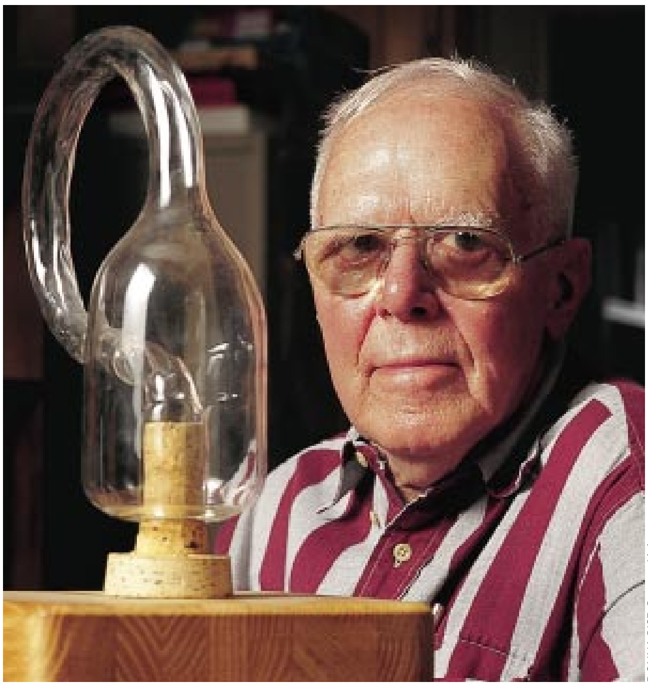
A Quarter-Century of Recreational Mathematics

The author of Scientific American*'s column "Mathematical Games" from 1956 to 1981 recounts 25 years of amusing puzzles and serious discoveries*

by Martin Gardner

"Amusement is one of the fields of applied math."
—William F. White,
A Scrapbook of Elementary Mathematics

My "Mathematical Games" column began in the December 1956 issue of *Scientific American* with an article on hexaflexagons. These curious structures, created by folding an ordinary strip of paper into a hexagon and then gluing the ends together, could be turned inside out repeatedly, revealing one or more hidden faces. The structures were invented in 1939 by a group of Princeton University graduate students. Hexaflexagons are fun to play with, but more important, they show the link between recreational puzzles and "serious" mathematics: one of their inventors was Richard Feynman, who went on to become one of the most famous theoretical physicists of the century.



MARTIN GARDNER continues to tackle mathematical puzzles at his home in Hendersonville, N.C. The 83-year-old writer poses next to a Klein bottle, an object that has just one surface: the bottle's inside and outside connect seamlessly.

At the time I started my column, only a few books on recreational mathematics were in print. The classic of the genre—*Mathematical Recreations and Essays,* written by the eminent English mathematician W. W. Rouse Ball in 1892—was available in a version updated by another legendary figure, the Canadian geometer H.S.M. Coxeter. Dover Publications had put out a translation from the French of *La Mathématique des Jeux* (*Mathematical Recreations*), by Belgian number theorist Maurice Kraitchik. But aside from a few other puzzle collections, that was about it.

Since then, there has been a remarkable explosion of books on the subject, many written by distinguished mathematicians. The authors include Ian Stewart, who now writes *Scientific American*'s "Mathematical Recreations" column; John H. Conway of Princeton University; Richard K. Guy of the University of Calgary; and Elwyn R. Berlekamp of the University of California at Berkeley. Articles on recreational mathematics also appear with increasing frequency in mathematical periodicals. The quarterly *Journal of Recreational Mathematics* began publication in 1968.

The line between entertaining math and serious math is a blurry one. Many professional mathematicians regard their work as a form of play, in the same way professional golfers or basketball stars might. In general, math is considered recreational if it has a playful aspect that can be understood and appreciated by nonmathematicians. Recreational math includes elementary problems with elegant, and at times surprising, solutions. It also encompasses mind-bending paradoxes, ingenious games, bewildering magic tricks and topological curiosities such as Möbius bands and Klein bottles. In fact, almost every branch of mathematics simpler than calculus has areas that can be considered recreational. (Some amusing examples are shown on the opposite page.)

Ticktacktoe in the Classroom

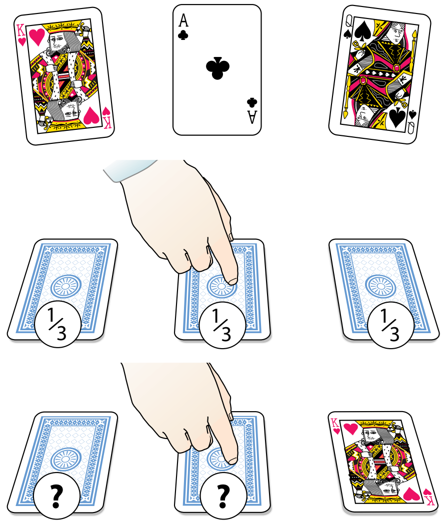
The monthly magazine published by the National Council of Teachers of Mathematics, *Mathematics Teacher,* often carries articles on recreational topics. Most teachers, however, continue to

Four Puzzles from Martin Gardner

(The answers are on page 75.)

ILLUSTRATIONS BY IAN WORPOLE

1



Mr. Jones, a cardsharp, puts three cards face down on a table. One of the cards is an ace; the other two are face cards. You place a finger on one of the cards, betting that this card is the ace. The probability that you've picked the ace is clearly 1/3. Jones now secretly peeks at each card. Because there is only one ace among the three cards, at least one of the cards you *didn't* choose must be a face card. Jones turns over this card and shows it to you. What is the probability that your finger is now on the ace?

2

28	26	30	27	~~29~~	25
~~34~~	~~32~~	~~36~~	~~33~~	35	~~31~~
16	14	18	15	~~17~~	13
4	2	6	3	~~5~~	1
10	8	12	9	~~11~~	7
22	20	24	21	~~23~~	19

The matrix of numbers above is a curious type of magic square. Circle any number in the matrix, then cross out all the numbers in the same column and row. Next, circle any number that has not been crossed out and again cross out the row and column containing that number. Continue in this way until you have circled six numbers.

