

GAMES KIT

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TEXT

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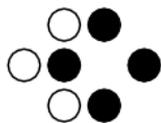


Gathering 4 Gardner.

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LUDUS

GAMES KIT

This volume contains the rule sets of all the games of the Portuguese Championship of Mathematical games (2004-2013) and some exercises on their strategy and tactics.

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Introduction

It is common knowledge that the practice of certain board games has a positive impact in learning mathematics. These games are supposed to help their players to think well. It is clear to all that these games have a relation with mathematics that some other educational activities, like football and painting, lack.

The connections between these board games and mathematics are manifold. Some have to do with board games in general, other with mathematics per se. We will try to pinpoint some of the most important:

Focus: Who fails to address all the relevant data related to each situation will not perform well in a game or in mathematics. To many a time the difficulty in the mathematical learning a child experiences is linked to the fact that she does not focus on the several parts of the material on the table.

Visualization: When playing a game it is very important to picture situations in your mind, so you can anticipate the near future. In mathematics this skill is paramount in graphic data, logic, geometry... In a game, the situations have a natural order: the sequence of moves; in mathematics there are sequences of arguments, of graphics, ...

Think first, act later: A well known public enemy: act first, think later. We must be able to support our decisions on sound justifications. This is very important in game playing and in mathematics, of course.

Assess pros and cons: Decision processes are based on evaluation of options. In problem solving (mathematical or ludic) we encounter several possible lines of action. Being able to choose the best ones shows deep understanding of the subjects involved.

Some other skills could be mentioned, as memorization, mental calculus, ...

Some game issues are addressed *directly* with mathematics. For example, *The Game of de Marienbad*, solved by the mathematician C. L. Bouton, or *Dots & Boxes*, which has been studied by the mathematician E. Berlekamp.

Another similarity lies on the theorem-like thinking of players that generalize and adapt particular game situations to more general contexts, much as mathematicians treat diverse examples with the same theoretical result. There is pattern recognition in both the gamer and the problem solver.

The emotional world could be addressed here as well. Competition, interaction with another human being, who opposes our intentions, is a plus of the gaming activity. Usually, kids adhere to games better than to mathematics because games are... games! They come as opportunities to just think, without no other

consequences behind victory or defeat. Thinking is a pleasurable activity. The better you think the closer you are of the glory of winning. An ideal situation for exercising the mind!

In mathematics, most of the times, the situation is not as exciting. The goal, in a game, is clear. In mathematics it can be obscure to the student. Even if the thinking processes are similar and equally nice, the context of games is clearly more appealing. We must emphasize that games and mathematics are two different areas. Mathematical games are a very nice activity, but must be understood as an after school one. Like swimming.

Trying to be a better player is a fundamental part of the enjoyment of playing games. If you don't care about loosing or winning, or about your level of sophistication, you are not a full gamer, and ruins the benefits of its practice. Mathematical games must be well thought, you must try to reach the optimal solutions so you can get closer to your goal. Otherwise you are not really playing.

The national championship of mathematical games (Portugal)

In Portugal, the agency Ciência Viva, the Ludus Association, APM, SPM, joined efforts to organize a National Championship of Mathematical Games (CNJM) which has been very successful, with annual participation of approximately 100,000. The choice of the games in each edition had in account their substance, variety, historical relevance, age targeted, and degree of difficulty of building play materials.

We focused on games with no chance factor (*Backgammon* is out), no hidden information (*Battle Ships* is out). We refer to these games as mathematical games. They include famous games as *chess*, *checkers*, and *tic tac toe*. We will show below some other games of this kind.

The Rule Sets

Amazons

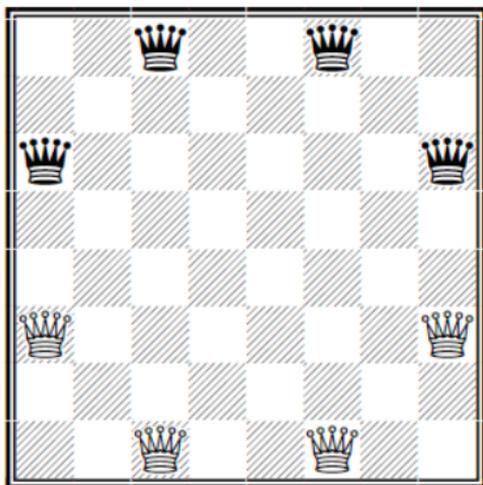
Author: Walter Zamkaskas, 1988

Equipment: A 8x8 square board, 4 white amazons, 4 black amazons and 56 neutral marks.

Goal: Be the last to move.

Rules:

Each player places her Amazons in the following setup:



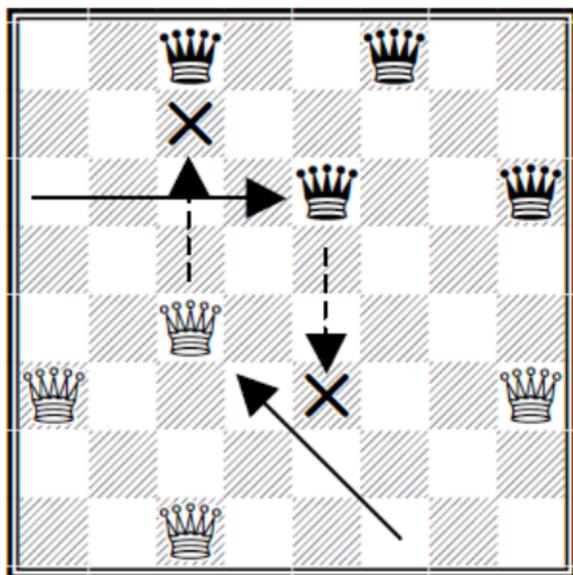
On each turn, each player makes two actions:

- 1) Moves a friendly Amazon over a straight line of empty cells, orthogonally or diagonally (like a chess queen).

2) Then, the player places a neutral mark on any empty square within moving range of the moved Amazon's current position.

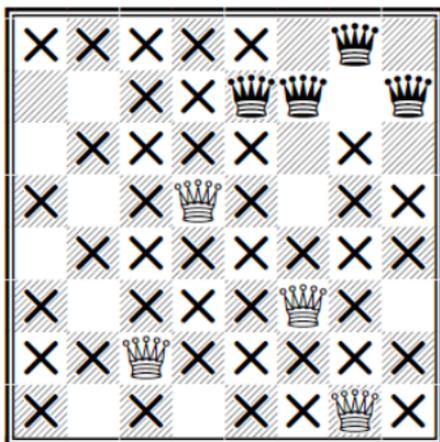
Since, on each move, a square is marked the match must end. Loses the player unable to perform the two required actions.

Below we can see an example of a starting match. White moves one of her Amazons diagonally three squares and then places a mark three squares north. Then Black moves one of his Amazons four squares to the east placing the neutral mark three squares to the south.

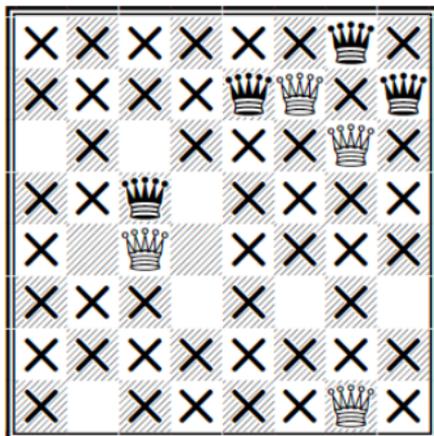


Let's see some problems that try to show some typical positions found during a match.

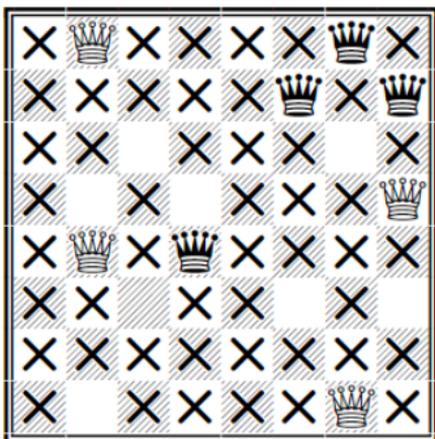
Problem 1: White plays and wins.



Problem 2: White plays and wins.



Problem 3: White plays and wins.



The solutions are found at the end of this document.

Breakthrough

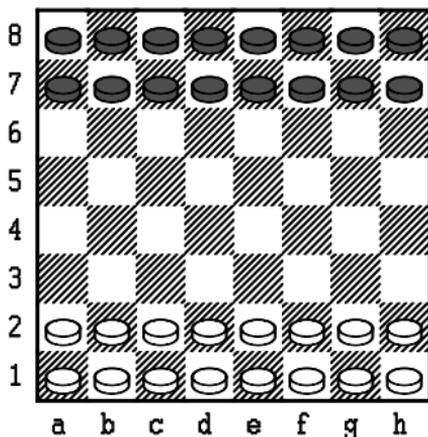
Author: Dan Troyka, 2000

Equipment: A 8x8 square board, 16 white pieces and 16 black pieces.

Goal: Wins the player that is the first to move one of his pieces to the board's last line (8th line for White, 1st line for Black).

Rules

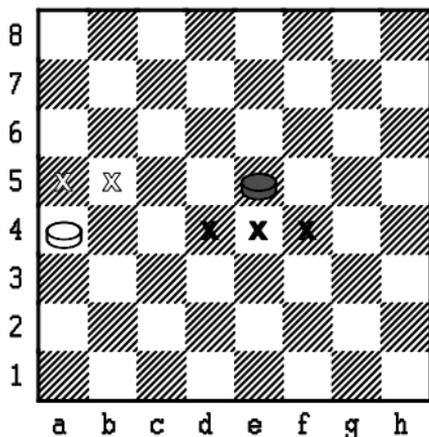
This is the initial setup:



On each turn, each player moves one of his pieces. White starts.

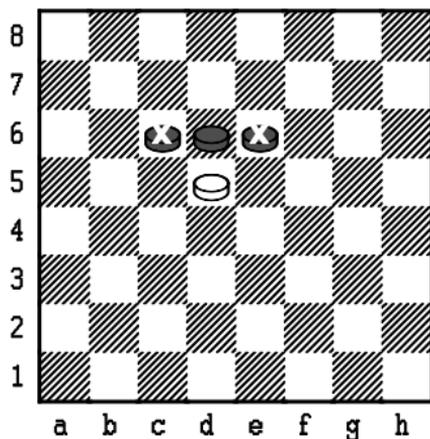
The pieces always move forwards, to an adjacent empty square either in front or in one of the diagonals.

The next diagram shows where either piece could move.



Pieces can capture enemy pieces that are at one of the adjacent diagonal front squares. Capture is by replacement (just like Chess pawns), optional and only one piece can be captured in the player's turn (multiple captures is not allowed).

The next diagram shows which black pieces could be captured by the white piece. Notice that the white piece cannot move to d6 (the square is occupied) not capture that black piece (diagonal captures only).

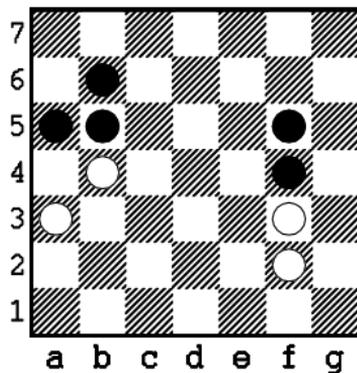


This is a race game, winner of the 2001 8x8 Game Design Competition. An important factor was how the game combines very simple rules with a non trivial tactical complexity. The game can also be played on any rectangular board. The size used in the Portuguese Championship was 7x7. But, even matches in size 5x5 can be interesting despite the small area of such board.

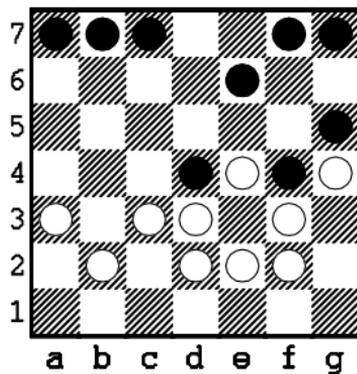
Breakthrough matches usually end quickly since the game converges to the end, since all pieces must always move forwards. Every player always has available moves – e.g., there is no way for an adversary to block the player’s most advanced piece. So, no draws are possible in Breakthrough.

Next we present some problems that show typical tactics that can be used in real matches.

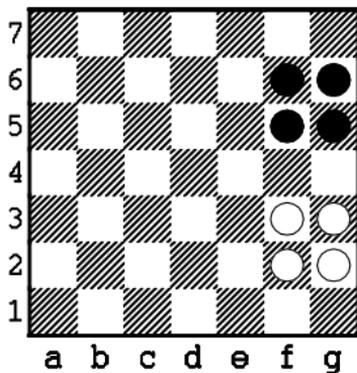
Problem 1: White moves and wins.



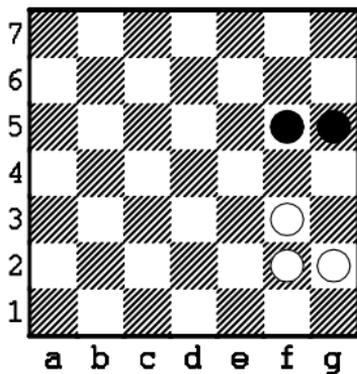
Problem 2: Black moves and wins.



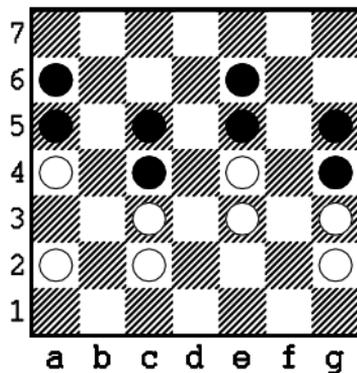
Problem 3: White moves and wins.



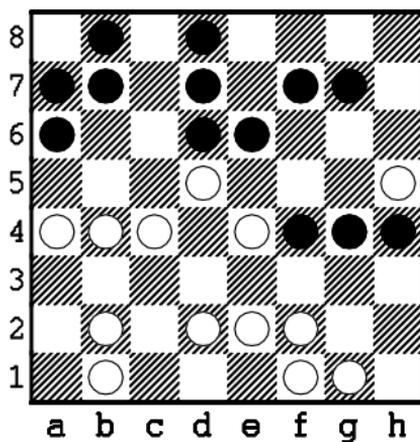
Problem 4: Black moves and wins.



Problem 5: White moves and wins.



Problem 6: Black moves and wins.



Go

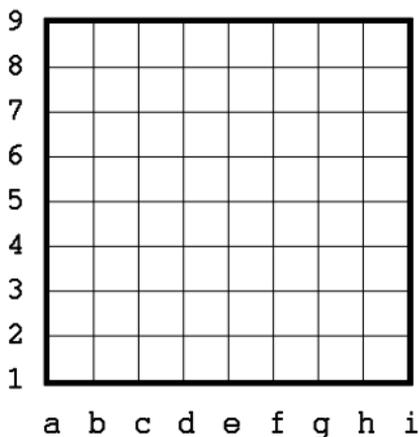
Traditional game. China, Japan, Korea.

