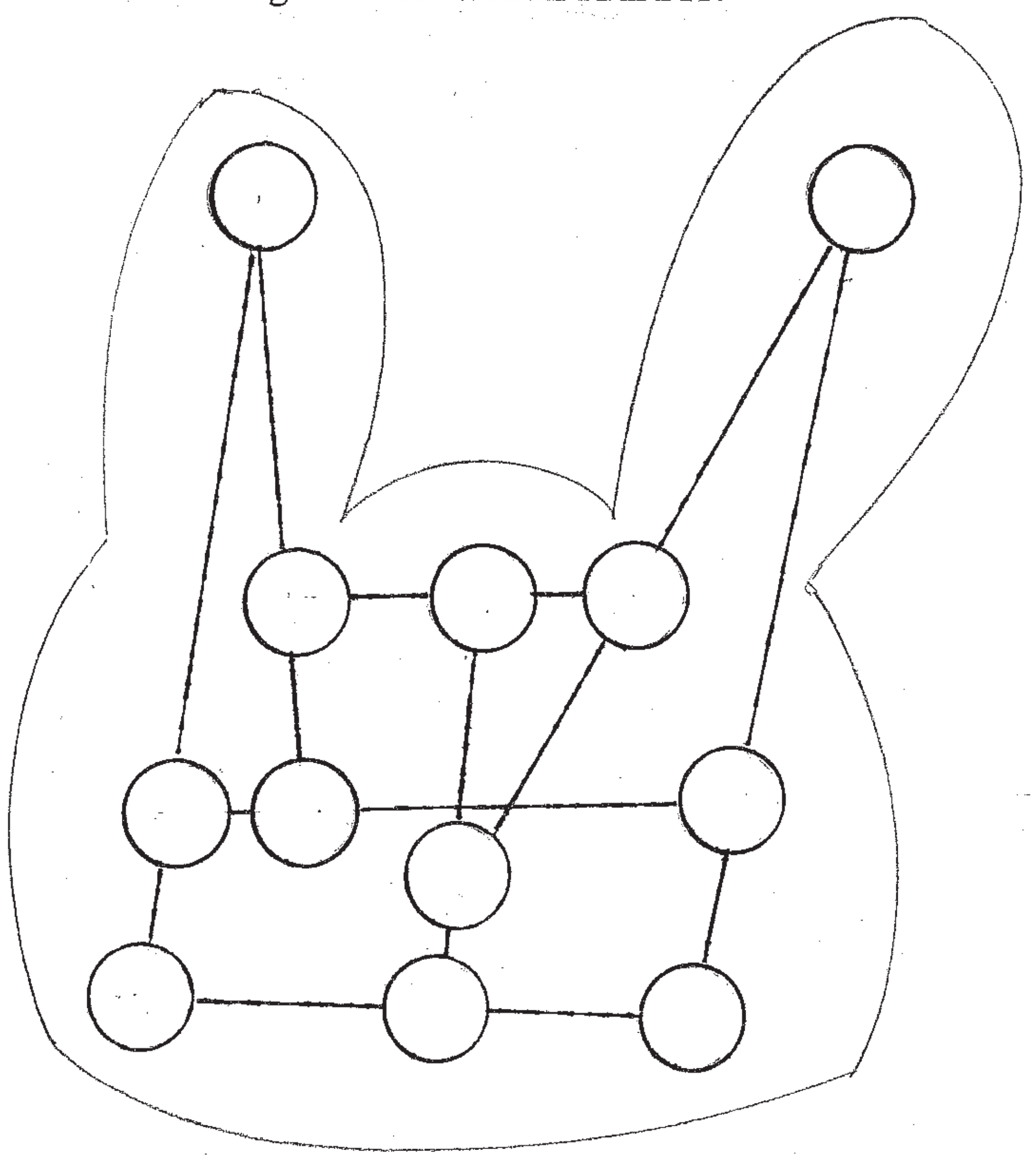
Just a thought. Best wishes, Allen Schwenk

An Exchange for
G4G12
Atlanta, March 2016

THE WHITE RABBIT 12-PUZZLE

By Chris Morgan
And
Jeremiah Farrell

Martin Gardner's fondness for the characters and themes of Lewis Carroll's "Alice" is well-known and to honor Gardner we offer two word puzzles to be played on the 12-node diagram of the WHITE RABBIT.



