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Author(s): Pradeep Mutalik

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How Puzzles Made Us Human

PRADEEP MUTALIK

Here's a simple mathematical puzzle. Multiply together the numbers of fingers on each hand of all the human beings in the world—approximately 7 billion in all. Is the answer approximately: A) $5^{7,000,000,000}$ B) $10^{7,000,000,000}$ C) $5^{14,000,000,000}$ D) Something else entirely?

While solving the above puzzle, did you get a flash of insight that led you to the correct answer without any trace of doubt whatsoever—a mini *Aha!*, or insight, moment? If you did not, read the hint at the bottom of the page and try again. The goal of this exercise is not so much to get the right answer, but to give you a small taste of the emotions of joy and certainty that accompany the *Aha!*, or insight, phenomenon that characterizes the cognitive act of solving some puzzles.

Here's another example from a completely different realm of thought. Make sense of the following sentence: "The haystack was important because the cloth ripped."

The answers to both puzzles are at the end of the article. If you solved one or both of these puzzles, or even if you just looked at and understood the answers, you may have experienced the sense of rightness or certainty—"Of course!"—and the positive emotion—"Cool!"—that accompanies the *Aha!* experience.

It is the contention of this article that this intrinsic emotional reward you may have experienced, linked to the cognitive act that you just performed, is an extremely important human characteristic. This cognitive-emotional link in the solving of puzzles, I contend, is one of the most important things that evolutionarily made us what we are today. We are different from other animals in many ways, but each of those differences requires or presupposes this cognitive-emotional link.

Hint: Some people may be missing some fingers.

To judge whether this seemingly grandiose claim is tenable, we need to isolate what characteristics of humans are *qualitatively* different from other intelligent animals and especially from our close ape relatives.

Does the difference lie in what we term our complex social human emotions—love, empathy, shame, jealousy, political intrigue, and the like? Not at all, as any pet lover knows—pets regularly exhibit such emotions, and

We seek out puzzles and learning for fun.

political intrigue is well known in apes. We share many behaviors with animals, and although we execute them with greater complexity and sophistication as a result of our greater intelligence, they do not define us.

Is it tool use or problem solving that makes us different? No. The use of simple tools and the ability to solve problems to obtain food or other extrinsic rewards is well known in animals.

What is different about human beings is *our underlying emotional attitude to problem solving*. We seek out puzzles and learning for fun. This makes us learning machines in the area of our choice, whether it be tracking prey or navigating difficult terrain. *Aha!* experiences help us master an area of learning unique to our species: spontaneous syntactic language. We enjoy art, music, and humor: cognitive experiences that seem to be without any short-term practical purpose. And we can form models of the world and understand it. "The most incomprehensible thing about the universe is that it is comprehensible," Albert Einstein famously declared. As we shall see, it is the cognitive-emotional links in our brains, of which the *Aha!* experience is the most dramatic manifestation, that makes all this possible.

Our brains have cognitive modules for language, face



Figure 1. What mathematical objects do you see in this picture? (The answer is at the end of the article.)

recognition, social interaction, numerical manipulations, motor planning, and so on. But as we just saw, even disparate cognitive processes have the same emotional concomitants when a solution is found. The modules all use the same reward mechanism.

What exactly is this unifying *Aha!* experience? At its strongest, it is a flash of insight that instantly shifts our worldview. It is accompanied by intense pleasure and the confident realization that the answer is right: No external validation is needed. There is a sense of rightness, of things falling into place, like a puzzle piece that can fit only one way. There is a strong memory of the insight, and the feeling is somewhat addictive: You want to come back for more.

Another important characteristic is that this feeling is an intrinsic, impersonal reward—it is not related to the utility of the result. This is perhaps most extremely illustrated in a statement made by the Cambridge mathematician G. H. Hardy to a friend, the philosopher Bertrand Russell: “If I could prove by logic that you would die in five minutes, I should be sorry you were going to die, but my sorrow would be very much mitigated by pleasure in the proof!”