Equipment: A 9x9 square board (played in the intersections), 40 white pieces and 40 black pieces.

Goal: Wins the player with the larger combined sum between her stones and her territory.

Rules

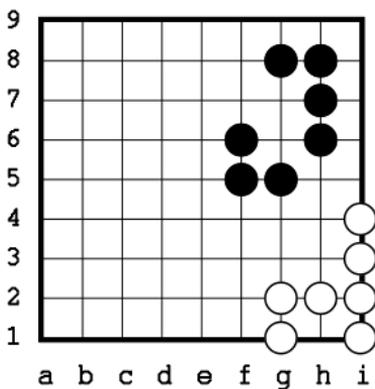
The game of Go is played in the intersection of a square board. Traditionally the board dimension has 19 rows and 19 columns, but the game is also played on board of size 13x13 or 9x9. Herein, we use the 9x9 board using the typical orthogonal coordinates (letters to represent columns, numbers to represent rows):



Each player possesses a sufficient number of stones of one color (there are black and white stones). Black starts.

To understand the rules, first we need to explain some concepts. The first one is the group. A group is a set of friendly stones (i.e., of the same color), that are connected horizontally or vertically one to each other (in Go there are no diagonal connections).

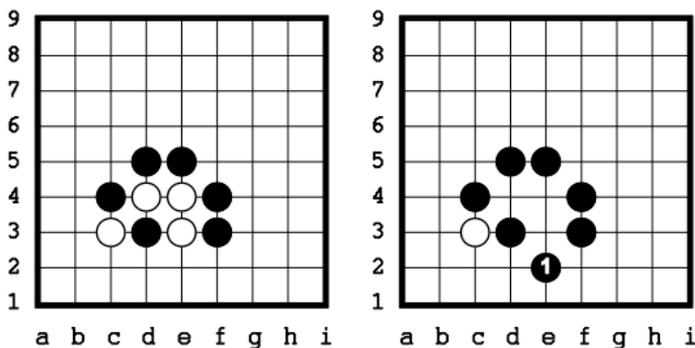
The next diagram show three examples of groups, one white group with seven stones, and two black groups, one with four and another with three stones:



Each group has an amount of liberty. A group's liberty is the number of horizontal and vertical adjacent empty intersections that the group has. In the previous diagram, the white group has seven liberties (f1, f2, g3, h3, h1, h4 e i5), the largest black group has nine liberties, and the second black group has seven.

A group at the board edge has potential fewer liberties than a group of the same size in the middle of the board. This is an important fact in how to play Go.

Players, alternatively, pass their turn or place a friendly stone on an empty intersection. If, after the placement, an opponent's group loses its liberties, than that group is captured and removed from the game. Let's see an example (initial position at the left diagram):



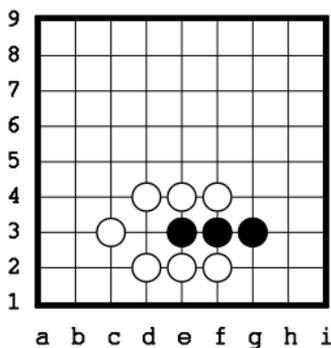
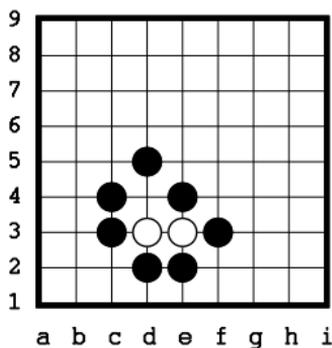
Black moved at [1]. The white group lost its last liberty and was captured. If it was White's turn, she could have played at d2 and captured the d3 black stone.

A group with only one liberty is said to be at *atari* (one of the many Japanese terms to describe a shape or situation in Go).

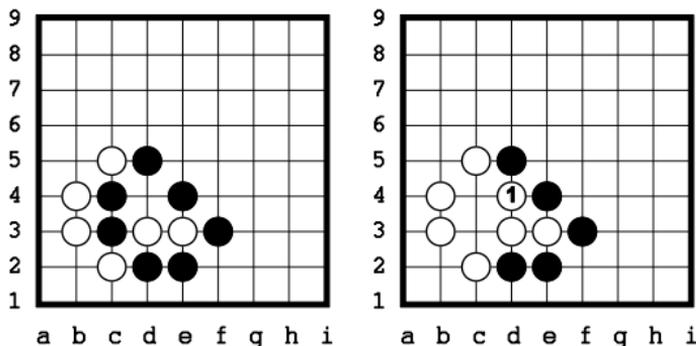
However, there are restrictions to the placement of stones.

One of such restrictions is that a placement is forbidden if it results on the immediate capture of the friendly group adjacent to the dropped stone (assuming there are no captures producing some extra liberties). In other words, suicides are forbidden.

In the left diagram, White cannot play at d4 because it would leave the group with no liberties. In the right diagram, however, Black can play at d3. Its group still has liberties left (g2,h3,g4).



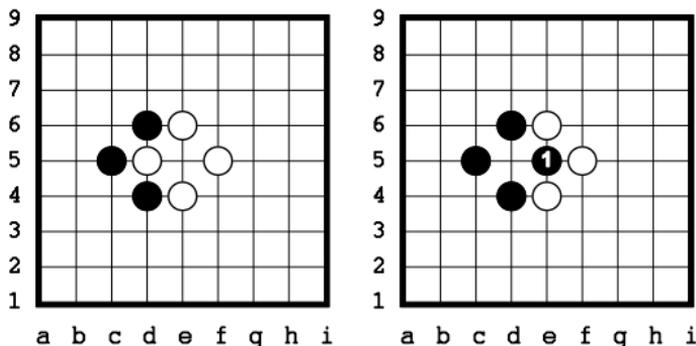
As hinted before, there is one situation where an (apparent) suicide is allowed. If the placement produces captures that gain new liberties to its group. The next diagram shows an example of this:



White moves in the left diagram. She decides to move [1] (right diagram). The white group lost its last liberty, but before that it capture the black group and regain new liberties. In this case, the move was legal.

There is a second restriction for placing stones. This restriction forbids the repetition of the previous board position. It is called the *Ko* rule.

In the following left diagram, Black must move. If he plays e5, he captures the white stone at d5 (right diagram). In the next move, White could (if there was no *Ko* restriction) move at d5 to capture e5. But if that would happen we would repeat the starting position (and risk a never ending sequence of moves).



With the Ko rule, these situation are automatically forbidden. After Black's e5, White must choose another move different from d5.

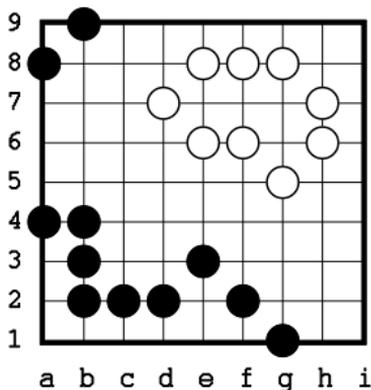
Notice however that the Ko rule only deals with the immediate previous position. If after White's move, Black does not play at d5, White could play at d5 and then the Ko restriction would favor her.

The game ends when both players pass in consecutive turns.

How to determine the winner? It seems that Go is a capture game, but in reality it is a territorial one.

A territory in Go is a set of connected intersections surrounded by stones of the same color and/or the board edges. The value of a territory is equal to the number of intersections it contains.

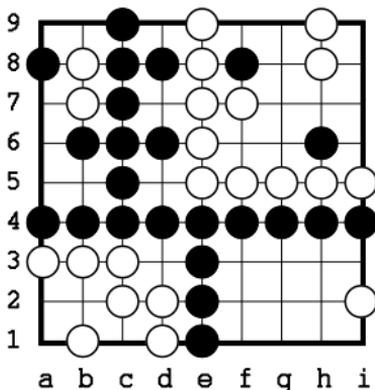
In the next diagram there are three territories: one white territory with four intersections (e7, f7, g7, g6), a black territory with nine intersections and a smaller one with one intersection (a9):



Usually at the end of the game, isolated enemy stones can be found within a player's territory. These stones are then removed from the board as if they were captured (doing it without explicitly executing the captures, speeds the game length).

The winner is the player with more the largest sum of (a) friendly stones on board, plus (b) the sum of all his territories value.

The next diagram shows an example of an endgame:



First, the isolated stones are removed. Here they are the white stone b7, b8, and the black stones f8 e h6. Then we count how many stones are left (20 black stones and 19 white stones). Next we check their territory sizes. White has two territories valuing 4 and 13 intersections. Black also has two territories with 8 and 12 intersections. Let's sum:

Black = 20 stones + (8+12) intersections = 40 points

White = 19 stones + (4+13) intersections = 36 points

So Black won the game.

It is important the each player only makes his final pass when there is no doubt about the ownership of all the territories in the board. If there are doubts the players should continue the game until they are resolved. This is common in beginner matches and should be seen with normality.

It is well known that the first player does have some advantage. With more experienced players there is a compensation for playing White. This compensation is called *Komi* and for the 9x9 board usually is 5.5 points (the decimal part is to prevent draws).

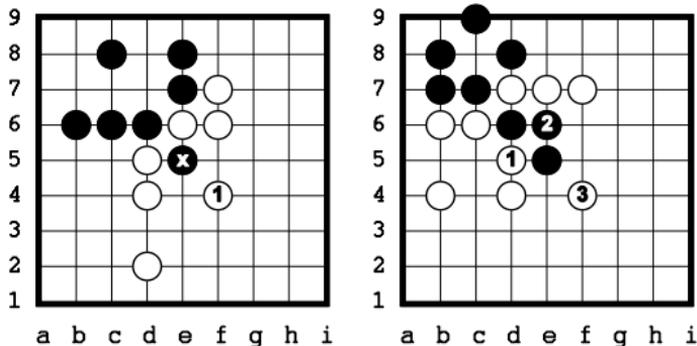
A last remark about the winning condition: the described method is used in the Chinese rule set. An alternative is the Japanese method which uses the number of captured stones and the size of the territory. In almost all cases – but not all – both methods determine the same winner.

Suggestions

Go does not have many different rules. Go's complexity derives from the interaction of these small set of rules. We'll show some important tactics that a beginner should know.

There are some common stone shapes that is useful to recognize. The study of Go shapes is called *Tesuji*. Let's see some of them:

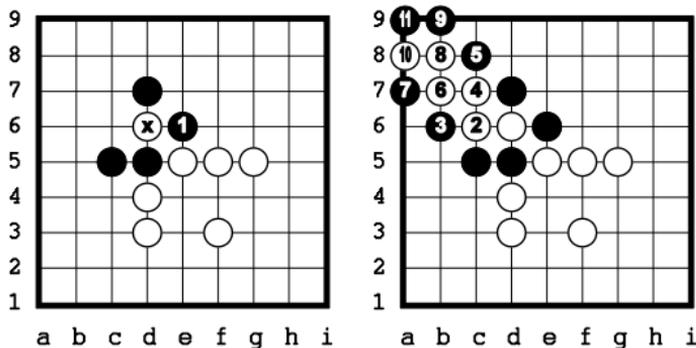
Nets: a net is an effective surrounding of a group by consecutively cutting its liberties. Two examples:



In the left diagram, White plays at [1] making a net over the marked black stone. If Black tries to escape by f5, White answers g5.

In the right diagram, after [1] to [3] White made a net condemning the black group.

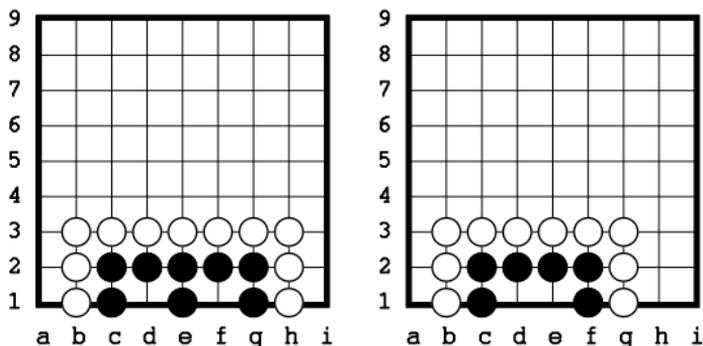
Ladders: a ladder is another common shape that should be understood to prevent big loses in a typical match. A ladder occurs when there is the systematic reduction of a group from two liberties to one.



In the left diagram Black notices a possible ladder and plays at [1]. From this moment, the marked white stone is already doomed. If White does not recognize this and tries to save, we will get something like the right diagram. At the end, five white stones would be captured, leaving Black in a very strong position.

An important Go lemma is to do not brood over your past mistakes. If some territory seems lost, players should move elsewhere, and do not insist by compounding his initial error.

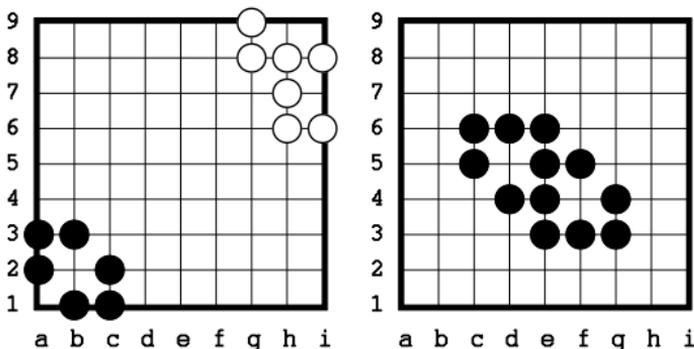
Perhaps the most important concept in Go is the possibility of making uncapturable groups. When this happens we say that that group is *alive*. Likewise, if a group cannot be saved from capture, we say the group is *dead*. The next diagrams show one black group which is alive (left diagram) and one that is dead (right diagram):



The black left group is alive because White cannot play in d1 or in f1 (both moves are illegal suicides). Since White cannot play two stones simultaneously, the black group cannot be captured.

However, the right black group can be captured. White plays at d1 make an Atari (a threat of immediate capture). Then Black could move at e1 (capturing d1) and then White plays again at d1 capturing the seven black stones.

Other examples of living groups:



It's vital for a Go player to know how to manage the living and death situations of her groups and also of the adversaries. Many games are won after subtle battles to capture or protect small groups of stones.

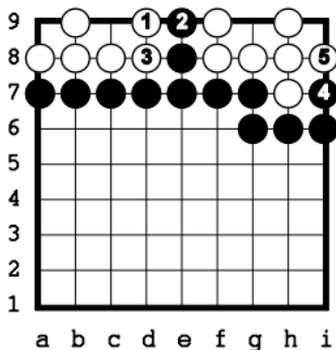
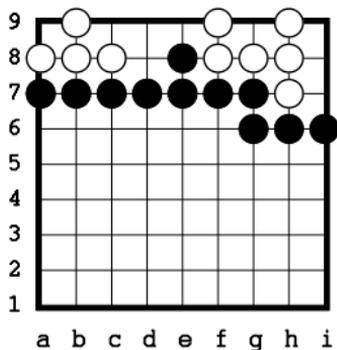
Living groups are strongholds for territory, influencing the nearby board zones (and sometimes, not so near zones). Friendly stones near living groups have powerful backups that help the player to dominate the board.