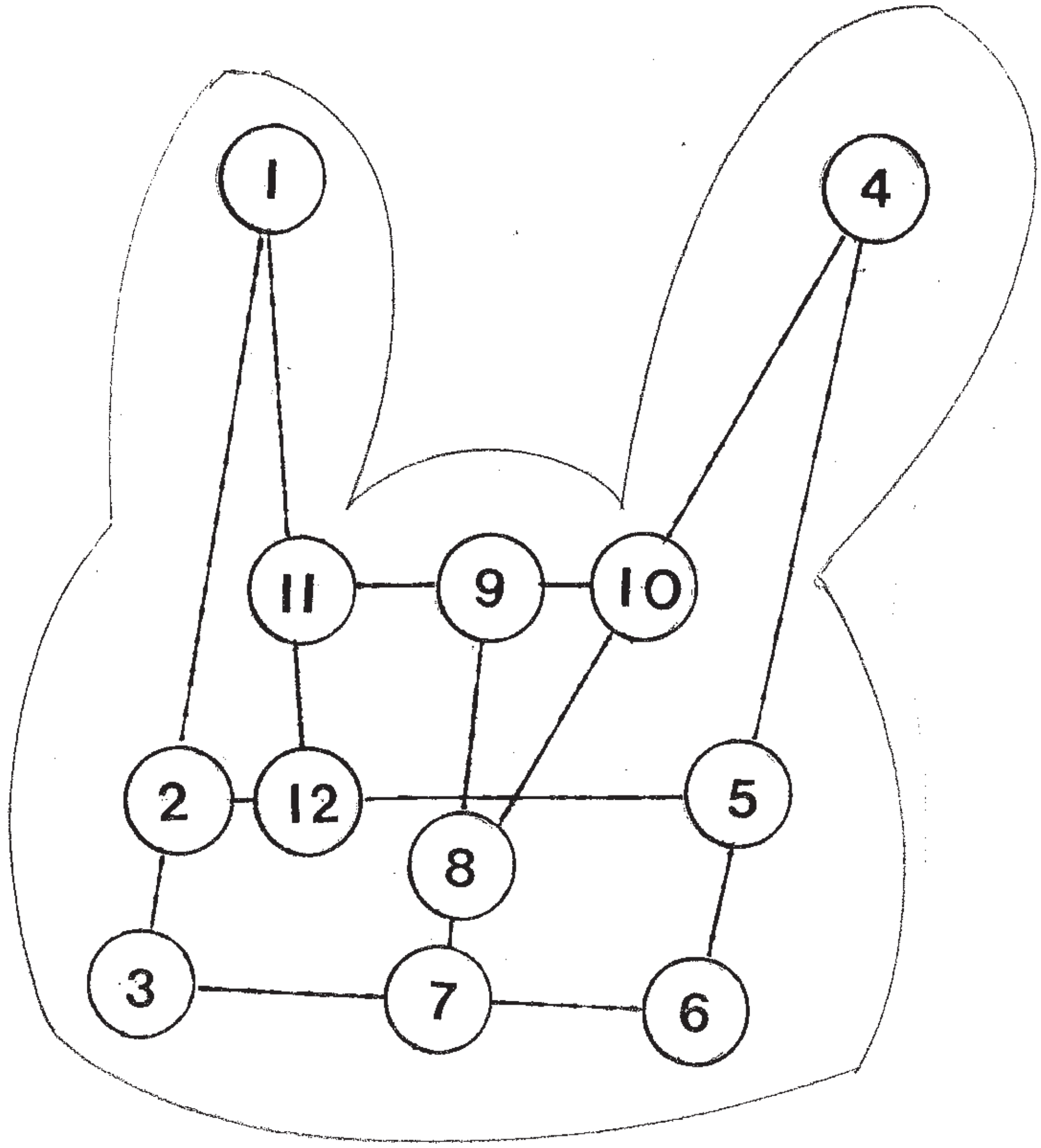
Puzzle 1. There are eight different letters in WHITE RABBIT and each is used exactly three times each to form these 12 words: AR (Argon), AW, BA (Barium), BE, BI (Bismuth), HE, HI, IT, TH (Thorium), TR (Teddy Roosevelt), WE, WR (White Rabbit).

The puzzle is to place these 12 words on the 12 nodes so that connected nodes have a common letter.