Clearly, each number has been randomly selected. But no matter which numbers you pick, they always add up to the same sum. What is this sum? And, more important, why does this trick always work?

3

In the beginning God created the
1 2 3 4 5 6

heaven and the earth.
7 8 9 10

And the earth was without form,
11 12 13 14 15 16

and void; and darkness was upon the
17 18 19 20 21 22 23

face of the deep. And the Spirit of God
24 25 26 27 28 29 30 31 32

moved upon the face of the waters.
33 34 35 36 37 38 39

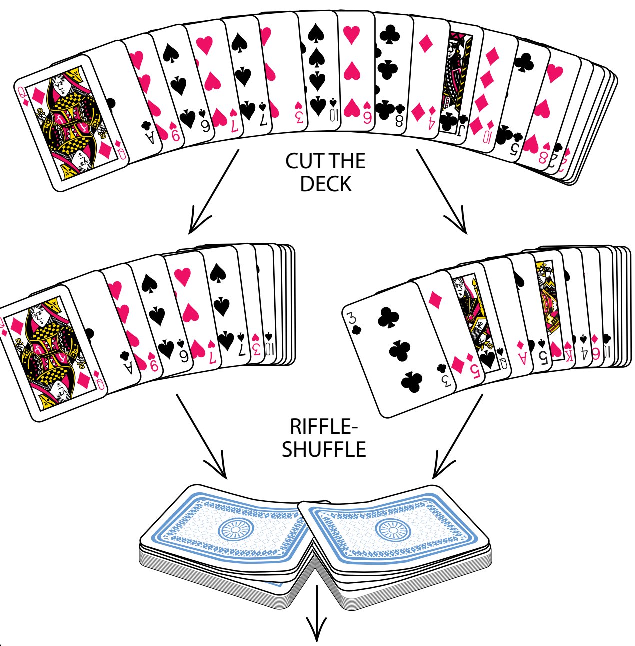
And God said, Let there be light:
40 41 42 43 44 45 46

and there was light.
47 48 49 50

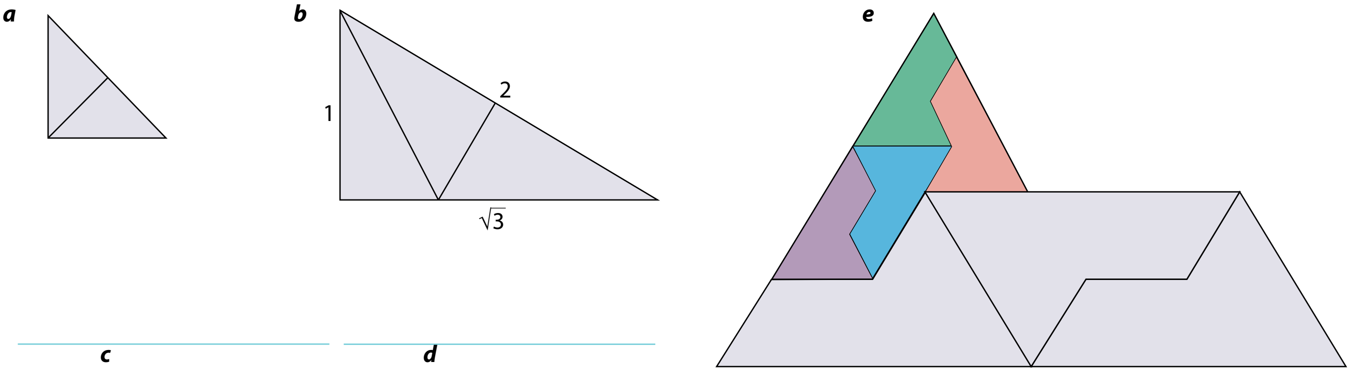
Printed above are the first three verses of Genesis in the King James Bible. Select any of the 10 words in the first verse: "In the beginning God created the heaven and the earth." Count the number of letters in the chosen word and call this number *x*. Then go to the word that is *x* words ahead. (For example, if you picked "in," go to "beginning.") Now count the number of letters in this word—call it *n*—then jump ahead another *n* words. Continue in this manner until your chain of words enters the third verse of Genesis.

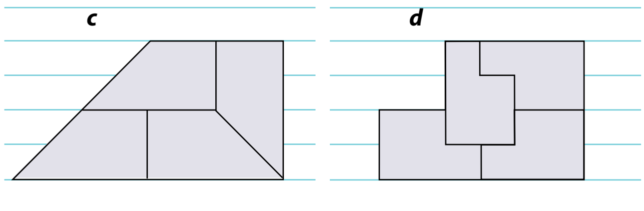
On what word does your count end? Is the answer happenstance or part of a divine plan?

4



A magician arranges a deck of cards so that the black and red cards alternate. She cuts the deck about in half, making sure that the bottom cards of each half are not the same color. Then she allows you to riffle-shuffle the two halves together, as thoroughly or carelessly as you please. When you're done, she picks the first two cards from the top of the deck. They are a black card and a red card (not necessarily in that order). The next two are also a black card and a red card. In fact, every succeeding pair of cards will include one of each color. How does she do it? Why doesn't shuffling the deck produce a random sequence?





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LOW-ORDER REP-TILES fit together to make larger replicas of themselves. The isosceles right triangle (*a*) is a rep-2 figure: two such triangles form a larger triangle with the same shape. A rep-3 triangle (*b*) has angles of 30, 60 and 90 degrees. Other rep-tiles include a rep-4 quadrilateral (*c*) and a rep-4 hexagon (*d*). The sphinx (*e*) is the only known rep-4 pentagon.

ignore such material. For 40 years I have done my best to convince educators that recreational math should be incorporated into the standard curriculum. It should be regularly introduced as a way to interest young students in the wonders of mathematics. So far, though, movement in this direction has been glacial.