This question of life and death is a derivation of two Go concepts: liberties and not allowing suicides. It is a perfect example of rule interaction that produces deep tactical and strategic complexities.

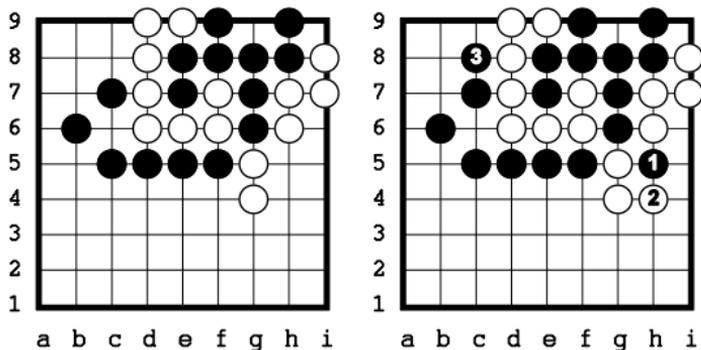
Let's analyze the following life and death problem (in Japan this is called a Tsumego).

In the left diagram, White moves to save her groups (please, hide the right diagram if you want to find the solution by yourself and stop reading).

In the right diagram we see the correct sequence that allows White to save both groups.



It is common that sacrifices must be made to obtain better results. In the next diagram, Black is having trouble to protect his group. There is a white threat in i9 followed by g9. Black moves.

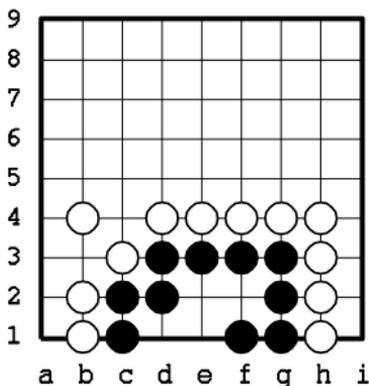


To stop those threats, Black must play h5. This prevents i9 (it would result in the capture of the white group) and threatens the right white group. So, White replies in h4 to attack h5.

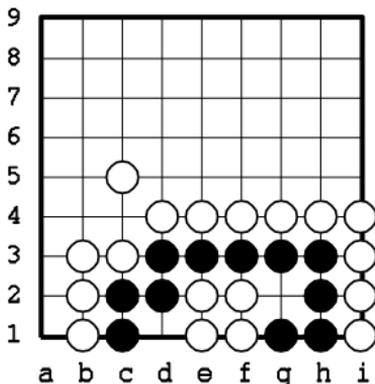
By doing this, Black won a tempo (since White must play at i5 to conclude the capture of h5), and takes the opportunity to play at c8 (move [3]). The situation has reverted! Now it is the big white group that risks capture.

We propose some life and death problems, where it is necessary to find the right move to keep your own groups or to capture adversary groups (solutions are found at the end of this text).

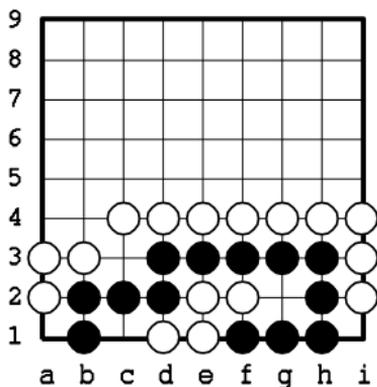
Problem 1: White moves. The black group is alive or dead?



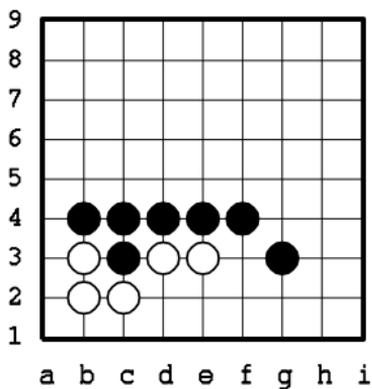
Problem 2. The black group is alive or dead?



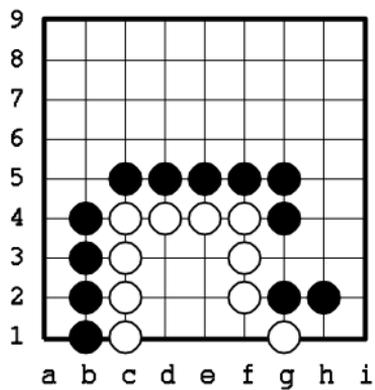
Problem 3. The black group is alive or dead?



Problem 4. Black moves and captures the white stones.



Problem 5. Black moves and captures the white stones.



Hex

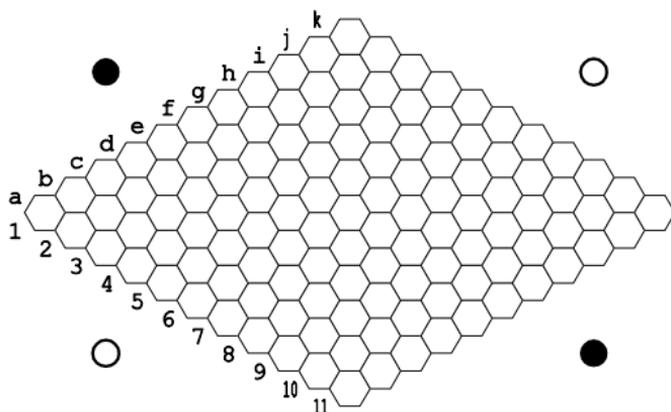
Author: Piet Hein, John Nash

Equipment: An 11 by 11 hexagonal board, 50 white stones, 50 black stones.

Goal: Make a connected path between the two opposite edges of its color.

Rules

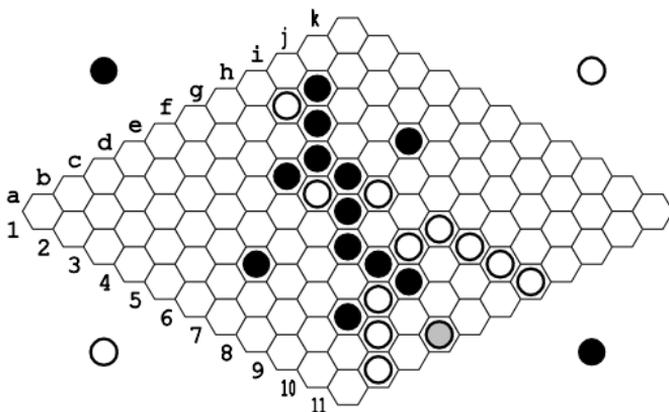
The game begins in the following empty board:



In each move, each player drops a friendly stone on an empty cell.

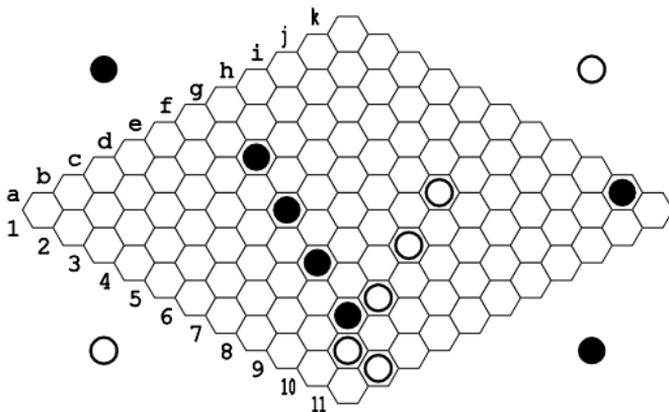
Black wins by creating a connected chain of black stones from the black margins (in the diagram, the northwest and southeast margins), while White wins by connecting the white edges by a white chain.

Problem 1: Check that White cannot prevent Black's victory.

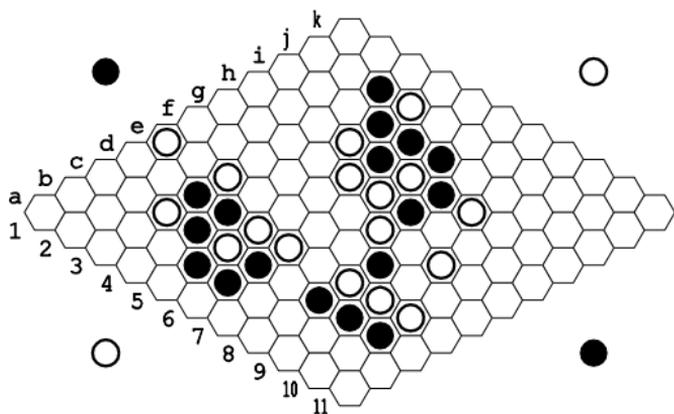


Analyze especially if White moves at d11 (here in gray).

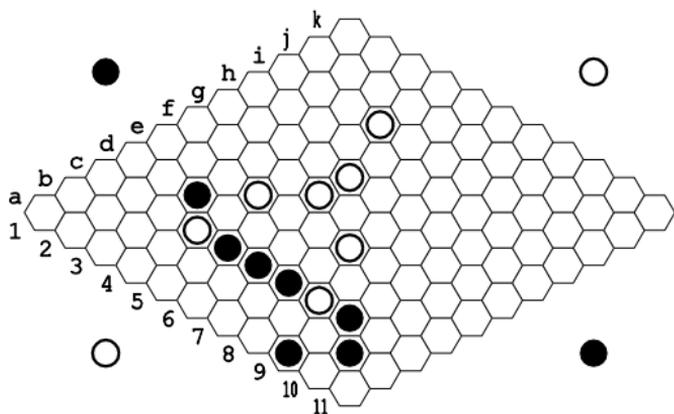
Problem 2. Black moves and wins.



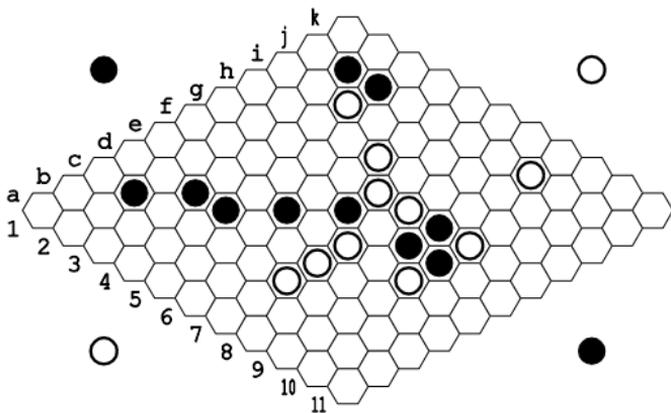
Problem 3. Black moves and wins.



Problem 4. What should Black do?



Problem 5. Black moves and wins.



Konane

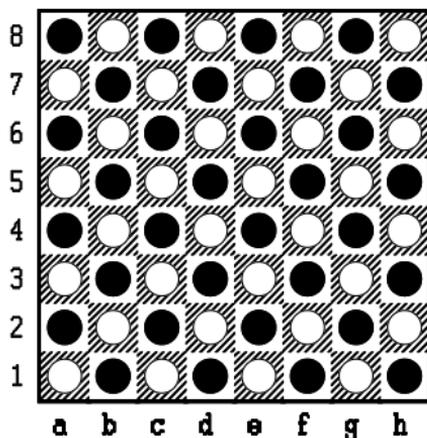
Traditional Hawaiian game

Equipment: A 8x8 square board, 32 white pieces and 32 black pieces.

Goal: Be the last to move.

Rules

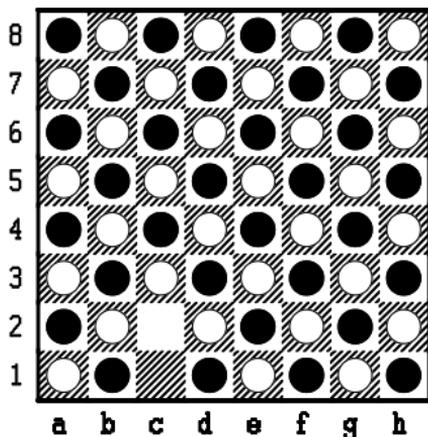
This is the starting position of Konane:



In the first move, White remove one piece of his central 2x2 square (on d4 or d5) or a piece of one side of the board.

Following, Black remove one horizontally or vertically adjacent to the removed piece.

For example, in the next diagram, White chosen the piece in c1 (one of the board edges) and Black chose to withdraw the adjacent piece of black c2. Thus, we obtain the following position:

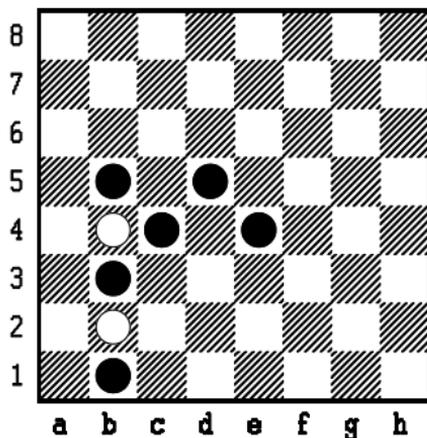


After the withdrawal of two parts, each player alternately moves its one piece. Start the White.

A piece can be moved if it is adjacent (horizontally or vertically but not diagonally) to an enemy piece and can jump over it (the square must be empty). A jumped piece is captured and removed from the board (like draughts). This means that captures must occur on all the moves.

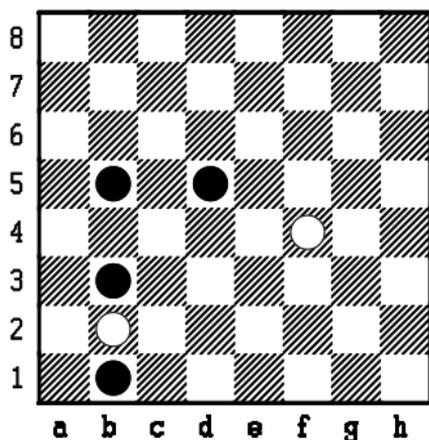
Multiple captures are allowed provided it is done in the same direction (ie, can not change the direction of the capture in a single move).

An example:



In the previous example, it is White to play. The white piece on b4 has several capture options: either move to b6 (jumping and capturing the black piece on b5); or moves d4 (capturing c4); or may continue to jump to f4 (capturing e4). Note that after the jump to d4, can not change the direction to capture d5 on the same move.