Puzzle 2. There are 12 different letters in the phrase DOWN THE RABBIT HOLE. Using these exactly two times each we form eight words:

BAN, BIT, HEW, LED, LOT, RAH, ROD, WIN

Place the 12 different letters on the nodes so that each line of three letters anagrams into one of the eight words.



ANSWERS.

Puzzle 1. The 12 words can be placed in order 1 to 12 thusly:
IT, HI, BI, WE, HE, BE, BA, AW, AR, WR, TR, TH
Notice that each line of three contains a common letter.

Puzzle 2. The 12 letters can be placed in order 1 to 12 thusly:
I, N, W, R, A, H, E, D, L, O, T, B

Both answers are word examples of mathematical geometric configurations.

WOW5

by Carl Hoff

WOW5 Description:

Wrap O-round Weave 5, or WOW5, is a 5 band puzzle ring where the weave is wrapped all the way around the circumference of the ring. The item in the exchange bag was printed in Polymide by i.Materialise. I can also obtain this ring printed in brass or silver using the Interlocking Metal pilot at Shapeways. As this is still a pilot, this option is only available to the designer, so I cannot yet make the metal versions available in my Shapeways shop. Contact me directly if you are interested in a metal copy of WOW5. All ring sizes are available. The story detailing how WOW5 was designed, and why it is the first puzzle ring to wrap the weave around the entire circumference of the ring, will be published in *Game & Puzzle Design* Vol. 2, no. 1, 2016.

Carl Hoff
Shapeways Shop: https://www.shapeways.com/shops/wwwmwww
Game & Puzzle Design: http://gapdjournal.com/issues/
carl.n.hoff@gmail.com

SCIENCE

Classical Mechanical Truth Table

$\langle p\vert$	$\vert q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}_C(\vee)$	$\hat{P}_C(\rightarrow)$	$\hat{P}_C(\leftarrow)$
$\langle R_C\vert$	$\vert R_C\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_C\vert$	$\vert L_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\vert$	$\vert R_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\vert$	$\vert L_C\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$

Quantum Mechanical Truth Table

$\langle p\vert$	$\vert q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}(\vee)$	$\hat{P}(\rightarrow)$	$\hat{P}(\leftarrow)$
$\langle R_Q^m\vert$	$\vert R_Q^m\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_Q^m\vert$	$\vert L_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$
$\langle L_Q^m\vert$	$\vert R_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$
$\langle L_Q^m\vert$	$\vert L_Q^m\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$

The Jin and Jang of Quantum Physics Truth Tables | Shannon Lieb & Jeremiah Farrell | Page 202

The Jin and Jang of Quantum Physics Truth Tables

"We don't see the world as it is, we see it as we are." –Anaïs Nin

Submitted to G4G12

By: Shannon G. Lieb and Jeremiah P. Farrell

At the turn of the 20th century, Max Planck uncovered a new physical constant that bears his name and turned Classical Physics upside down. Instead of allowing all possible energy states to be accessed, Max Planck did the unthinkable of transforming an integral over all energy states times the probability of occupation of the energy states of matter into a Geometric Sum of discrete energy states. In the well known experimental but theoretically unexplained results of the Blackbody radiation curve, Planck introduced one constant to the experimental curve of the emitted light intensity versus the frequency of light. Five years later, Albert Einstein was able to explain the Photoelectric Effect by transforming the wave nature of light into a particle description of light. The essence of the Photoelectric Effect is measuring the electrical current of a metal as a function of frequency of the light incident on its surface. The experimental results are linear with a slope of Planck's constant.

The simultaneously discovered quantum theories of Werner Heisenberg and Erwin Schrödinger evolved to explain interactions of light with matter, thus theoretically explaining the line spectra of atoms and molecules. The double slit experiment demonstrates the dual wave/particle nature of light (photons). It is the results of this experiment that defines a Jin and Jang of the Quantum Physics Truth Table.