I have often told a story from my own high school years that illustrates the dilemma. One day during math study period, after I'd finished my regular assignment, I took out a fresh sheet of paper and tried to solve a problem that had intrigued me: whether the first player in a game of ticktacktoe can always win, given the right strategy. When my teacher saw me scribbling, she snatched the sheet away from me and said, "Mr. Gardner, when you're in my class I expect you to work on mathematics and nothing else."

The ticktacktoe problem would make a wonderful classroom exercise. It is a superb way to introduce students to combinatorial mathematics, game theory, symmetry and probability. Moreover, the game is part of every student's experience: Who has not, as a child, played ticktacktoe? Yet I know few mathematics teachers who have included such games in their lessons.

According to the 1997 yearbook of the mathematics teachers' council, the latest trend in math education is called "the new new math" to distinguish it from "the new math," which flopped so disastrously several decades ago. The newest teaching system involves dividing classes into small groups of students and instructing the groups to solve problems through cooperative reasoning. "Interactive learning," as it is called, is substituted for lecturing. Although there are some positive aspects of the new new math, I was struck by the fact that the yearbook had nothing to say about the value of recreational mathematics, which lends itself so well to cooperative problem solving.

Let me propose to teachers the following experiment. Ask each group of students to think of any three-digit number—let's call it ABC. Then ask the students to enter the sequence of digits twice into their calculators, forming the number ABCABC. For example, if the students thought of the number 237, they'd punch in the number 237,237. Tell the students that you have the psychic power to predict that if they divide ABCABC by 13 there will be no remainder. This will prove to be true. Now ask them to divide the result by 11. Again, there will be no remainder. Finally, ask them to divide by 7. Lo and behold, the original number ABC is now in the calculator's readout. The secret to the trick is simple: ABCABC = ABC × 1,001 = ABC × 7 × 11 × 13. (Like every other integer, 1,001 can be factored into a unique set of prime numbers.) I know of no better introduction to number theory and the properties of primes than asking students to explain why this trick always works.

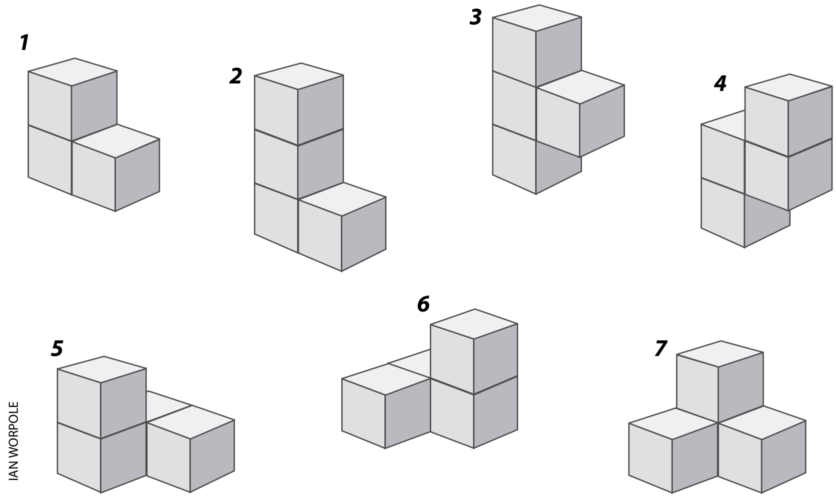
Polyominoes and Penrose Tiles

One of the great joys of writing the *Scientific American* column over 25 years was getting to know so many authentic mathematicians. I myself am little more than a journalist who loves mathematics and can write about it glibly. I took no math courses in college. My columns grew increasingly sophisticated as I learned more, but the key to the column's popularity was the fascinating material I was able to coax from some of the world's best mathematicians.

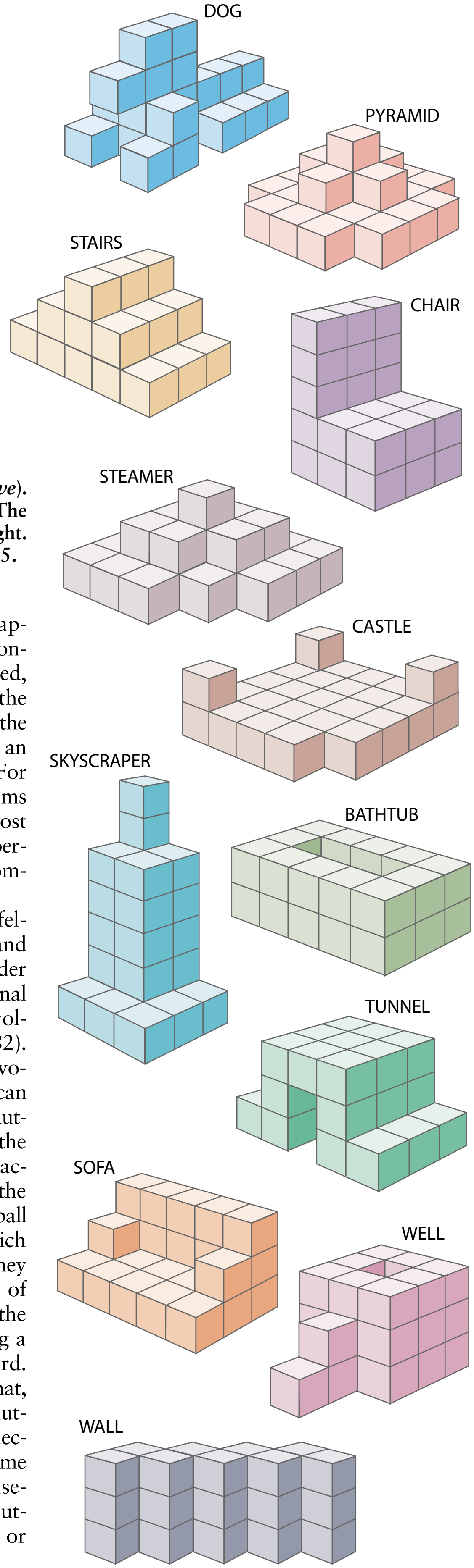
Solomon W. Golomb of the University of Southern California was one of the first to supply grist for the column. In the May 1957 issue I introduced his studies of polyominoes, shapes formed by joining identical squares along their edges. The domino—created from two such squares—can take only one shape, but the tromino, tetromino and pentomino can assume a variety of forms: Ls, Ts, squares and so forth. One of Golomb's early problems was to determine whether a specified set of polyominoes, snugly fitted together, could cover a checkerboard without missing any squares. The study of polyominoes soon evolved into a flourishing branch of recreational mathematics. Arthur C. Clarke, the science-fiction author, confessed that he had become a "pentomino addict" after he started playing with the deceptively simple figures.