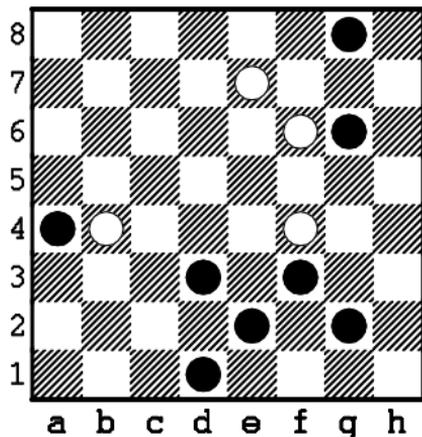
Consider the same diagram. The piece b4 captured the two black pieces on c4 and e4 with a double jump. After the move we get the following position:



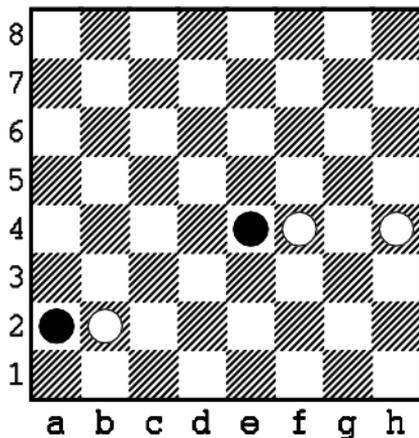
Is now Black to move and there is not a single move available, the game ends with a victory of White.

The following exercises represent some typical strategies of Konane. The solutions are at the end of this volume.

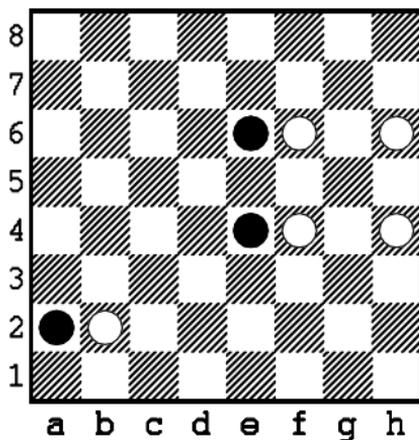
1) Black to play and win.



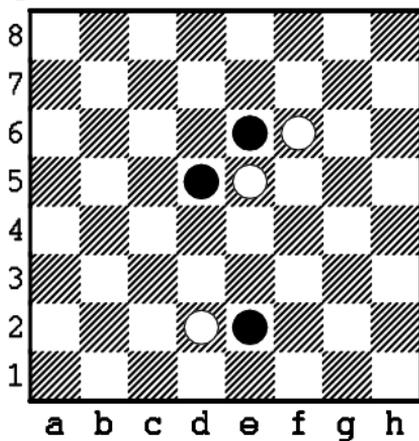
2) Analyze the position whether is Black or White to play.



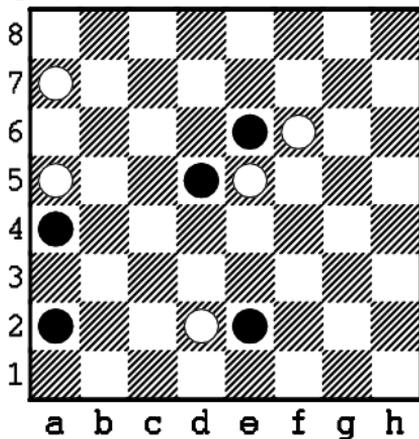
3) Analyze the position whether is Black or White to play.



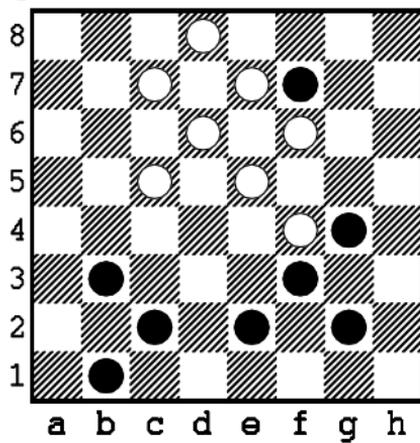
4) Black to play and win.



5) Black to play and win.



6) Black to play and win.



Wari

African traditional game.

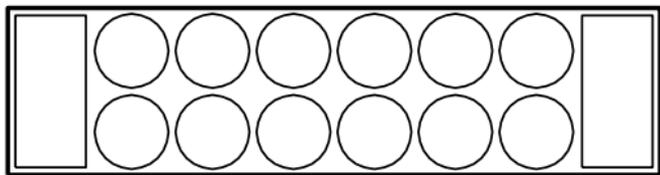
Material: 48 seeds, board with 14 holes.

Goal: Wins the player that captures most seeds. Since there are a total of 48 seeds, capturing 25 seeds is enough to secure victory.

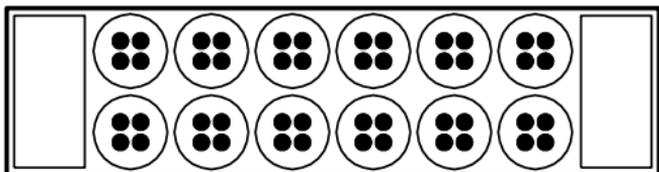
Rules

In the 14 hole board, the extremes are called deposits and are where the captures of each player are kept. Usually, each player has the deposit on her right.

The remaining holes are divided in two rows of six, each row belonging to a different player. Unless, stated otherwise, hole will mean non-deposit holes.



Initially, each hole has four seeds, in a total of $12 \times 4 = 48$ seeds. This is the board setup:



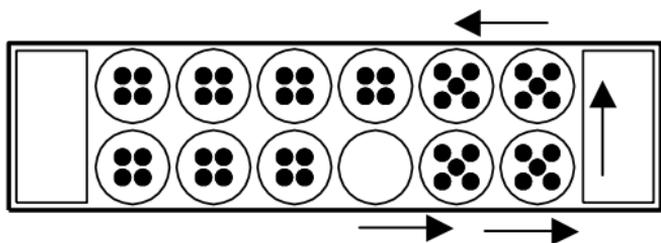
Wari is a capture game. Since the total number of seeds is an even number, it is possible for the game to end in a draw (however, draws are not common in wari matches).

On each turn, each player performs a seeding.

A seeding consists of picking a friendly non-empty hole, taking its seeds into the player's hand, and placing one seed per hole on a counter-clockwise movement until all seeds are again on the board. Notice that deposits do not receive seeds while the seeding is being made.

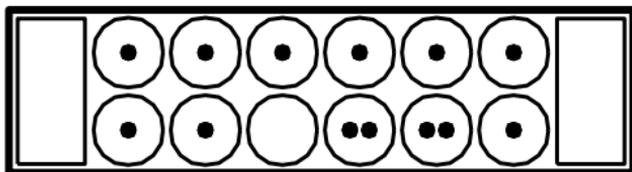
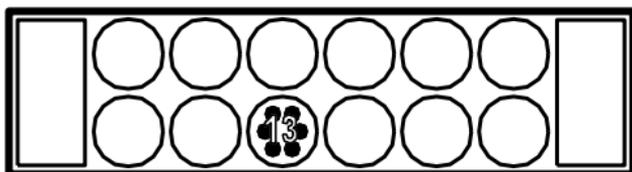
Let's assume, in the following diagrams, that the first player owns the bottom row, while the second player owns the top row.

An example: if the first player, on her first move, seeds her fourth hole, this would be the result after the seeding ends:



If the player is seeding a hole with 12 or more seeds, the seeding will travel the entire board. In these cases, the original hole will not receive a seed. The player must jump that hole and continue seeding on the next hole.

The next two diagrams show the before and after of seeding a hole with 13 seeds

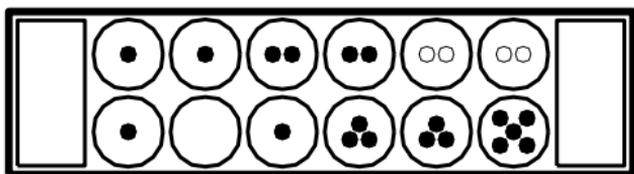
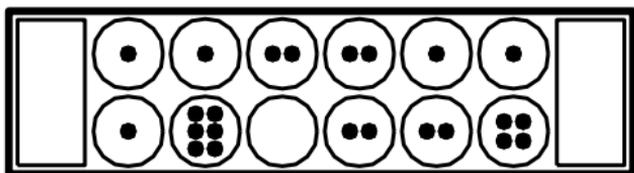


There is a restriction on the choosing of the seeding hole. If there are holes with two or more seeds, the player cannot select holes with just one seed. In the

first diagram of the next page, the second player, if he was the one to move, could only select one of the two holes with a pair of seeds.

After the seeding ends, captures might happen. How? If the last hole to receive a seed has two or three seeds those are captured and placed in the player's deposit. If the previous hole also has two or three seeds, those are also captured. This is repeated until the process reaches a hole on the player's row, or a hole with less than two or more than three seeds.

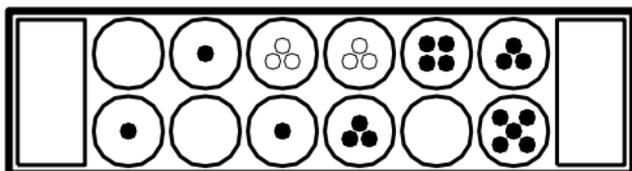
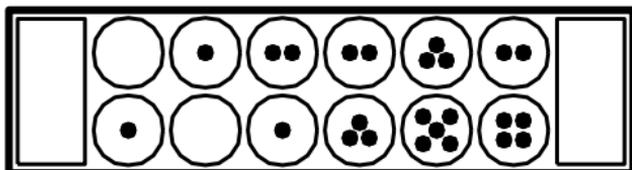
In the next example, the first player decides to seed her hole with six seeds.



The second diagram shows the board after the seeding ends. Since the last seed fell on a hole with two seeds, those are captured. And since the previous hole also has two seeds, those are also captured. Then, the first

player's captured a total of four seeds (the captured seeds are the white stones in the second diagram).

In the next example, the first player chooses her fifth hole, the one with five seeds. As the second diagram show, that move was able to capture six seeds.



Any seeding that ends on an enemy hole with less than two or more than three seeds does not produce any captures. Also, if the seeding ends on a friendly hole, there are no captures, no matter the number of final seeds.

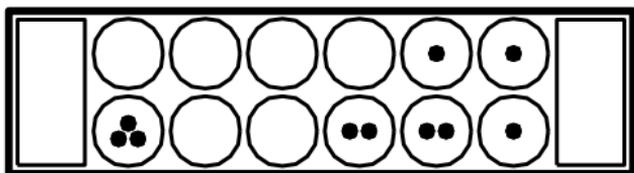
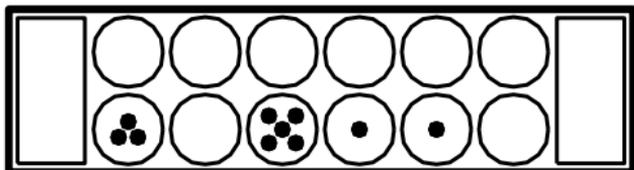
These are the fundamental rules of Wari.

The following rules handle some particular situations.

If an adversary has no seeds of his row, the current player must select a seeding that provides him with at

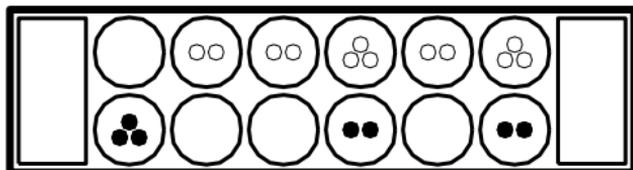
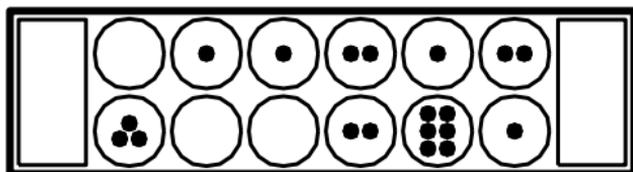
least one seed. If that is not possible, the game ends, and all the seeds are captured by the current player.

In the next example, the first player is forced to move her third hole, the only hole that seeds the adversary row:



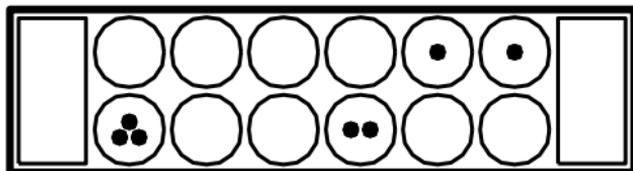
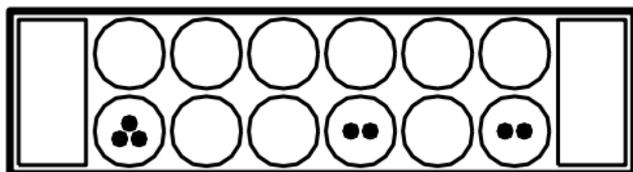
If a player, after her seeding, captures all adversary seeds, she must play again and place at least one seed on the adversary row. Again, if she has no seeding that performs that, she captures all her seeds and the game ends.

In the next example, the first player can capture all adversary seeds, by seeding her fifth hole. Why?



As we can see in the second diagram, all adversary holes have either two or three seeds, which mean that all are captured (i.e., the white seeds).

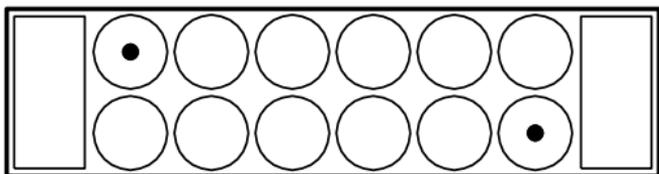
After the capture, this is the current board position:



Since the second player cannot move, the first player moves again and is forced to place some seeds in the adversary row. The only possible move is the seeding of her sixth hole.

There are some rare positions not dealt with the previous rules.

If a certain position enters a loops, like in the next diagram:



If this happens, the players must reach a consensus, select a moment to end the game and share the available seeds.

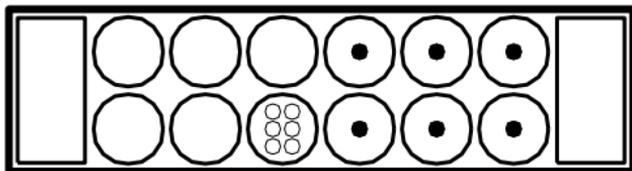
We leave the rules description with a puzzle. In the previous diagram, considering that all the remaining seeds were capture, what is the current capture score?

Some suggestions

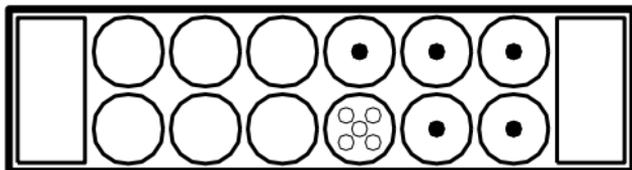
In Wari, it is very important to count the number of seeds on a hole before deciding to move it. Only that way it's possible to know where the seeding will end.

For example, a hole with six stones will end its seeding on the right diagonal hole on the opposite row.

The next diagram show the previous position (on white) and how the seeding went (the black stones).