The results of the experiment are based on a comparison of the Classical Physics results of particle and wave behaviors passing through a single and double slit. If you shoot small paint balls through a fence that has open slots in front of a screen, you will find individual marks on the screen. Those marks would correspond to well defined trajectories (paths) that would lead back to the paint ball gun's angle with respect to the fence and the amount of force with which the paint ball was released from the gun. If you shoot through two slits in the fence you will find two single fence slot patterns of individual paint balls. If you pass a beam of light waves through a single slit, you will observe a diffuse band of light right in the region of the screen where you expect to see the paint balls land. If you pass a monochromatic beam of light through a double slit of the proper geometric proportions, you will find an interference pattern, which is comprised of multiple bands of light with the most intense band lining up with the region halfway between the two slits. Clearly there is a distinction between wave behavior and particle behavior. The particle behavior is described by a trajectory leaving distinct marks on a screen. This gives information about the path between the source of the particle and the point of impact of the particle. Waves, on the other hand, have no defined trajectory and different parts of the wave "interfere" with one another. When the crest (or trough) of one wave reinforces and amplifies the crest (or trough) of another wave, the result is constructive interference. Destructive interference is produced when the crest of one wave meets the trough of another wave, causing them to cancel each other out and leave a node or place with no intensity.

With this introduction, particle and wave behavior are clearly distinguished from one another in Classical Mechanics. Based on Einstein's explanation of the photoelectric effect, experiments have been created in which the single and double slit experiments can be carried out using photons as our light particles. As anticipated, when individual photons pass through a single slit, the screen on the other side shows a single band of individual photons. However, when the both slits are opened, the interference pattern emerges from the pattern of dots showing up on the screen. The eminent physicist, Paul Dirac explained this by stating that the photon interferes with itself as it passes through both slits. The wave/particle duality of quantum-sized entities does not admit of a trajectory when a wave experiment is performed.

First, when setting up truth tables, one can choose to assume that a three valued logic is appropriate in which the Law of the Excluded Middle is set aside. One can create a true, false and maybe table or a true, false, undetermined and indeterminate, thus creating a three or four valued logic table, respectively. Careful examination of these alternatives reveals an imposition of a trajectory on the quantum-sized objects under study. To avoid this inherent implicit assumption, the use of the Heisenberg representation of the state of a system is undertaken. The state of the system is represented as a vector. Since one of the quantum postulates set forth by von Neumann states that the probability of state of the system is the square of the state vector, one can represent the state of an arbitrary vector, $|g\rangle$ in terms of the complete set of vectors describing the pure states of the system. In the case of the truth table, the complete set of vectors is either true or false. Either the particles or photons hit a particular region of the screen or they do not hit other portions of the screen, irrespective of whether there is a trajectory or not.

The classic truth table can be constructed in the following way:

p	q		p∧q	p∨q	p→q	p←q
T	T		T	T	T	T
T	F		F	T	F	T
F	T		F	T	T	F
F	F		F	F	T	T

The p∧q column has only one T value and the rest are F. The p∨q column reveals that there is only one F value and the rest are T. If the particular trajectory from the p (source) to the q (screen) is ignored, p∧q is only T when both p and q are both T. Likewise p∨q is only F when both p and q are F. These are the only necessary rows of the classic truth table. The other two columns of the truth table are only F when p→q has p = F and p←q has q = F. Once again only two rows of the truth table are needed to describe the probability of the truth of the outcome. Those two rows are two different rows than the ∧ and ∨ rows. Further more, the p→q column can be replaced by its equivalent of p or **not** q and p←q can be replaced by **not** p or q. The following postulates are set forth. Another quantum physics postulate is that the expectation value of a particular operation is equal to the following: $\langle f|\hat{O}|g\rangle$ = scalar value dependent on the particular operator "$\hat{O}$". The notation $|g\rangle$ is called a ket vector representing the state g and because the vectors are in general representative of complex functions. The other notation of $\langle g|$ (the bra vector) is the complex conjugate of the ket vector. The particular representation that is adopted here is the following. In the case of the double slit experiment, the representation of the state in which the particle passes through the right slit will be: $|R\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $\langle R| = \begin{bmatrix}1 & 0\end{bmatrix}$. A particle passing through the left slit will be: $|L\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ and $\langle L| = \begin{bmatrix}0 & 1\end{bmatrix}$. Note the following properties of these two representations of classical particles:

$$\langle R_C|R_C\rangle = \langle L_C|L_C\rangle = 1 \text{ and } \langle R_C|L_C\rangle = \langle L_C|R_C\rangle = 0$$

where the subscript C is added to make a distinction between Classical Physics and Quantum Physics. This follows another postulate of von Neumann for quantum physics and that is that the square of the state vector equals the probability of finding a particle in a particular state. The probability of a particle aimed at the right slit and showing up at the right region of the screen is certain, but the probability of a particle aimed at the left slit and showing up at the right region of the screen is zero.