Golomb also drew my attention to a class of figures he called "rep-tiles"—identical polygons that fit together to form larger replicas of themselves. One of them is the sphinx, an irregular pentagon whose shape is somewhat similar to that of the ancient Egyptian monument. When four identical sphinxes are joined in the right manner, they form a larger sphinx with the same shape as its components. The pattern of rep-tiles can expand infinitely: they tile the plane by making larger and larger replicas.

The late Piet Hein, Denmark's illustrious inventor and poet, became a



SOMA PIECES are irregular shapes formed by joining unit cubes at their faces (*above*). The seven pieces can be arranged in 240 ways to build the 3-by-3-by-3 Soma cube. The pieces can also be assembled to form all but one of the structures pictured at the right. Can you determine which structure is impossible to build? The answer is on page 75.



good friend through his contributions to "Mathematical Games." In the July 1957 issue, I wrote about a topological game he invented called Hex, which is played on a diamond-shaped board made of hexagons. Players place their markers on the hexagons and try to be the first to complete an unbroken chain from one side of the board to the other. The game has often been called John because it can be played on the hexagonal tiles of a bathroom floor.

Hein also invented the Soma cube, which was the subject of several columns (September 1958, July 1969 and September 1972). The Soma cube consists of seven different polycubes, the three-dimensional analogues of polyominoes. They are created by joining identical cubes at their faces. The polycubes can be fitted together to form the Soma cube—in 240 ways, no less—as well as a whole panoply of Soma shapes: the pyramid, the bathtub, the dog and so on.

In 1970 the mathematician John Conway—one of the world's undisputed geniuses—came to see me and asked if I had a board for the ancient Oriental game of go. I did. Conway then demonstrated his now famous simulation game called Life. He placed several counters on the board's grid, then removed or added new counters according to three simple rules: each counter with two or three neighboring counters is allowed to remain; each counter with one or no neighbors, or four or more neighbors, is removed; and a new counter is added to each empty space adjacent to exactly three counters. By applying these rules repeatedly, an astonishing variety of forms can be created, including some that move across the board like insects. I described Life in the October 1970 column, and it became an instant hit among computer buffs. For many weeks afterward, business firms and research laboratories were almost shut down while Life enthusiasts experimented with Life forms on their computer screens.

Conway later collaborated with fellow mathematicians Richard Guy and Elwyn Berlekamp on what I consider the greatest contribution to recreational mathematics in this century, a two-volume work called *Winning Ways* (1982). One of its hundreds of gems is a two-person game called Phutball, which can also be played on a go board. The Phutball is positioned at the center of the board, and the players take turns placing counters on the intersections of the grid lines. Players can move the Phutball by jumping it over the counters, which are removed from the board after they have been leapfrogged. The object of the game is to get the Phutball past the opposing side's goal line by building a chain of counters across the board. What makes the game distinctive is that, unlike checkers, chess, go or Hex, Phutball does not assign different game pieces to each side: the players use the same counters to build their chains. Consequently, any move made by one Phutball player can also be made by his or her opponent.

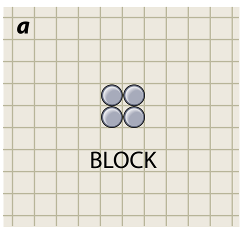
Other mathematicians who contributed ideas for the column include Frank Harary, now at New Mexico State University, who generalized the game of ticktacktoe. In Harary's version of the game, presented in the April 1979 issue, the goal was not to form a straight line of Xs or Os; instead players tried to be the first to arrange their Xs or Os in a specified polyomino, such as an L or a square. Ronald L. Rivest of the Massachusetts Institute of Technology allowed me to be the first to reveal—in the August 1977 column—the "public-key" cipher system that he co-invented. It was the first of a series of ciphers that revolutionized the field of cryptology. I also had the pleasure of presenting the mathematical art of Maurits C. Escher, which appeared on the cover of the April 1961 issue of *Scientific American,* as well as the nonperiodic tiling discovered by Roger Penrose, the British mathematical physicist famous for his work on relativity and black holes.

Penrose tiles are a marvelous example of how a discovery made solely for the fun of it can turn out to have an unexpected practical use. Penrose devised two kinds of shapes, "kites" and "darts," that cover the plane only in a nonperiodic way: no fundamental part of the pattern repeats itself. I explained the significance of the discovery in the January 1977 issue, which featured a pattern of Penrose tiles on its cover. A few years later a 3-D form of Penrose tiling became the basis for constructing a previously unknown type of molecular structure called a quasicrystal. Since then, physicists have written hundreds of research papers on quasicrystals and their unique thermal and vibrational properties. Although Penrose's idea started as a strictly recreational pursuit, it paved the way for an entirely new branch of solid-state physics.