For a seeding to end on the opposite hole, the initial number of seeds must be an odd number. In the next diagram, five stones were needed.



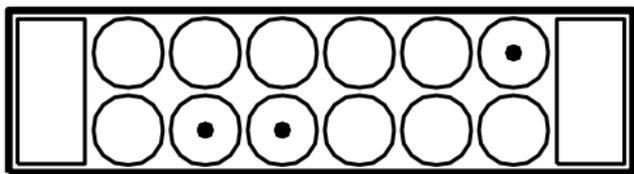
To count holes with 12 or more seeds, subtract 11 (remember, the original hole is jumped during such big seedings). For hole with 23 or more stones, subtract 22.

It is very useful to know how many seeds are on each deposit. A victory is achieved once a player captures 25 seeds. No more is needed.

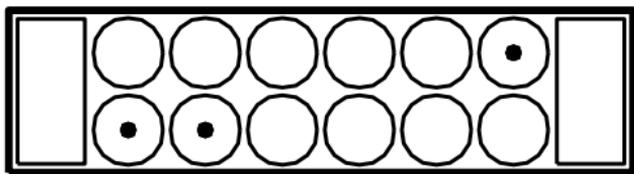
An important strategic point is to have control of how many moves are still available without seeding the adversary row. Let's call these move «home seeding».

At the endgame, there will be very little seeds still not captured. A good number of home seeding options can mean the possibility of depriving the adversary of moves, and if there's no option to seed the adversary row, the game is won.

In the next diagrams, we observe very balanced matches (on both, the current score is 22-23, i.e., there is one seed advantage for the second player). First player moves. Can the first player still win on either match?



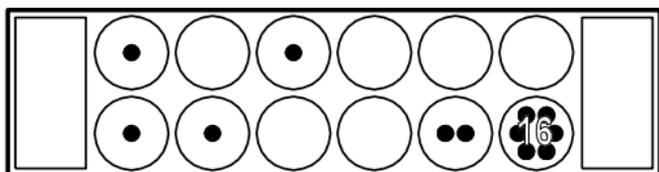
In the first match, the first player is able to make seven home seedings. These are not enough, to allow the six adversary moves, and still have the possibility to prevent the mandatory seeding rule. This match will end on a 24-24 draw.



On the second match, the first player has eighth home seedings. This extra move is enough for the first player to create a position where he can prevent the mandatory reseeding, and win the game by 25-23.

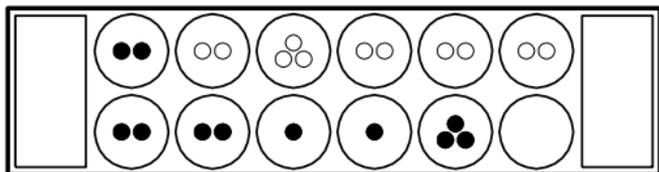
Another strategic point in Wari is to build holes with lots of seeds enough to make two rounds on the board, In certain situation, this will imply the capture of all adversary seeds!

The current score, in next diagram, is 8-18. First player moves.



The first player, a good Wari player, observes that (a) there are enemy holes that can be captured, (b) her first, second and fifth hole can be captured, (c) the current score disadvantage.

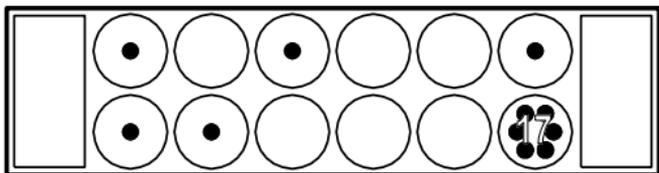
If the player decides to seed her 16 seed hole, this would be the result (the white stones are captured):



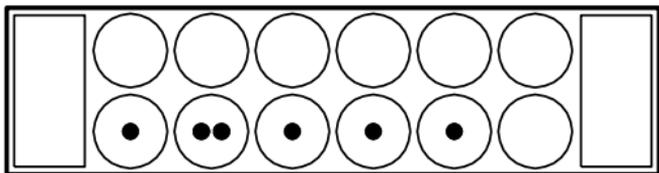
The first player just captured 11 seeds, leaving the current score at 19-18. However, in his next, the

second player could capture 6 stones, securing, at least, a draw.

Can the first player do better? The problem with the previous move was that the first player tried to maximize the number of captures without considering the weaknesses of her own row. A better move would be to seed her fifth hole:

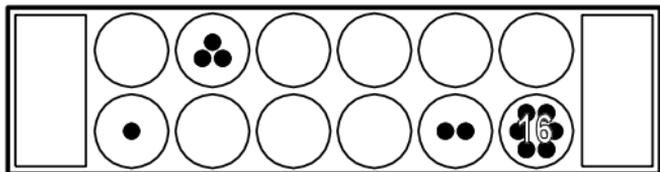


The second player seeded his leftmost hole (current score 8-20). And now it is the appropriate moment for the big seed, capturing 14 seeds (current score 22-20) resulting in the following position:

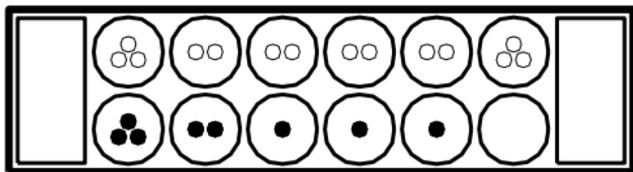
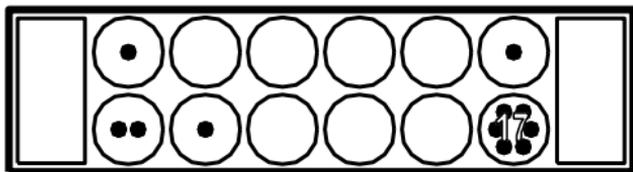


The second player is forced to pass (he has no seeds left) but the first player cannot reseed the enemy row. So the game ends. The first player collects the remaining seeds and wins 28-20.

Consider the next similar position. Can the first player capture all seeds? Try to find the answer before reading the next paragraph.



The best move is to accumulate an extra seed on her sixth hole. If the second player seeds his three seed hole, the first player seeds the 17 seed hole and captures all seeds:

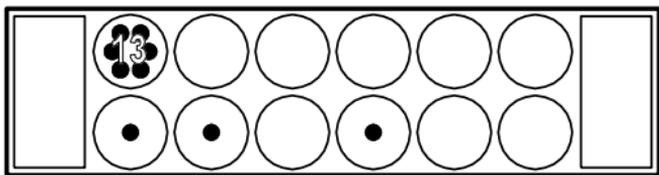


The second player passes, the first player cannot reseed him, thus ending the game and collecting the remaining seeds.

These two last examples indicate that if a player reaches endgame with a hole with lots of seeds, and the

adversary with a small number of seeds, it is possible to create a position to captures all or almost all available seeds.

In the next diagram, can the first player prevent the adversary to capture all her seeds?

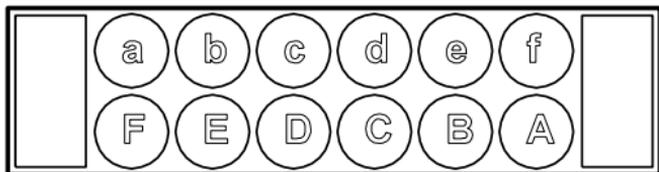


To prevent the danger of capture from an entire seeding cycle, it is critical to have an hole with two seeds. So the appropriate move would be to seed the first hole into the second. That way, after the adversary big seeding, the second hole would have four seeds, thus preventing a capture.

Wari Opening

Herein we'll suggest some good opening moves, and guiding principles to improve your playing ability.

In the next sentences, we'll use a notation to identify each board hole. The bottom row will use uppercase letters, while the top row will use lowercase letters:



Player should prefer to seed right holes (starting from A is better than starting from F). Even if we are giving seeds to the adversary row, we are providing more moves in the medium-term. Players should not start the game by playing E or F.

Try not to seed two consecutive holes at the beginning. This prevents initial double captures, since the adversary can seed both holes with one seed each, and next turn threaten to capture them. So, after the initial seeding at A, the player could choose C, D or E.

Do not forget that the adversary moves will also change the board state, and his decisions will affect your next moves. Try to prevent great score disadvantages at the beginning, which might be difficult to recover later on.

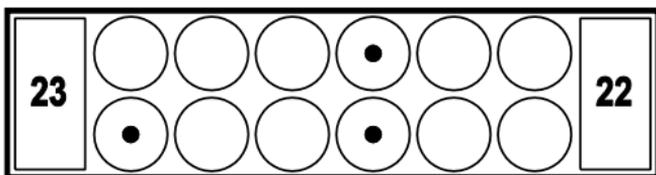
It is important to keep a good number of seeding options, so that we can have answers to the adversary choices.

An advantage of having holes with lots of seeds is that if a player has a reduced number of choices, a big seeding will provide lots of new seeding options.

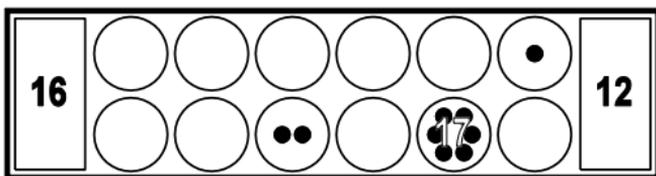
Captures should not be rushed. An available capture can be an adversary trap, or a preliminary move can improve the position for larger later gains.

The next diagrams provide some Wari problems for the reader to find the appropriate solution.

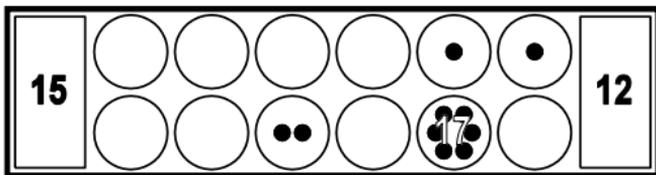
Problem 1: First player moves and wins (she already captured 22 seeds).



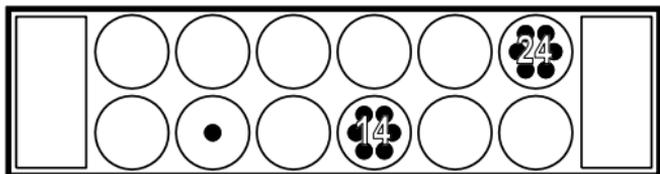
Problem 2: First player moves.



Problem 3: First player moves.



Problem 4: If first player moves, and moves C how to rapidly know where the seeding ends? And if the second player moves f?



Find the solutions are the end of this text.

Pawns

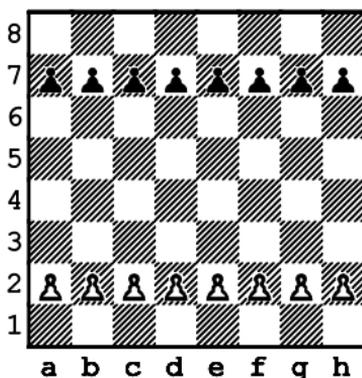
Author: Bill Taylor, 2001

Material: An 8x8 square board, 8 white pieces and 8 black pieces.

Goal: Be the first to reach with one of your pawns to the last line or be the last to move.

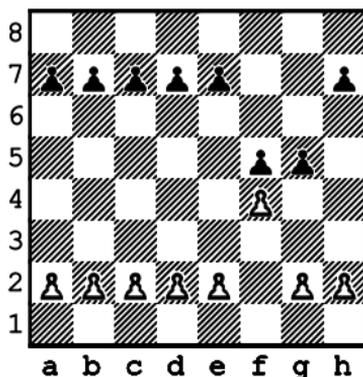
Rules

This is the starting position:

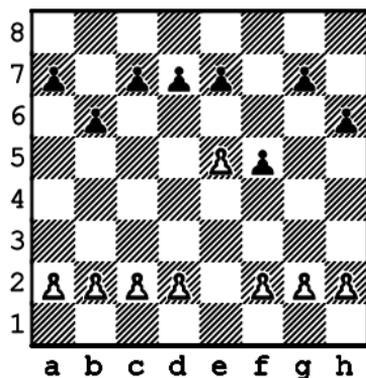


White starts. On each turn, each player moves a pawn. Pawns move like in chess, ie:

- Moving forward vertically if the square is empty (except at its first movement, which can move two squares).
- Capturing an opponent pawn if it is in diagonally (see the following diagram: the pawn on f4 can capture in g5 but can not capture in f5). Captures are not mandatory.

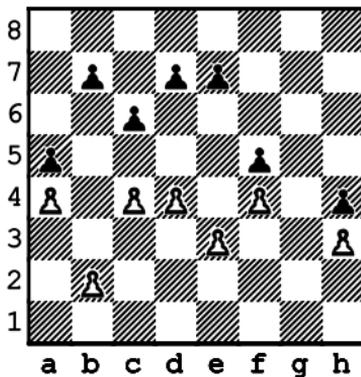


- Captures can be *en-passant*, that is, a pawn attacking an empty square crossed by an enemy pawn that advance two squares (ie, still had not moved) can capture this pawn as if it had moved only one square. This capture may be made only on the next move of this advance. In the next diagram, Black moved the pawn f7 to f5; White can capture that pawn with the pawn in e4, moving walkway to f6. This can only be made in this move, after it is not possible.

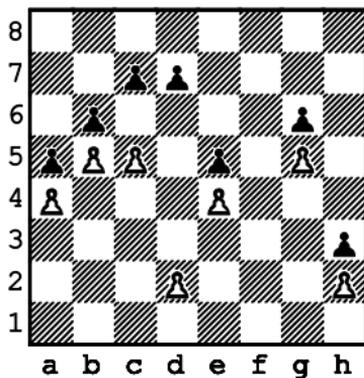


Now, we propose a number of problems showing some interesting features of this game. The solutions are at the end of this volume.

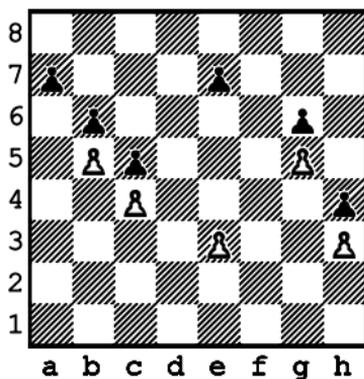
- 1) Black to play and win in 7 moves.



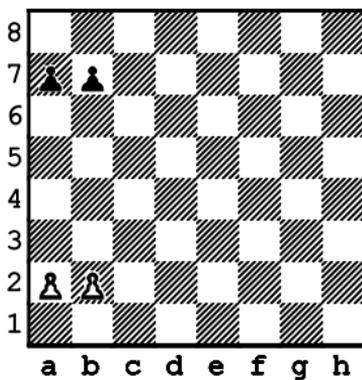
2) White to play and win in 6 moves.



3) Black to play and win in 2 moves.



4) White to play. Who wins?