Math enthusiasts know that puzzle solving is intrinsically fun, but seeking out puzzles is not a universal activity by any means. What relevance does the *Aha!* experience have to the vast number of human beings who don't care for puzzles, mathematical or otherwise?

Here's the kicker: The same emotional reaction of joy and certainty is experienced when the brain solves a puzzle that is subconscious—when a person is not even aware that he or she has solved a puzzle!

Such puzzles are constantly being solved by the cognitive, visual, and auditory systems of all humans in day-to-day activities. The cognitive puzzles we need to solve all the time require abstraction, pattern recognition, generalization, the solving of equations, and rule-based induction—things that mathematicians do consciously. And when these puzzles are solved, our brains reward themselves by a similar positive emotional reaction.

As *Gestalt* psychology has shown, some functions of the brain are global: common across modules. The brain has general algorithms that can recognize good solutions to any kind of problem. Let's look at some examples to try to understand what these are.

Figure 1 shows a stereogram puzzle of the type popularized by the *Magic Eye* book series. When you relax your eyes, allowing the two guide circles at the top to come together, and staying focusing on the pattern, some hidden three-dimensional objects emerge. Finding this image elicits the same emotional elements as the *Aha!* experience—positive reinforcement with no doubts at all.

In fact, every act of recognition—whether visual, auditory, or conceptual—is an *Aha!* experience. Cognitively, it is triggered by a change in an initially disordered internal



Gabriel Calderón, <https://www.flickr.com/photos/searingheat>

Figure 2. Beautiful woodwork on the ceiling of the Alhambra in Granada, Spain.

representation to one that makes sense. Order is created out of disorder; the new representation is more compact and coherent. It is much easier to have a bunch of splotches coherently organized into the shape of a recognized object than to account for them individually.

Thus, what brings on the *Aha!* experience is something that can be termed a *decrease in cognitive entropy*. Our brains appear to have a built-in algorithm that triggers the familiar emotional *Aha!* reaction whenever a simple coherent explanation fits disorderly input. The famous principle of parsimony in problem solving—Occam's razor—is apparently built in to our brains.

This powerful principle also helps us learn language. When a child learns to speak, the number of words he or she knows grows slowly at first, and then at around 18 months, suddenly takes off at an exponential rate. The reason seems to be that every child inductively discovers the rule that every object has a name. From then on, the child hounds its parents into feeding it names . . . and the rest is history.

The experience of discovering the name rule occurs too early for most of us to remember, but Helen Keller had it at the age of seven and here's how she described it: "I knew then that 'w-a-t-e-r' meant the wonderful cool something that was flowing over my hand. That living word awakened my soul, gave it light, hope, joy, set it free!"

The certainty and joy she describes clearly identify this as a true *Aha!* experience. *This certitude and plea-*

sure is extremely important to learning language because the child cannot turn to anyone else for validation of its conclusions: It still has to learn language! Cognitively, the unification of independent representations caused by this induced rule represents a large decrease in cognitive entropy quite similar to the visual case. Mini *Aha!* experiences continue to guide language learning and, in fact, all independent learning throughout childhood.

This emotional reaction that favors low cognitive entropy in the solution of unconscious problems gives a natural explanation for those uniquely human aesthetic pursuits: art and music. We find regular visual patterns like the one in figure 2 pleasing. We love symmetry. Our visual system makes recognized patterns pop out. Symmetry and observed patterns reduce the representational requirement of a visual object, triggering pleasurable reactions.

Music is pleasurable for the same reason. Musical scales consist of notes in simple integer ratios: 1:2, 1:3, 5:4, and so on. The pleasure associated with such ratios is based on the fact that sound-makers in the environment essential to our survival, such as predators, prey, and vibrating inanimate objects, give out resonant frequencies in integer ratios.