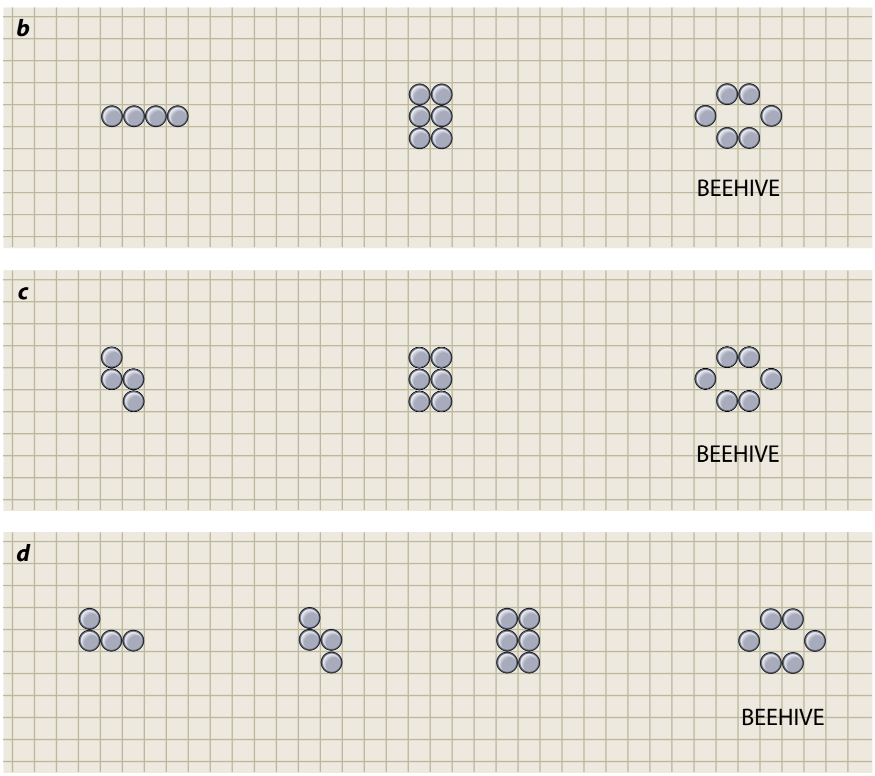
Leonardo's Flush Toilet

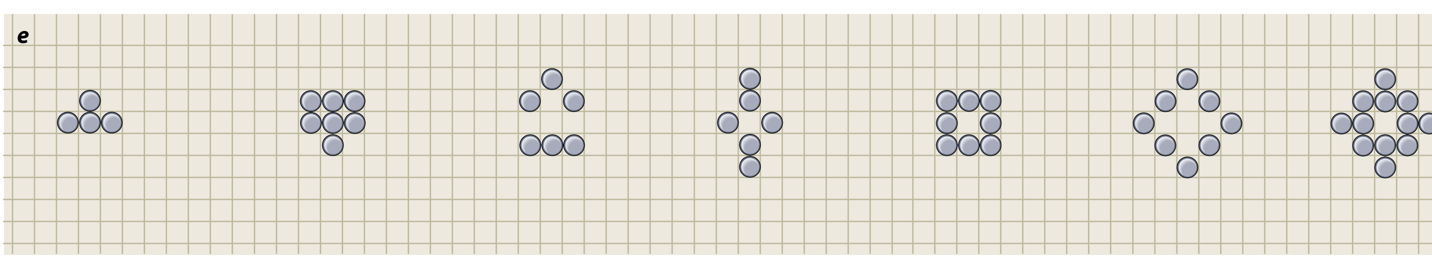
The two columns that generated the greatest number of letters were my April Fools' Day column and the one on Newcomb's paradox. The hoax column, which appeared in the April 1975 issue, purported to cover great breakthroughs in science and math. The startling discoveries included a refutation of relativity theory and the disclosure that Leonardo da Vinci had invented the flush toilet. The column also announced that the opening chess move of pawn to king's rook 4 was a certain game winner and that e raised to the power of $\pi \times \sqrt{163}$ was exactly equal to the integer 262,537,412,640,768,744. To my amazement, thousands of readers failed to recognize the column as a joke. Accompanying the text was a complicated map that I said required five colors to ensure that no two neighboring regions were colored the same. Hundreds of readers sent me copies of the map colored with only four colors, thus upholding the four-color theorem. Many readers said the task had taken days.

Newcomb's paradox is named after physicist William A. Newcomb, who originated the idea, but it was first described in a technical paper by Harvard University philosopher Robert Nozick. The paradox involves two closed boxes, A and B. Box A contains $1,000. Box B contains either nothing or $1 million. You have two choices: take only Box B or take both boxes. Taking both obviously seems to be the better choice, but there is a catch: a superbeing—God, if you like—has the power of



IN THE GAME OF LIFE, forms evolve by following rules set by mathematician John H. Conway. If four "organisms" are initially arranged in a square block of cells (*a*), the Life form does not change. Three other initial patterns (*b*, *c* and *d*) evolve into the stable "beehive" form. The fifth pattern (*e*) evolves into the oscillating "traffic lights" figure, which alternates between vertical and horizontal rows.