Dots and Boxes

Traditional game

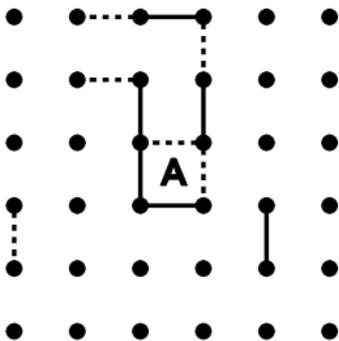
Material: Sheet of dotted paper (as pictured below), a pencil.

Goal: Have more squares with the player's initials.

Rules

On each turn, each player connects two neighboring points with a horizontal or vertical segment. When one completes a square, write its initial inside the square and plays again. Closing squares is not mandatory.

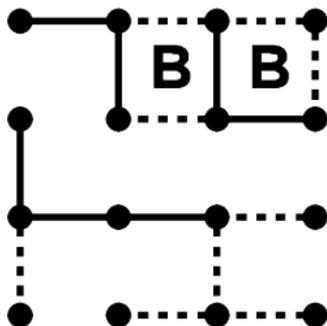
In the following example, the first player was able to close a square (marking it, for example, with an "A"). Since it plays again, has the possibility to close three more.



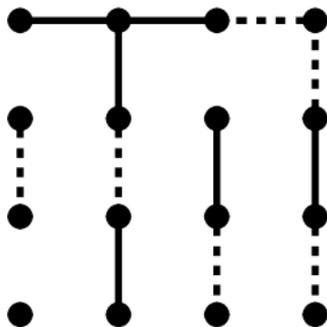
Following are some dots and boxes problems as a challenge to the readers and the solutions talk about the strategy of this game. The first player is designated

by A (which uses solid lines) and the second player by B (using dotted lines). The solutions are at the end of this volume.

1) Player A to move and win.



2) Player A to move and win.



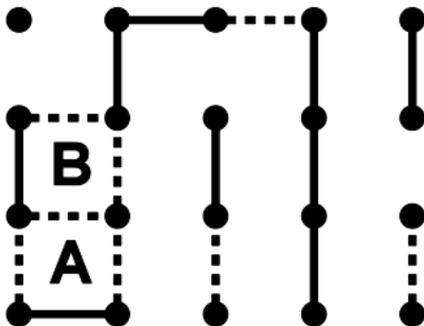
3) Consider the following settings:



Explain why, in not too small boards, leave these settings to your opponent usually is not a good strategie.

Explique porquê que, em tabuleiros não muito pequenos, deixar estas configurações ao adversário não costuma ser uma boa prática.

4) Player A to move and win.



Slimetrail

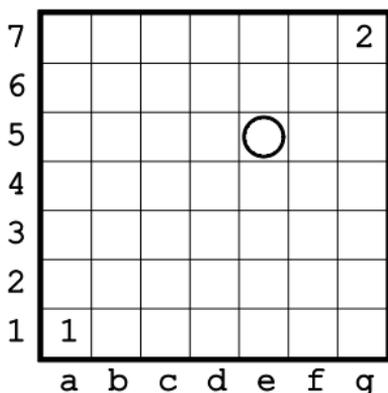
Autor: Bill Taylor, 1992

Material: A 7×7 square board, 1 white piece, and 40 black pieces.

Goal: A player wins if the white piece moves to its goal square, or if it is able to block the opponent preventing him from playing.

Rules

In the starting position of the game, the white piece is placed in e5:

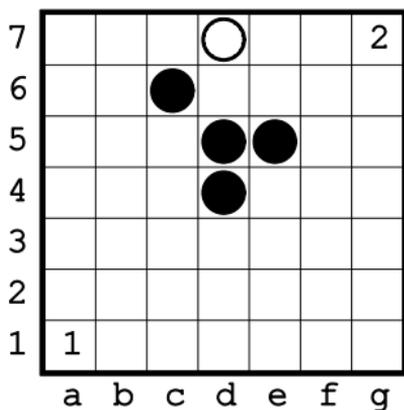


The first and second player's goal are the squares a1 and g7, marked with [1] and [2], respectively.

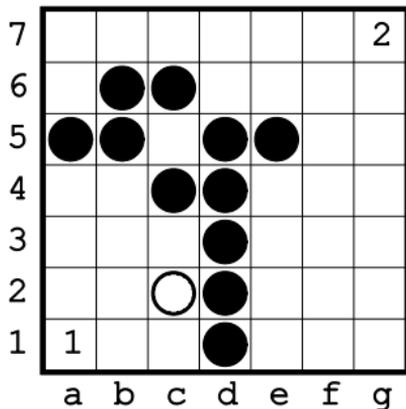
Each player takes turns moving the white piece around the board. The piece can take one step (vertical,

horizontal or diagonal), and may not re-visit a visited square. In each move, the square abandoned by the white piece leaves a trail, which means, is occupied by a black piece.

In the course of the game, the board will become increasingly occupied by black pieces, reducing the number of choices for each player. The following diagram shows the first four moves of a game (1.e5-d4, d4-d5 2.d5-c6 2.c6-d7):



In the next diagram is the first player to play. What should he do?

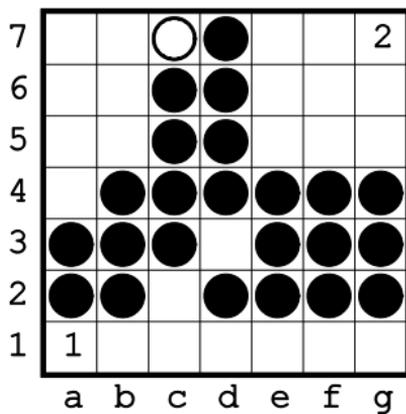


If he plays for b1 loses the ability to reach his goal. How? The move against b1 is the sequence c1, b2, c3. After this, if the first player moves to b3, a4 is the response. In the initial position, the move to b2 has a similar problem.

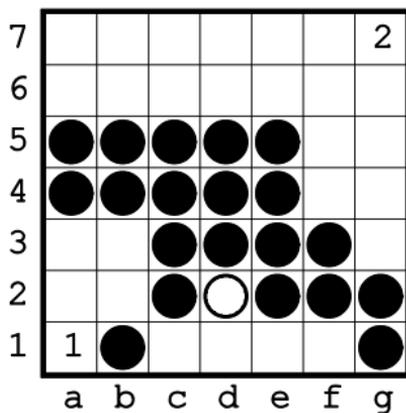
The best way to reach the first player's goal square is to move to c1. The opponent's next move is mandatory to b1 or b2, which will result in an automatic win for the first player.

Next we present some problems as a challenge to the reader. The solutions are at the end of this volume.

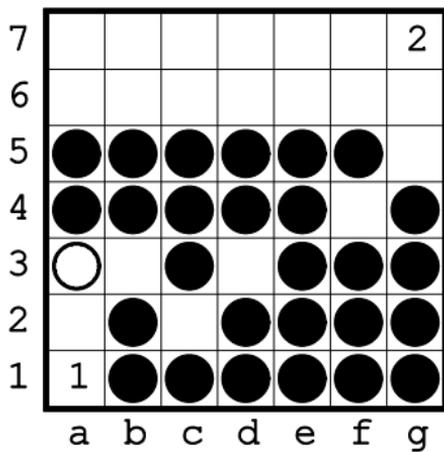
1) How do you assess the situation?



2) If the first player's turn, what to do? And if it is second player's turn?



3) If the first player's turn, what to do?



Traffic Lights

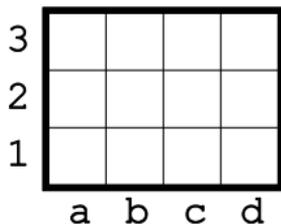
Author: Alan Parr, 1998

Material: A 3×4 square board; 8 green pieces, 8 yellow pieces and 8 red pieces to be shared by both players.

Goal: Be the first to get three in a row (horizontal, vertical or diagonal) of the same color.

Rules

The game takes place on the next board, initially empty:



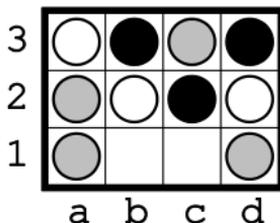
On each turn, each player performs one of the following actions:

- Puts a green piece on an empty square;
- Replaces a green piece by yellow;
- Replaces a yellow piece by a red.

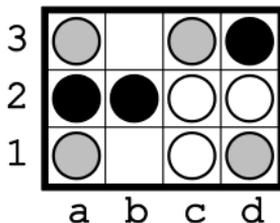
Note that the red pieces can not be replaced. This means that the game ends: as the board gets red pieces, inevitably emerges a three in a row.

In the following diagrams the colours white, gray and black are used to represent respectively the green, yellow and red.

The following diagram shows a position with three possibilities of win with one move: 1. Replace the green piece on a3 (creates three vertical yellow line); 2. Replace the yellow part d1 (one three diagonal line of red); 3. Drop a green piece c1 (a three in diagonal green line).

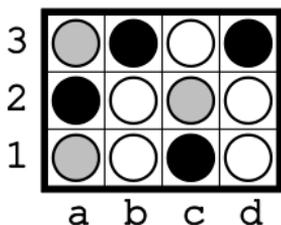


The following example is an ending position. Looking at the board, we see that there are only two moves that don't lose immediately: (a) drop a green piece in b1; (b) replace the green piece on d2. This means that this is a losing position for the next player. When playing one of these options, the opponent plays the other.

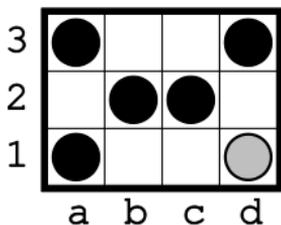


Following are some Traffic Lights problems as a challenge to the reader. The solutions are at the end of this volume.

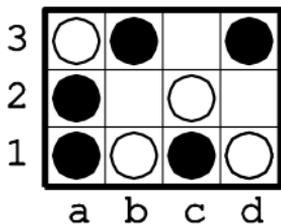
1) The next player wins. How?



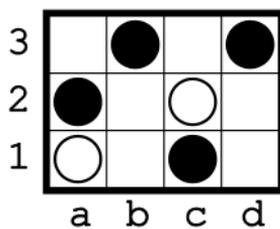
2) The next player wins. How?



3) The next player wins. How?



4) The next player wins. How?



Cats and Dogs

Author: Simon Norton, 1970s

Material: An initially empty 8x8 square board, 28 cat pieces and 28 pieces (represented respectively by black and white pieces).

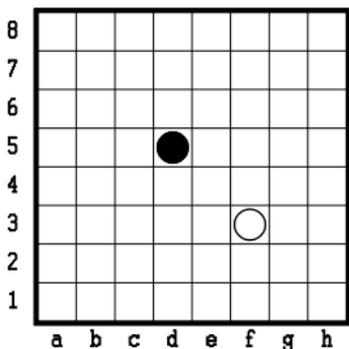
Goal: Be the last move to move.

Rules

Each player, in turn, puts his piece in an empty house. Start the Cats. The cat must first be placed in the central zone (indicated in the figure) and the first dog must be placed outside the central zone.

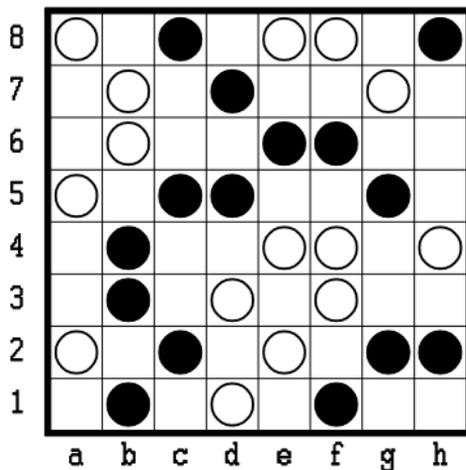
When making a new pet on the board, players can not put a cat next to a dog to the (horizontal or vertical) or beside a dog a cat.

In the diagram we see a valid beginning of starting, the cat was placed in the central area and the dog out of the same central zone.



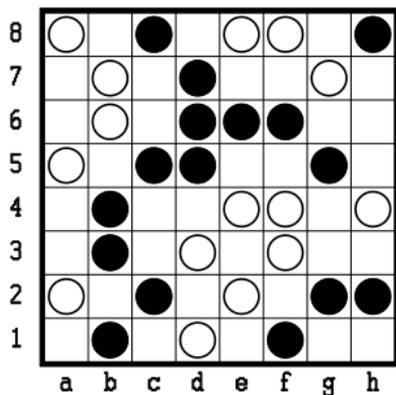
For example, the dog could not be put into c5 because, despite being outside the central area, would be adjacent to the cat already in the tray.

In the next diagram are the Cats play. The Cats have the guaranteed future moves in c4, g1 and h1. Dogs have the guaranteed future moves in e3, a6 and a7. The only house in dispute is the h6 home. Playing first, the Cats put there for a cat and gain (if the turn of Dogs, placing a dog in this house would also guarantee victory).

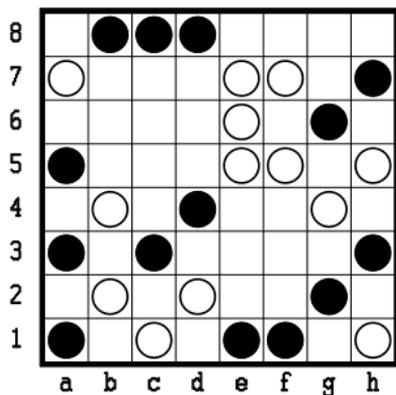


Let us now consider some problems of Cats and Dogs.

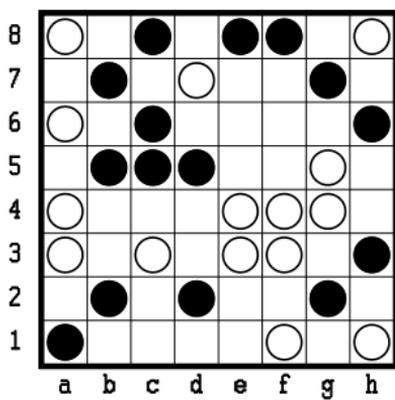
1) Regardless of the player, where the next move must occur?



2) Analyze the situation. Where should play either the Cats or Dogs?



3) Analyze the following situation for both players.



Product

Author: Nick Bentley, 2008.

Rules adapted by João Neto and Bill Taylor, 2011.

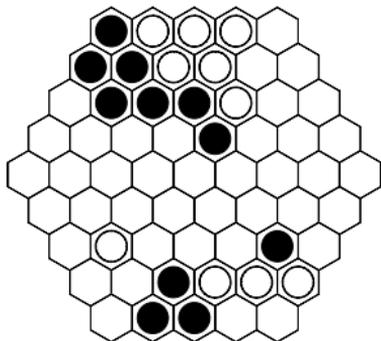
Material: A hexagonal board with five houses away. 45 white pieces and 45 black pieces.

Goal: When the board is full, we calculate the product of the sizes of the two largest groups of each color (who has less than two groups gets the value zero). Wins who obtains the highest product. If these are equal, the winner will have fewer pieces of your color into play.