To complete the description the quantum mechanical state, one has to evoke the notion that solutions are most readily described as vectors in the complex plane. The pure states that are orthogonal to the real axis are:

$$|R_Q\rangle = \begin{bmatrix}e^{i\pi/2}\\0\end{bmatrix} \text{ and } |L_Q\rangle = \begin{bmatrix}0\\e^{-i\pi/2}\end{bmatrix}$$

Note, once again, that The subscript Q refers to a quantum state, but as is readily noted the designation of right and left are completely arbitrary. As in the classical physics representation the following results are:

$$\langle R_Q|R_Q\rangle = \langle L_Q|L_Q\rangle = 1 \text{ and } \langle R_Q|L_Q\rangle = \langle L_Q|R_Q\rangle = 0.$$

Based on the experimental results of the double slit experiment, the trajectory is undefined for photons and photons can only be represented as linear combinations of their pure states, referred to as mixed states. The mixed quantum states in this representation would be:

$$\left|R_Q^m\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\pi/2} \\ e^{-i\pi/2} \end{bmatrix} \text{ and } \left|L_Q^m\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\pi/2} \\ -e^{-i\pi/2} \end{bmatrix}.$$

As before, these mixed states are orthogonal to one another, but a factor of the square root of 2 is needed to keep the square of the vector equal to unity.

Having the states of the system defined, the definition of the operators of ∧, ∨, → and ← need to be defined. Once again, a quantum projection operator technique is going to be used that is based on the classical state vectors. Since the experiment uses a classical sized experimental apparatus (i.e., the double slit), the projection operators for each of the four symbols of ∧, ∨, → and ← will be based on the classical physics pure left and right slit vectors. The projection operators for each of the operators are defined as:

$$\hat{P}_C(\wedge) = \left|R_C\right\rangle\left\langle R_C\right| + \left|L_C\right\rangle\left\langle L_C\right| = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{P}_C(\vee) = \left|R_C\right\rangle\left\langle R_C\right| \oplus \left|L_C\right\rangle\left\langle L_C\right| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{P}_C(\rightarrow) = \left|\bar{R}_C\right\rangle\left\langle R_C\right|(-) \oplus \left|\bar{L}_C\right\rangle\left\langle L_C\right|(-) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}(-) \oplus \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}(-)$$

$$\hat{P}_C(\leftarrow) = (-)\left|R_C\right\rangle\left\langle \bar{R}_C\right| \oplus (-)\left|L_C\right\rangle\left\langle \bar{L}_C\right| = (-)\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \oplus (-)\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

The plus sign in the ∧ operator means the arithmetic plus. The plus within a circle is a designation like the plus sign linking the real and imaginary parts of a complex number. The classic projection operator created by the $\left\langle \bar{R}_C\right| = [-1\ 0]$ or $\left|\bar{R}_C\right\rangle = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ produces the matrices with negative ones on the diagonal element. The (–) to the left or the right of the projection operators means to change the phase of the vector that is multiplying either from the left or right of the operator. In terms of the classical mechanics particles this has no effect on the outcome of the experimental results, but in the case of the mixed quantum states representing the wave/particle duality of particles there is an effect when using the classical mechanics projection operator.

The final step in the construction of our truth table for classical and quantum particles is to apply the four operators to combinations of the two pure classical particle states and to the two mixed quantum particle states. The first test is to see if the projection operator for a single slit (either the right or left slit only) would give the experimental results for the small paint ball and photon passing through a single slit. In this case, the projection operators are:

$$\hat{P}_R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \hat{P}_L = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The results of the experiments based on the expectation values produced by these operators follows:

$$\langle R_C|\hat{P}_R|R_C\rangle = \langle L_C|\hat{P}_L|L_C\rangle = 1; \quad \langle R_C|\hat{P}_R|L_C\rangle = \langle R_C|\hat{P}_L|L_C\rangle = \langle R_C|\hat{P}_L|R_C\rangle = \langle L_C|\hat{P}_R|L_C\rangle = 0$$

Also:

$$\langle R_Q^m|\hat{P}_R|R_Q^m\rangle = \langle L_Q^m|\hat{P}_L|L_Q^m\rangle = \langle R_Q^m|\hat{P}_R|L_Q^m\rangle = \langle R_Q^m|\hat{P}_L|L_Q^m\rangle = 1; \quad \langle R_Q^m|\hat{P}_L|R_Q^m\rangle = \langle L_Q^m|\hat{P}_R|L_Q^m\rangle = 0$$