To parcel out environmental sounds accurately, the brain has to be able to identify integer ratios in the mish-mash of frequencies that we hear. So in effect, our auditory system tries to solve Diophantine equations. When

it does so, *Aha!* There is a reduction of cognitive entropy and we feel pleasure. Also, musical rhythm is a compact organization of time intervals, creating, essentially, symmetric patterns in time. Of course, there is a lot more to aesthetics than these basic elements, but the underlying intrinsic pleasure of low cognitive entropy motivates us to follow these pursuits.

The same drive to detect existing patterns in aesthetics extends to finding hitherto unknown patterns in humor and creativity. As Arthur Koestler outlined in his brilliant book *The Act of Creation*, humor and creativity are linked because they both arise from finding new patterns of reasoning that are intrinsically appealing: those that decrease cognitive entropy. Once we find such

new patterns, we can celebrate those that are valid and weed out those that don't quite work in the real world and are therefore funny.

Koestler tells the joke about the man who came home to find his wife in bed with a priest and, instead of reacting angrily, went out onto the balcony and pretended to bless an imaginary congregation. His explanation to the priest was "You are doing my job, so let me do yours." This creative pattern of thinking—reciprocity—is valid in many situations, but not in this one. So we find it funny: Humor is the brain's way of saying, "Nice try, but you are reasoning on thin ice here."

Neuroimaging studies confirm that both cognition and emotion are involved in the *Aha!* effect. There is increased brain activity in the more recently evolved brain structures of the cerebral cortex—specifically, the anterior superior temporal gyrus and the right hemisphere—during the *Aha!* effect. But there is also increased activation of the right hippocampus, which is involved in memory, and of more primitive brain structures that are powerfully involved in emotion, motivation, and even addiction, such as the amygdala.

It is a signal achievement of human brain evolution that it has managed to link the results of our most sophisticated cognitive processes with our most primitive pleasure centers. It makes evolutionary sense: if you were to make an animal with no imposing physical traits that had to live off its wits, you would provide it an internal reward when it solved a problem. And that's exactly what evolution has done.

All primitive human societies have experts that excel in particular fields of knowledge: language, reckoning, navigating by the stars, tracking, and so on. Unlike, say, insect societies, this expertise is not innate but self-culti-

vated. *Aha!* experiences in childhood in a particular field can accentuate variations in intrinsic ability, leading the child to seek problems in, and master, a particular field. The almost addictive nature of the *Aha!* experience can set a child's course for life. This phenomenon likely gave human societies the specialists that helped them survive and thrive. In the words of Jacob Bronowski, "The most powerful drive in the ascent of man is his pleasure in his own skill. He loves to do what he does well and, having done it well, he loves to do it better."

We are finally in a position to respond to Einstein's observation that the universe is comprehensible to us. Occam's razor is a part of our conscious and subconscious problem solving: We experience joy in finding

simple elegant representations of complexity. *This is adaptive because the universe has evolved by self-assembly and natural selection, gradually growing more complex from simple beginnings. In such a process, the simplest mechanisms of complexity are encountered first and hence are the most probable.* We conceptually run this process of complexification in reverse when we

find simple explanations. Hence, the patterns we find attractive are likely to correspond to the workings of the world. That's all there is to it, Albert.

Although it is heartening to know that the quest for mathematical elegance is hard-wired in our brains, it is humbling—and satisfying—to know that it is not unique to mathematicians. ■

Pradeep Mutalik, a research scientist at the Yale Center for Medical Informatics, has been addicted to the Aha! experience since early childhood and has sought it everywhere: in mathematics, animal behavior, neuroscience, radiology, computer science, artificial intelligence, and consciousness. He founded the New York Times puzzle blog "Numberplay" and authored it from 2009 through 2012. He would love to hear from readers about their memorable Aha! moments and whether they influenced their choice of major or career.

Email: pradeep.mutalik@yale.edu

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Answers to the puzzles:
 1. The product is zero, because there is at least one person in the world who has no fingers on one hand.
 2. A parachute ripped.
 3. The Platonic solids: the tetrahedron, the octahedron, the icosahedron, the cube, and the dodecahedron.