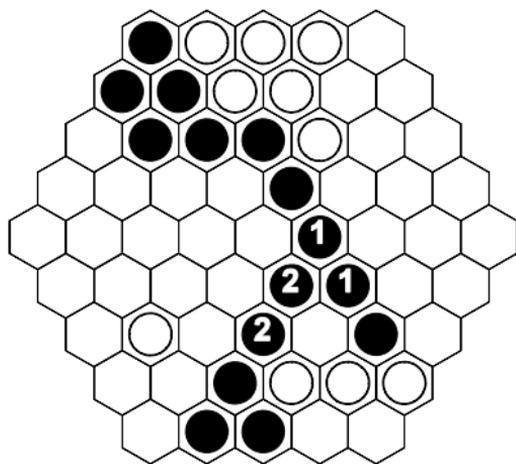
Rules

In turn, the player must place two pieces of any color in two empty houses. Begin the Dark. On the first throw, the Black play only one piece.

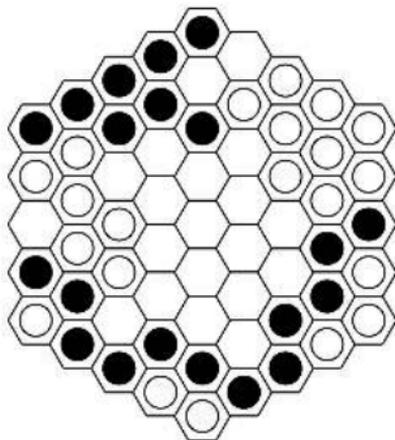
In the following diagram the score is 21 points for the Black (7 Pieces 3 times the larger group pieces in your second largest group) with the White 18 points:



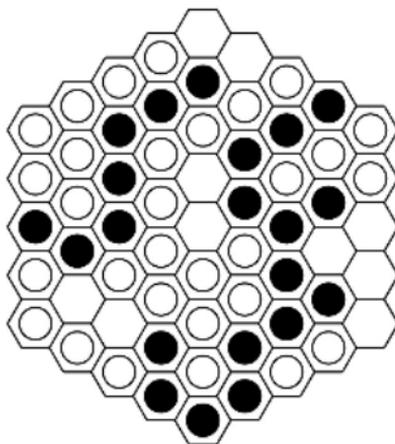
In the following diagram, if Black then throw the pieces [1], the White could reply with [2] (players can put pieces of the opposite color) to create a single Negro group. Despite having a very large group, the Black would now have difficulty making a second group with an isolated enough to win the final product that White will achieve dimension.



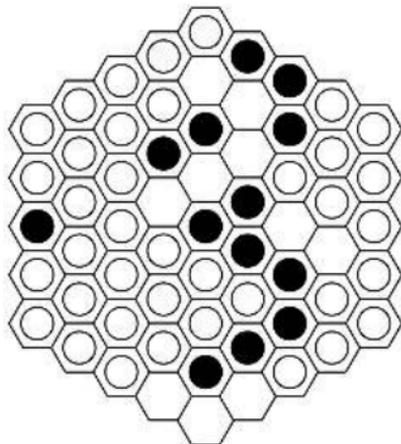
3) White to play and win.



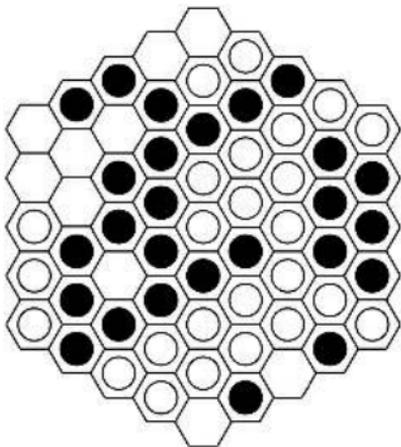
4) Black to play and win.



5) Black to play and win.



6) White to play and win.



Solutions

Amazons

1) Almost all of the times, an Amazon in a closed area is able to move **as many times as there are empty squares in that area**. This position is no exception. The Amazon at c2 is able to move 7 times, while the Amazons in the up right corner have 6 moves. White victory can be obtained using Amazon at c2 with this precise sequence of moves: c2-a4 (marker at d1) [or c2-b3 (d1)], a4-c2 (b1), c2-b3 (c2).

2) Another important concept is that territorial wars from the enemy Amazons in the same closed area are very delicate. The only winning move is c4-d4 (d3).

3) While quite simple, this problem shows a not uncommon type of position. At the left part of the board is bad to move (in Chess is what is called a *zugzwang*). So, to win White must move in another zone. The solution is h5-g6 (h5).

Breakthrough

1) The most common tactic is using sacrifices to advance into the enemy lines. This is one of such positions. White's winning move is 1.a4! followed by 1...bxa4 (if 1...axb4 2.axb5 wins) 2.b5.

2) This is also a sacrifice even if subtler. The correct move is 1...de3! (1...fe3! is similar) followed by 2.dxe3 fxe3 3.fxe3 e5 and then the advance in f4 wins.

3) Another important tactical tools in race analysis: who wins the race first, when it starts? Herein, the winning move is 1.f3! assuring that the piece wins the race. Instead, the bad move 1.g4? is answered by 1...fe5! and the black piece wins the race.

4) It is usual for one player to have more area than his adversary. This problem is an elegant example of it. The only winning move is 1...gf4!! 2.g3 fg4! The move 1...g4? is bad because of either:

- a) 2.f4;
- b) 2.fxg4 fxg4 3.fg3!

5) Just like in Chess there are *zugzwang* positions (i.e., it is bad to be the first to move). The right move is 1.a3! creating a *zugzwang*.

6) This is a complex example taken from a real match by email, A.Perkis – D.Troya, in 2005. There are sacrifices and race analysis. The winning move is 1.bc5!! with the following variants:

- a) 1...fg3 2.fxg3 hxg3 3.hg6! fxg6 4.dxe6 dxe6 5.cxd6 wins;
- b) 1...dxc5 2.dxe6 dxe6 3.hg6 fxg6 4.cd5 wins;
- c) 1...bc7 2.dxe6 dxe6 3.cd5 d7 4.cxd6 cxd6 5.e5! wins.

Go

1) It is alive. Even if White plays e1, Black lives with e2 (or vice-versa)

2) It is dead. White can play g2 anytime, threatening to capture. Even if Black captures with d1, White moves to f2, the black group's vital intersection.

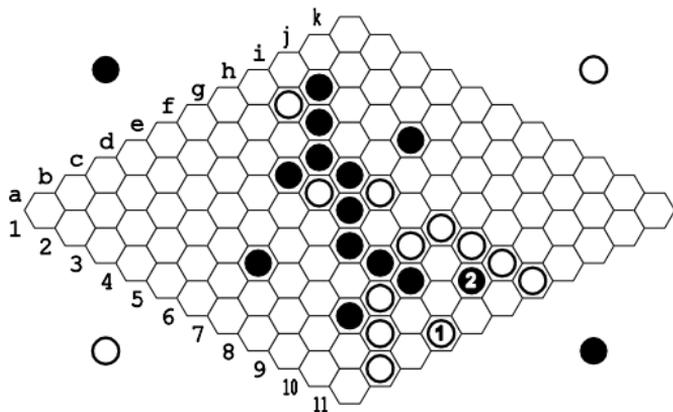
3) It is alive. Black can capture the inner white stones playing in g2 and c1, because they have two vital intersections (e1, e2) which allow the group to be alive.

4) Black should move e2. After white's d2, Black replies f2 e there is no space left for a white living group. If Black had played f2, White would answer e2 and would be safe.

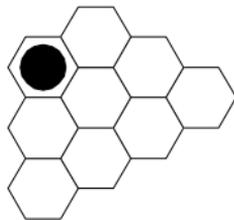
5) Black should move f1. After its capture in e1, Black plays d2 cutting the vital points that would give life to the white group. If White initially answers e2, Black would play d3.

Hex

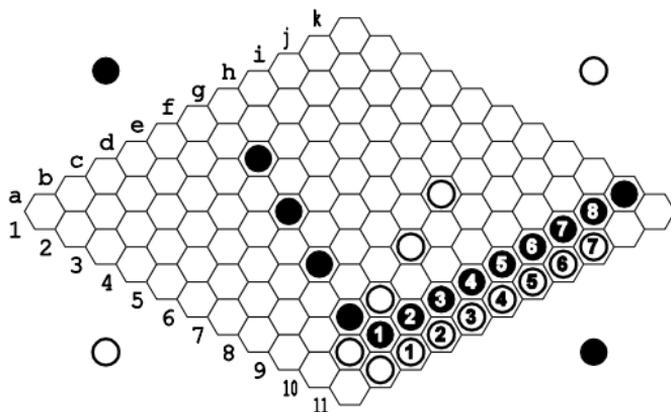
1) If White moves [1], Black answers with [2]:



A Hex player must know the following pattern that, near one of his edges, guarantees the connection to it:

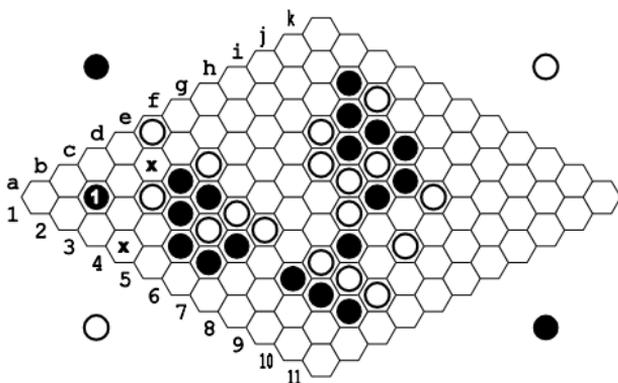


2) This position illustrates one of the most important tactics in Hex: the ladder. The first move is c10 which is followed by a sequence of replies until the k10 stone finally decides the match.

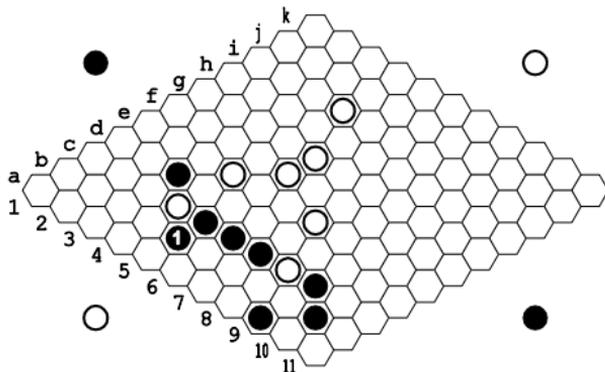


In Hex the stones' influence is a vital part of the game dynamics.

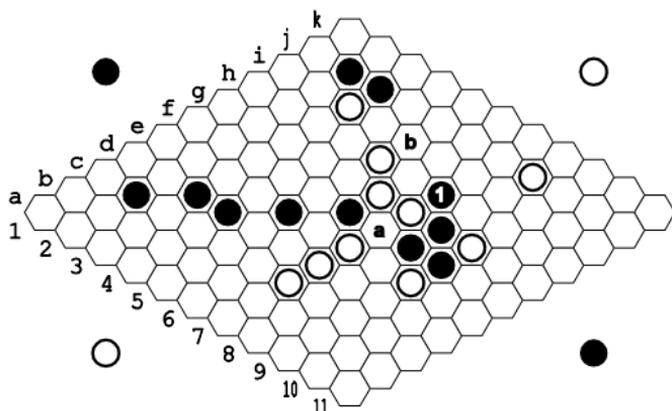
3) The answer lies in a ladder preparation. The vital move is in [1]. This move is a ladder for the threat in d2, while making a second threat in a5. White cannot reply to both threats and loses.



4) The answer is [1]. It prevents a ladder associated with b8. Also, after White's d4 it follows b4. This move is called a counter-ladder.

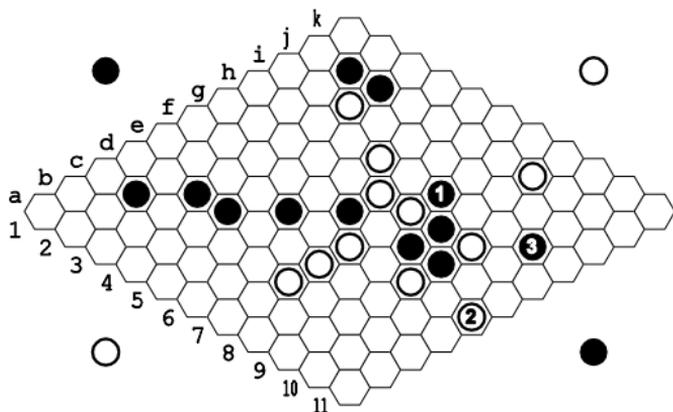


5) A move like f10 guarantees a connection with the south edge. However there are several options, and Black should choose one that helps as much as possible to connect also the north edge. The best move is [1].



In this case, [1] by creating a second threat (b) of a north connection - the first one was (a) - thus wins the game.

Notice that the south connection is still maintained. If White moves e11, then h10 and we have a position similar to problem 3:



Konane

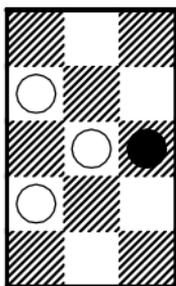
1) This exercise is related to the recognition of some advantageous structures. Here are some examples:



one point advantage for Black;



one point advantage for Black;



two points of advantage for Black.

In the case of exercise, the analysis of the structures in the game leads to the discovery of solution 1 ... f3-f5.

2) If the Dark, win with 1 ... e4-f4 2.h4 g4-a2-c2. If the White, 1.f4-d4 a2-c2 winning.

Conclusion: The Black always win. The supremacy that White has the right side is not as strong as they have in the Black left.

3) Notes easily in this case, those who play lost. Chess in the manner called zugzwang situations of this type.

Conclusion: The supremacy that White has in each of the devices on the right corresponds to $\frac{1}{2}$ bidding advantage! It takes two to cancel the bidding advantage that Negras have on the left side.

4) Any of the plays 1 ... e6-g6-d5 and 1 ... f5 is winning. 1 ... e2-c2 would be a serious mistake because of 2.e5-e7.

5) This, together with the above, discloses one of very common in games such as combinatorial Konane, good plays vital characteristics depend on the global context. Just no longer a factor, even if completely separate, so a bad move going to be good. The solution in this case is 1-e2 c2 ...! (only winning move).

6) There is the powerful threat of f2-f4 White. Thus, the piece f4 must be captured.

E4 g4-1 ...! with the following assumptions:

a) 2.f6 f8-E4-E8; b) 2.e7-e4-e6 g7 c) 2.e5-e3-f7 d7!
Mau would be 1 ... f3-f5? due to 2.e5-g5.

Wari

1) The number of home seedings each player can make can be decisive. In this position, the first player has more moves available and wins with 1. C d, 2. F c, 3.E b, 4.D a (capturing all seeds).