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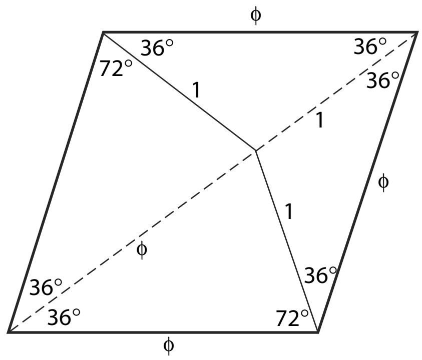
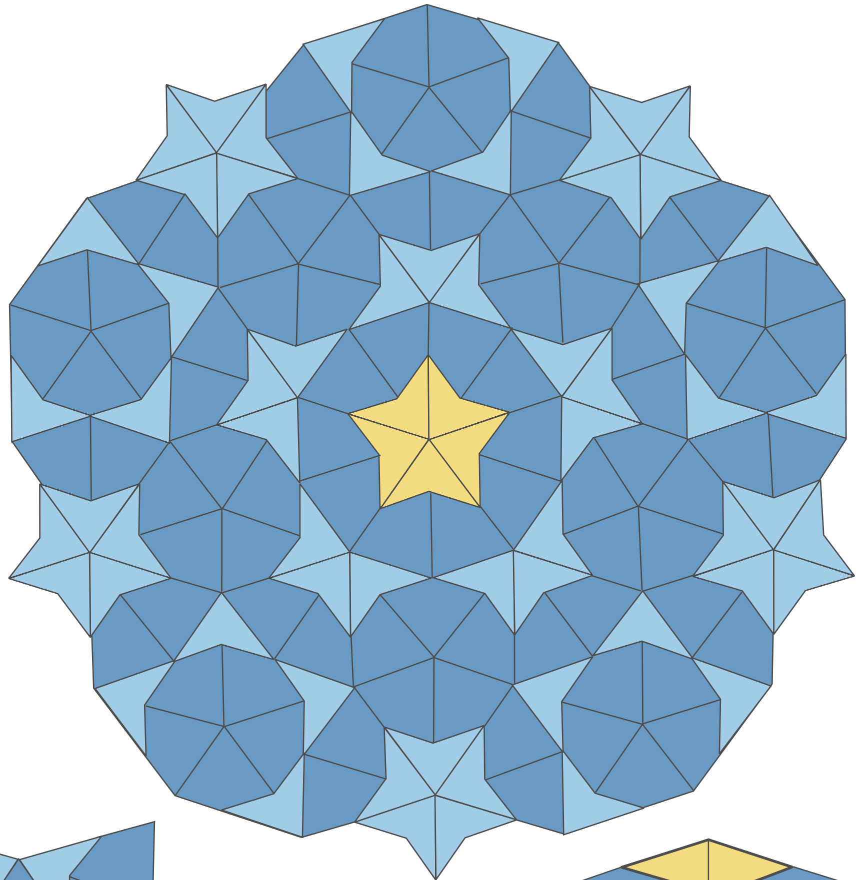
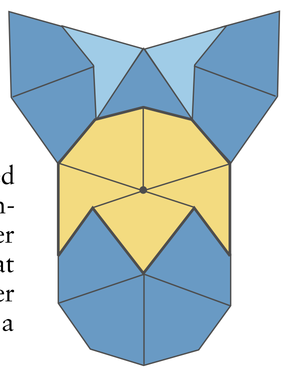


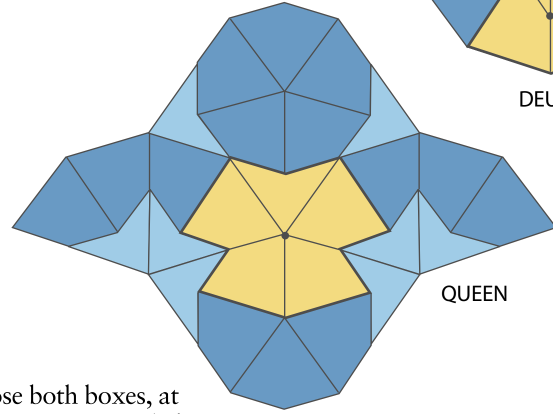
$$\phi = \frac{1+\sqrt{5}}{2}$$

PENROSE TILES can be constructed by dividing a rhombus into a "kite" and a "dart" such that the ratio of their diagonals is phi (φ), the golden ratio (*above*). Arranging five of the darts around a vertex creates a star. Placing 10 kites around the star and extending the tiling symmetrically generate the infinite star pattern (*right*). Other tilings around a vertex include the deuce, jack and queen, which can also generate infinite patterns of kites and darts (*below right*).

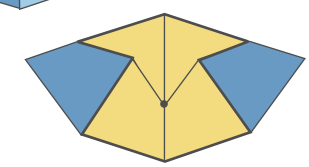




JACK



QUEEN



DEUCE

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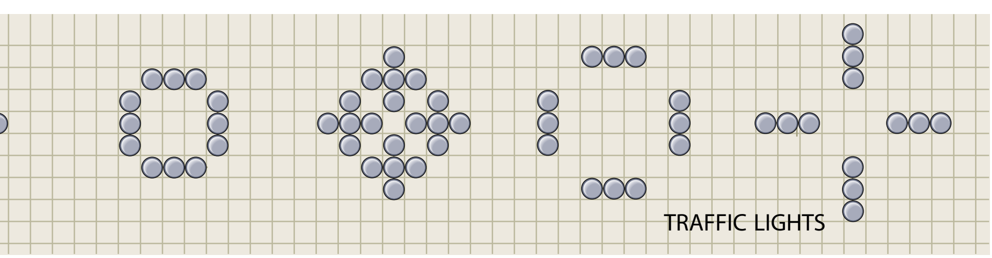
knowing in advance how you will choose. If he predicts that out of greed you will take both boxes, he leaves B empty, and you will get only the $1,000 in A. But if he predicts you will take only Box B, he puts $1 million in it. You have watched this game played many times with others, and in every case when the player chose both boxes, he or she found that B was empty. And every time a player chose only Box B, he or she became a millionaire.

How should you choose? The pragmatic argument is that because of the previous games you have witnessed, you can assume that the superbeing does indeed have the power to make accurate predictions. You should therefore take only Box B to guarantee that you will get the $1 million. But wait! The superbeing makes his prediction *before* you play the game and has no power to alter it. At the moment you make your choice, Box B is either empty, or it contains $1 million. If it is empty, you'll get nothing if you choose only Box B. But if you choose both boxes, at least you'll get the $1,000 in A. And if B contains $1 million, you'll get the million plus another thousand. So how can you lose by choosing both boxes?