The last result may seem surprising, because of the labeling of L and R on the projection operator and the state vectors not matching. As noted earlier, the labeling of the quantum state vector is arbitrary and irrespective of the labeling the projection operator, the expectation value for the probability of the L_Q mixed state going through the projection operator for the right slit is the same as the probability of the R_Q mixed state going through the projection operator for the left slit. Otherwise, the passage of a R_Q mixed state and a L_Q mixed state through the same L or R projection operator causes a destructive interference of the two out of phase states. These results are in complete agreement with the experimental results.

Applying the right and left projection operators for the double slit experiment to the classical mechanical particles, yield the following results:

$$\langle R_C|\hat{P}_C(\wedge)|R_C\rangle = 1; \ \langle L_C|\hat{P}_C(\wedge)|L_C\rangle = 1; \ \langle R_C|\hat{P}_C(\wedge)|L_C\rangle = \langle L_C|\hat{P}_C(\wedge)|R_C\rangle = 0$$

$$\langle R_C|\hat{P}_C(\vee)|R_C\rangle = 1\oplus 0; \ \langle L_C|\hat{P}_C(\vee)|L_C\rangle = 0\oplus 1; \ \langle R_C|\hat{P}_C(\vee)|L_C\rangle = \langle L_C|\hat{P}_C(\vee)|R_C\rangle = 0\oplus 0$$

$$\langle R_C|\hat{P}_C(\rightarrow)|R_C\rangle = 1\oplus 0; \ \langle L_C|\hat{P}_C(\rightarrow)|L_C\rangle = 0\oplus 1; \ \langle R_C|\hat{P}_C(\rightarrow)|L_C\rangle = \langle L_C|\hat{P}_C(\rightarrow)|R_C\rangle = 0\oplus 0$$

$$\langle R_C|\hat{P}_C(\leftarrow)|R_C\rangle = 1\oplus 0; \ \langle L_C|\hat{P}_C(\leftarrow)|L_C\rangle = 0\oplus 1; \ \langle R_C|\hat{P}_C(\leftarrow)|L_C\rangle = \langle L_C|\hat{P}_C(\leftarrow)|R_C\rangle = 0\oplus 0$$

The interpretation of these equations is that if the probability of the classical particle is certain to go through the right side of the double slit, then it will end up on the right side of the screen. But, if one shoots our tiny paint ball toward the left slit, then it will go only show up on the left side of the screen.

Applying the classical mechanics projection operators for the double slit experiment to the quantum mechanical wave/particle properties of photons, yield the following results.

$$\langle R_Q^m|\hat{P}_C(\wedge)|R_Q^m\rangle = 1; \langle L_Q^m|\hat{P}_C(\wedge)|L_Q^m\rangle = 1; \langle R_Q^m|\hat{P}_C(\wedge)|L_Q^m\rangle = \langle L_Q^m|\hat{P}_C(\wedge)|R_Q^m\rangle = 0$$

$$\langle R_Q^m|\hat{P}_C(\vee)|R_Q^m\rangle = 1\oplus 0; \langle L_Q^m|\hat{P}_C(\vee)|L_Q^m\rangle = 0\oplus 1; \ \langle R_Q^m|\hat{P}_C(\vee)|L_Q^m\rangle = \langle L_Q^m|\hat{P}_C(\vee)|R_Q^m\rangle = \frac{1}{2}\oplus\frac{1}{2}$$

$$\langle R_Q^m|\hat{P}_C(\rightarrow)|R_Q^m\rangle = 1\oplus 0; \langle L_Q^m|\hat{P}_C(\rightarrow)|L_Q^m\rangle = 0\oplus 1; \langle R_Q^m|\hat{P}_C(\rightarrow)|L_Q^m\rangle = \frac{1}{2}\oplus -\frac{1}{2}; \langle L_Q^m|\hat{P}_C(\rightarrow)|R_Q^m\rangle = -\frac{1}{2}\oplus\frac{1}{2}$$

$$\langle R_Q^m|\hat{P}_C(\leftarrow)|R_Q^m\rangle = 1\oplus 0; \langle L_Q^m|\hat{P}_C(\leftarrow)|L_Q^m\rangle = 0\oplus 1; \langle R_Q^m|\hat{P}_C(\leftarrow)|L_Q^m\rangle = -\frac{1}{2}\oplus\frac{1}{2}; \langle L_Q^m|\hat{P}_C(\leftarrow)|R_Q^m\rangle = \frac{1}{2}\oplus -\frac{1}{2}$$