Each argument seems unassailable. Yet both cannot be the best strategy. Nozick concluded that the paradox, which belongs to a branch of mathematics called decision theory, remains unresolved. My personal opinion is that the paradox proves, by leading to a logical contradiction, the impossibility of a superbeing's ability to predict decisions. I wrote about the paradox in the July 1973 column and received so many letters afterward that I packed them into a carton and personally delivered them to Nozick. He analyzed the letters in a guest column in the March 1974 issue.

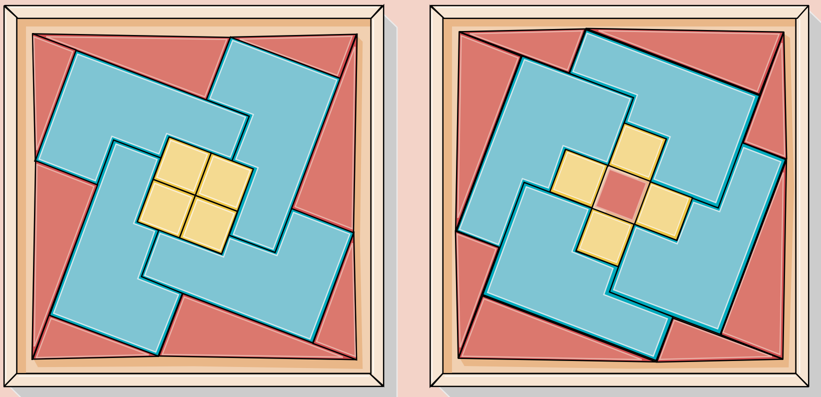
Magic squares have long been a popular part of recreational math. What makes these squares magical is the arrangement of numbers inside them: the numbers in every column, row and diagonal add up to the same sum. The numbers in the magic square are usually required to be distinct and run in



TRAFFIC LIGHTS

The Vanishing Area Paradox

Consider the figures shown below. Each pattern is made with the same 16 pieces: four large right triangles, four small right triangles, four eight-sided pieces and four small squares. In the pattern on the left, the pieces fit together snugly, but the pattern on the right has a square hole in its center! Where did this extra bit of area come from? And why does it vanish in the pattern on the left?



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The secret to this paradox—which I devised for the "Mathematical Games" column in the May 1961 issue of *Scientific American*—will be revealed in the Letters to the Editors section of next month's issue. Impatient readers can find the answer at www.sciam.com on the World Wide Web. —*M.G.*

consecutive order, starting with one. There exists only one order-3 magic square, which arranges the digits one through nine in a three-by-three grid. (Variations made by rotating or reflecting the square are considered trivial.) In contrast, there are 880 order-4 magic squares, and the number of arrangements increases rapidly for higher orders.

Surprisingly, this is not the case with magic hexagons. In 1963 I received in the mail an order-3 magic hexagon devised by Clifford W. Adams, a retired clerk for the Reading Railroad. I sent the magic hexagon to Charles W. Trigg, a mathematician at Los Angeles City College, who proved that this elegant pattern was the only possible order-3 magic hexagon—and that no magic hexagons of any other size are possible!

What if the numbers in a magic square are not required to run in consecutive order? If the only requirement is that the numbers be distinct, a wide variety of order-3 magic squares can be constructed. For example, there is an infinite number of such squares that contain distinct prime numbers. Can an order-3 magic square be made with nine distinct square numbers? Two years ago in an article in *Quantum,* I offered $100 for such a pattern. So far no one has come forward with a "square of squares"—but no one has proved its impossibility either. If it exists, its numbers would be huge, perhaps beyond the reach of today's fastest supercomputers. Such a magic square would probably not have any practical use. Why then are mathematicians trying to find it? Because it might be there.

The Amazing Dr. Matrix

Every year or so during my tenure at *Scientific American,* I would devote a column to an imaginary interview with a numerologist I called Dr. Irving Joshua Matrix (note the "666" provided by the number of letters in his first, middle and last names). The good doctor would expound on the unusual properties of numbers and on bizarre forms of wordplay. Many readers thought Dr. Matrix and his beautiful, half-Japanese daughter, Iva Toshiyori, were real. I recall a letter from a puzzled Japanese reader who told me that Toshiyori was a most peculiar surname in Japan. I had taken it from a map of Tokyo. My informant said that in Japanese the word means "street of old men."

I regret that I never asked Dr. Matrix for his opinion on the preposterous 1997 best-seller *The Bible Code,* which claims to find predictions of the future in the arrangement of Hebrew letters in the Old Testament. The book employs a cipher system that would have made Dr. Matrix proud. By selectively applying this system to certain blocks of text, inquisitive readers can find hidden predictions not only in the Old Testament but also in the New Testament, the Koran, the *Wall Street Journal*—and even in the pages of *The Bible Code* itself.

The last time I heard from Dr. Matrix, he was in Hong Kong, investigating the accidental appearance of π in well-known works of fiction. He cited, for example, the following sentence fragment in chapter nine of book two of H. G. Wells's *The War of the Worlds:* "For a time I stood regarding…" The letters in the words give π to six digits! SA

The Author

MARTIN GARDNER wrote the "Mathematical Games" column for *Scientific American* from 1956 to 1981 and continued to contribute columns on an occasional basis for several years afterward. These columns are collected in 15 books, ending with *The Last Recreations* (Springer-Verlag, 1997). He is also the author of *The Annotated Alice, The Whys of a Philosophical Scrivener, The Ambidextrous Universe, Relativity Simply Explained* and *The Flight of Peter Fromm,* the last a novel. His more than 70 other books are about science, mathematics, philosophy, literature and his principal hobby, conjuring.

Further Reading

RECREATIONS IN THE THEORY OF NUMBERS. Albert H. Beiler. Dover Publications, 1964.

MATHEMATICS: PROBLEM SOLVING THROUGH RECREATIONAL MATHEMATICS. Bonnie Averbach and Orin Chein. W. H. Freeman and Company, 1986.

MATHEMATICAL RECREATIONS AND ESSAYS. 13th edition. W. W. Rouse Ball and H.S.M. Coxeter. Dover Publications, 1987.

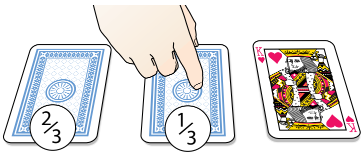
PENGUIN EDITION OF CURIOUS AND INTERESTING GEOMETRY. David Wells. Penguin, 1991.

MAZES OF THE MIND. Clifford Pickover. St. Martin's Press, 1992.

Answers to the Four Gardner Puzzles

(The puzzles are on page 69.)

1. Most people guess that the probability has risen from $1/3$ to $1/2$. After all, only two cards are face down, and one must be the ace. Actually, the probability remains $1/3$. The probability that you *didn't* pick the ace remains $2/3$, but Jones has eliminated some of the uncertainty by showing that one of the two unpicked cards is not the ace. So there is a $2/3$ probability that the other unpicked card is the ace. If Jones gives you the option to change your bet to that card, you should take it (unless he's slipping cards up his sleeve, of course).



I introduced this problem in my October 1959 column in a slightly different form—instead of three cards, the problem involved three prisoners, one of whom had been pardoned by the governor. In 1990 Marilyn vos Savant, the author of a popular column in *Parade* magazine, presented still another version of the same problem, involving three doors and a car behind one of them. She gave the correct answer but received thousands of angry letters—many from mathematicians—accusing her of ignorance of probability theory! The fracas generated a front-page story in the *New York Times*.

2. The sum is 111. The trick always works because the matrix of numbers is nothing more than an old-fashioned addition table (*below*). The table is generated by two sets of numbers: (3, 1, 5, 2, 4, 0) and (25, 31, 13, 1, 7, 19). Each number in the matrix is the sum of a pair of numbers in the two sets. When you choose the six circled numbers, you are selecting six pairs that together include all 12 of the generating numbers. So the sum of the circled numbers is always equal to the sum of the 12 generating numbers. These special magic squares were the subject of my January 1957 column.

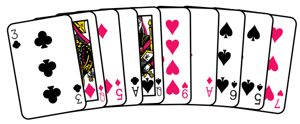
	3	1	5	2	4	0
25	28	~~26~~	~~30~~	~~27~~	~~29~~	~~25~~
31	~~34~~	~~32~~	~~36~~	~~33~~	35	~~31~~
13	~~16~~	~~14~~	18	~~15~~	~~17~~	~~13~~
1	~~4~~	~~2~~	~~6~~	3	~~5~~	~~1~~
7	~~10~~	8	~~12~~	~~9~~	~~11~~	~~7~~
19	~~22~~	~~20~~	~~24~~	~~21~~	~~23~~	19

3. Each chain of words ends on "God." This answer may seem providential, but it is actually the result of the Kruskal Count, a mathematical principle first noted by mathematician Martin Kruskal in the 1970s. When the total number of words in a text is significantly greater than the number of letters in the longest word, it is likely that any two arbitrarily started word chains will intersect at a keyword. After that point, of course, the chains become identical. As the text lengthens, the likelihood of intersection increases.

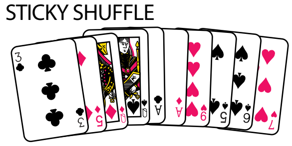
I discussed Kruskal's principle in my February 1978 column. Mathematician John Allen Paulos applies the principle to word chains in his upcoming book *Once upon a Number*.

4. For simplicity's sake, imagine a deck of only 10 cards, with the black and red cards alternating like so: BRBRBRBRBR. Cutting this deck in half will produce two five-card decks: BRBRB and RBRBR. At the start of the shuffle, the bottom card of one deck is black, and the bottom card of the other deck is red. If the red card hits the table first, the bottom cards of both decks will then be black, so the next card to fall will create a black-red pair on the table. And if the black card drops first, the bottom cards of both decks will be red, so the next card to fall will create a red-black pair. After the first two cards drop—no matter which deck they came from—the situation will be the same as it was in the beginning: the bottom cards of the decks will be different colors. The process then repeats, guaranteeing a black and red card in each successive pair, even if some of the cards stick together (*below*).

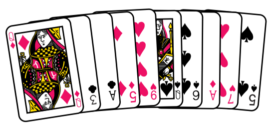
THOROUGH SHUFFLE



STICKY SHUFFLE

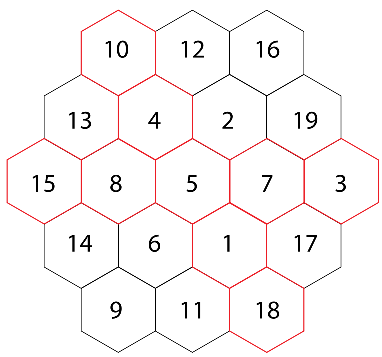


OR

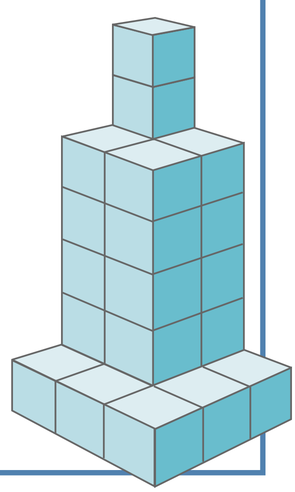


This phenomenon is known as the Gilbreath principle after its discoverer, Norman Gilbreath, a California magician. I first explained it in my column in August 1960 and discussed it again in July 1972. Magicians have invented more than 100 card tricks based on this principle and its generalizations.

—*M.G.*



MAGIC HEXAGON
has a unique property: every straight row of cells adds up to 38.



SKYSCRAPER
cannot be built from Soma pieces.
(The puzzle is on page 71.)

IAN WORPOLE