In some respects, the results look similar to the classical mechanics particle vector results. However, the quantum mixed vectors carry both of the pure imaginary states. The results of the $\left\langle X_Q^m \middle| \hat{P}_C(\wedge) \middle| Y_Q^m \right\rangle = 1$ when $X_Q = Y_Q$ and $\left\langle X_Q^m \middle| \hat{P}_C(\wedge) \middle| Y_Q^m \right\rangle = 0$ when $X_Q \neq Y_Q$ does not have the same interpretation as the classical double slit results. In the classical regime, the right and left vectors are pure states; whereas, in the quantum regime, the right and left vectors are not pure quantum states, but mixed states, which makes the labels of R_Q^m and L_Q^m meaningless in terms of trying to put a Newtonian label on the quantum particles that have no trajectories. Viewing the totality of both the classical particle and the quantum particle, gives a quite different interpretation of the quantum regime. The $\left\langle X_Q^m \middle| \hat{P}_C(\wedge) \middle| Y_Q^m \right\rangle = 1$ when $X_Q = Y_Q$ is interpreted as a linear superposition principle of both pure states that constructively interfere with one another. The $\left\langle X_Q^m \middle| \hat{P}_C(\wedge) \middle| Y_Q^m \right\rangle = 0$ when $X_Q \neq Y_Q$ means that the interference of these two different, orthogonal, linear superpositions destructively interfere with one another, thus, setting up an interference pattern.

The same argument applies to the other three projection operators for the case where $X_Q = Y_Q$. Moving on to the cases where $X_Q \neq Y_Q$, the $\hat{P}_C(\vee)$ projection operator shows that going through the double slit with two orthogonal mixed states, is equally probable for these two different, orthogonal, mixed states to constructively interfere with one another. This is another way of expressing constructive interference of the two pure states as they pass through the double slit. Now for the $\hat{P}_C(\rightarrow)$ and $\hat{P}_C(\leftarrow)$ projection operators. Their results not only show the equal probability but the relative phases of the two pure states.

Classical Mechanical Truth Table

$\langle p\|$	$\|q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}_C(\vee)$	$\hat{P}_C(\rightarrow)$	$\hat{P}_C(\leftarrow)$
$\langle R_C\|$	$\|R_C\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_C\|$	$\|L_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\|$	$\|R_C\rangle$	0	$0\oplus 0$	$0\oplus 0$	$0\oplus 0$
$\langle L_C\|$	$\|L_C\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$

Quantum Mechanical Truth Table

$\langle p\|$	$\|q\rangle$	$\hat{P}_C(\wedge)$	$\hat{P}(\vee)$	$\hat{P}(\rightarrow)$	$\hat{P}(\leftarrow)$
$\langle R_Q^m\|$	$\|R_Q^m\rangle$	1	$1\oplus 0$	$1\oplus 0$	$1\oplus 0$
$\langle R_Q^m\|$	$\|L_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$
$\langle L_Q^m\|$	$\|R_Q^m\rangle$	0	$\frac{1}{2}\oplus\frac{1}{2}$	$-\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus-\frac{1}{2}$
$\langle L_Q^m\|$	$\|L_Q^m\rangle$	1	$0\oplus 1$	$0\oplus 1$	$0\oplus 1$

List of Authors | VOLUME 2

List of Authors (cont.)

G4G12 Exchange Book

VOLUME 2

Atlanta, Georgia

MARCH 30 - APRIL 3, 2016

It's not often for a conference to be so culturally diverse that its presenters and patrons include mathematicians, physicists, philosophers, logicians, jugglers, puzzle designers, artists, card players, and knitters. Yet it happens every two years at the Gatherings for Gardner, held in honor of Martin Gardner, author of *Scientific American*'s mathematical recreation column for nearly a quarter-century. The papers of these two volumes are write-ups of the presentations at the twelfth conference in this series: G4G12.

- excerpt from the Preface by Robert P. Crease

Published by:
Gathering 4 Gardner™