2) A typical strategy consists on having a hole with lots of seeds, in order to make a fatal move. However, that move must be precisely calculated. In this example, the best sequence is 1. D f, 2. B e wins the game. This is better than starting with 1.B.

3) Here, the best move is to start with the big seed. The correct move is 1.B. If 1.D, there is the defensive reply 1...f.

4) Subtract 11 seeds. So $14-11=3$, from C we reach f. With 23 or more seeds subtract 22. So $24-22=2$, from f we reach d.

Pawns

1) The key move is $1 \dots d7-d6!$ If White-c5 $2.c4$ respond then $2 \dots d6-d5$, $3.c2-c3$, $e7-e6$ and White lose. If White then respond $2.d4$ $d5-2 \dots c5-c6$ -like solution.

2) The winning move is $1.c5$ $c6-!$ The game continues with $1 \dots d7: c6$, $2.b5: c6$ and after the exchange, the Black come in a position that would rather spend the time but have to play: $2 \dots b6-b5$ $3.a4 : b5$, $a5-a4$, $4.b5-b6$, $c7$, $b6$, $b7-5.b6$ and win the next round.

3) A player can also lose by not being able to move. Let's see how the Dark can do that in this diagram: $1 \dots e7-e6$, $2. e3-e4$, $e6-e5$ and White lost because they have no legal move. This is an example of the power that even a pawn can not be played due to the option that gives the player to move one or two squares on their first move.

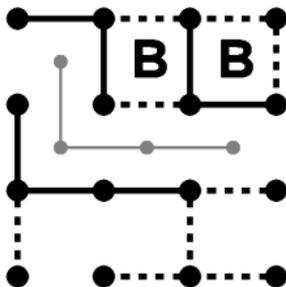
4) The Black wins the match. White has four options as they move two pawns, each with the option to move one or two squares. But for every one of them, the Black can proceed appropriately one of his pawns who are also in their initial positions.

The four hypotheses to consider are: (i) if $a2-a4$ then $a7-a5$, $b2-b3$, $b7-b6$, $b3-b4$, $a5:b4\dots$ (ii) if $b2-b4$ then $b7-b5$, $a2-a3$, $a7-a6$, $a3-a4$, $b5:a4\dots$ (iii) if $a2-a3$ then

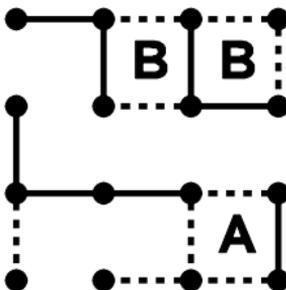
b7-b5, b2-b3, a7-a5, a3-a4, b5-b4 and White have no legal moves (iv) if b2-b3 then a7-a5, a2-a4, b7-b6, b3-b4, a5:b4...

Dots and Boxes

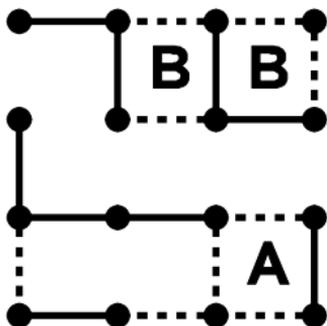
1) We begin by noting the area to try to conquer:



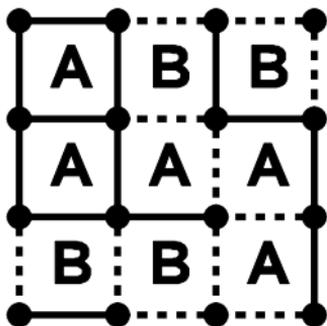
To achieve this goal, Player 1 must begin by closing the square in the lower right corner:



Then, should **give the turn** to the second player even sacrificing the two squares in the bottom left corner. This type of movement is called *doublecrossed* move. This type of tactic is the first weapon that a Dots and Boxes player should learn.



The game ends 5-4 in favor of Player A.



2) Can be demonstrated mathematically, with processes outside the scope of this text, which given a position, the number of possible moves is equal to

$$\text{number of points} + \text{number of long chains} - 1$$

where long chain means a chain with three or more squares.

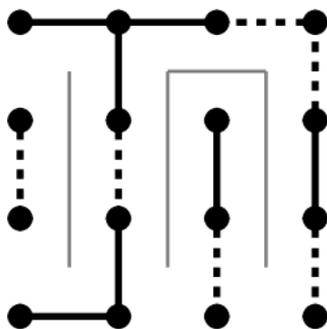
Since, usually, it is important to be the last play, the first player who interests

$$\text{number of points} + \text{number of long chains} \text{ is even}$$

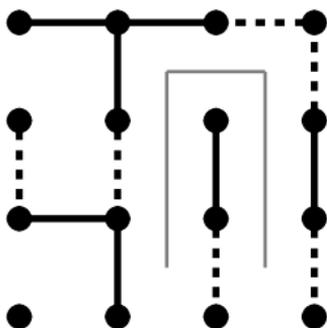
while the second player interest that

$$\text{number of points} + \text{number of long chains} \text{ is odd}$$

Using this concept, the Player can play horizontally on the bottom left and make number of points + number of long chains equal to $16 + 2 = 18$, which is an even number:



If player A had thrown up a horizontal



there would be just one long chain, making the odd sum. In fact, Player B now has victory in his hands.

3) It may be noted that since the play, and having at its disposal one of the exposed settings, a player has two options: either make the two squares and then play the rest of the board, or let the opponent make the two squares and play the rest of the board. The choice will be made by analyzing the rest of the board. Drop one of these settings to the opponent is called by experts one loony moves and often lead to defeat, since it gives the choice to the opponent.

4) Let us remind the reader of the classic Game of Marienbad. Start by grouping several matches on hills. Each move consists of removing some (or all) of one of the hills. Earns the player to remove the last match. Imagine the following position:



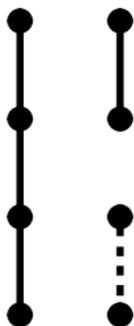
If you think carefully, will verify that the winning play for whoever is playing this position is to remove a lot of the match which has two. It is interesting to note that sometimes can be used for similar reasoning dots and squares.

Consider that the position was:

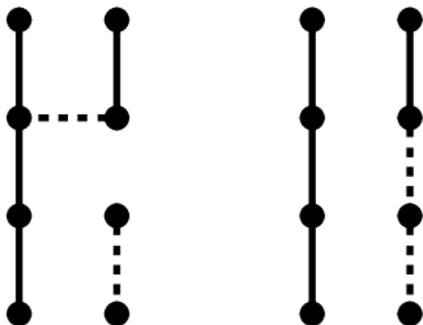


...and the purpose was to avoid being the last to play. The position was equal to a hill with a match in Marienbad game.

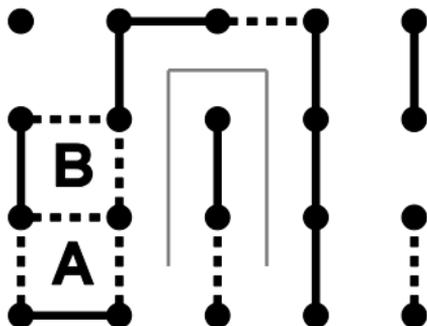
Now consider the following position:



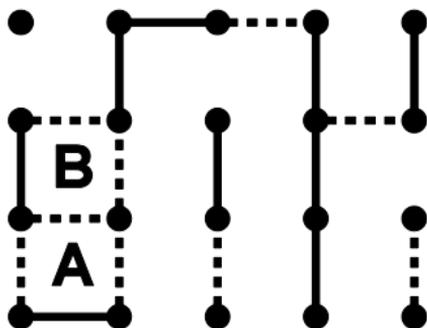
This case is already equal to a hill with two matches. Two hypotheses are (a) 'draw a match "(above diagram), or (b)' removing two matches" (below diagram)



Returning to the solution of the exercise, there is a long string that "nobody wants to open":



Therefore, in the outer zones, the main objective is to **avoid being the last to play**, leaving the background to close the matter or not square. Consequently, the position is equal to the position of the game with a lot of Marienbad with a match (the left end of the board) and a lot with two (extreme right). The winning move is to get a match of lot two, ie,



Slimetrial

1) In this exercise what is at issue is the treatment of a zone that involves no goal. Here we must take into account the **parity of the number of houses in dispute**. Since 7 is an odd number, if you make no blunder, the player who has the right to bid will win. You can play with the following criterion: if there are no vacant square on top, throws to the side; if there are no vacant square or above, or alongside, plays down. This will ensure the occupation of all squares. Take, for example, two possible lines:

1.b8 a8 2.a7 b7 3.b6 a6 4.a5 winning

1.b8 a7 2.a8 b7 3.b6 a6 4.a5 winning

2) In exercising the area in question involves beacon 1. In this case the player should have the No. 1 **concern of not getting closed with negative parity**. The winning move is 1.d1 giving the choice to the second player two hypotheses bad for him. 1.c1 would hardly be played once with 1 ... d1 the second player closed position with a favorable parity. If the first player to play # 2 is also lost. Against any move, Player # 1 **answer on goal**. This is an example of a position that is won for the first player, whoever is playing.

3) This example illustrates a fact which is recorded many times in this game: **not always move in the direction of the goal is the best**. 1.b3 is the winning move.

Traffic Lights

1) The solution is based on the simplest strategy that is in the traffic lights: **count the movements that are still available**. The solution is to move on c2, leaving two moves available in d2. The other options are bad. For example, the roll B1 is poor since only one combination remains available d1.

2) This exercise uses the concept of **symmetry**. Optimal move is on d1, repeated from then on what the first player does in symmetrical houses from the central point.

3) In this example we use a more elaborate thinking: **change to red**. A good move is in c3. From then on, for the sake of organization, the simplest is to **change all the yellow to red** (you may have to challenge the hypothetical in green with a green b2 d2 and vice versa).

4) This is the most difficult example and it is necessary to understand very well the position to find the proper throw. The second player has plans to make the house red or red c2 a1 home. If you do, you can follow the strategy of **change to red** until the end. Thus, the only way down to putting green is in b1. Thus, when changing from green to yellow in a1 or c2, can dispute with the change from green to yellow in b1, avoiding the danger. The reader can confirm that the first player able to win if you play carefully.

Cats and Dogs

1) The boxes marked with x can no longer be playing:

8	○	x	●	x	○	○	x	●
7		○	x	●	x	x	○	x
6		○	x	●	●	●	x	
5	○	x	●	●	x	x	●	x
4	x	●		x	○	○	x	○
3	x	●	x	○		○	x	x
2	○	x	●	x	○	x	●	●
1	x	●	x	○	x	●		
	a	b	c	d	e	f	g	h

We notice that both have three unique possibilities:

8	○	x	●	x	○	○	x	●
7	c	○	x	●	x	x	○	x
6	c	○	x	●	●	●	x	
5	○	x	●	●	x	x	●	x
4	x	●	g	x	○	○	x	○
3	x	●	x	○	c	○	x	x
2	○	x	●	x	○	x	●	●
1	x	●	x	○	x	●	g	
	a	b	c	d	e	f	g	h

The one who is playing should do it in the double square available on h6, thus win the game.

2) The forbidden squares are the following:

8	x	●	●	●	x			
7	○	x		x	○	○	x	●
6	x				○	x	●	x
5	●	x		x	○	○	x	○
4	x	○	x	●	x		○	x
3	●	x	●	x			x	●
2	x	○	x	○	x		●	x
1	●	x	○	x	●	●	x	○
	a	b	c	d	e	f	g	h

In this position, the empty squares have the same value for both players. For example, a move in f3 becomes exclusive two adjacent houses. We will mark the correspondent values of these squares:

8	x	●	●	●	x		1	
7	○	x		x	○	○	x	●
6	x		3		○	x	●	x
5	●	x		x	○	○	x	○
4	x	○	x	●	x		○	x
3	●	x	●	x		2	x	●
2	x	○	x	○	x		●	x
1	●	x	○	x	●	●	x	○
	a	b	c	d	e	f	g	h

Thus, the next player must place his piece on c6 that allows you to get three exclusive homes and thus win the game.

3) The squares marked with x can no longer be played:

8	○	x	●	x	●	●	x	○
7	x	●	x	○	x		●	x
6	○	x	●	x		2	x	●
5	x	●	●	●	x		○	x
4	○	x	x	x	○	○	○	x
3	○	x	○	x	○	○	x	●
2	x	●	x	●	x	x	●	x
1	●					○	x	○
	a	b	c	d	e	f	g	h

The option f6 worths two points for both players, ie, allows acquiring two exclusive houses where only the player can place his pieces.

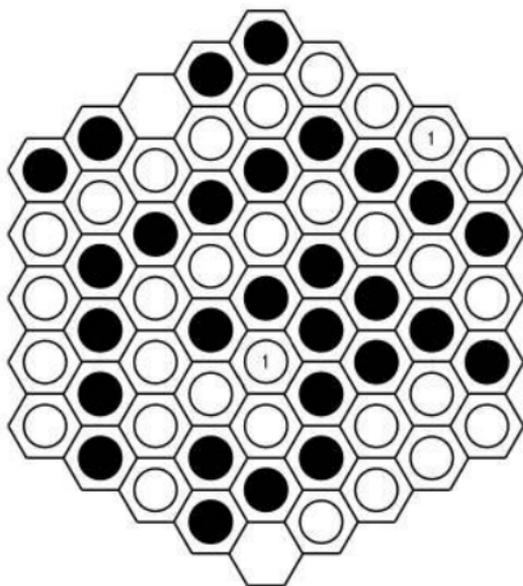
But in the 1st line the situation is not symmetric. The home d1 corresponds to two points for the Cats (where the dogs can not play). Dogs throw up in the house c1, could eliminate the moves of the cats in this sector.

Both must play at home f6, giving them two points. If the Cats play, the game is won, because the dogs have only a single bid on c1. But if the dogs are starting to f6, the Cats respond with d1 and can win the game, as are both the same number of houses exclusive but will be the last to play.

In summary: the Cats win regardless of who is the next turn.

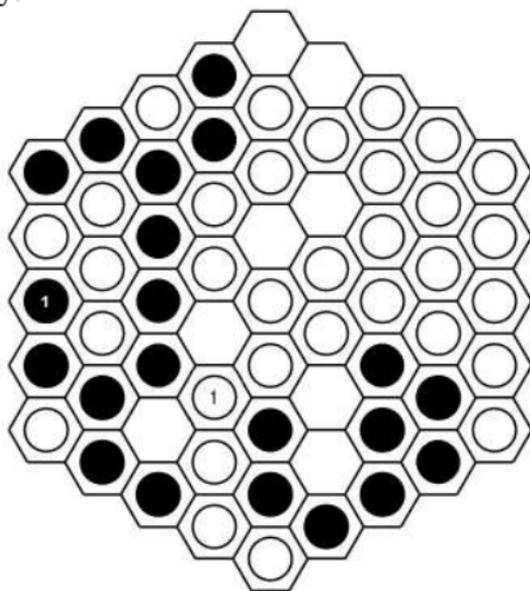
Product

1) The correct plays are as follows.



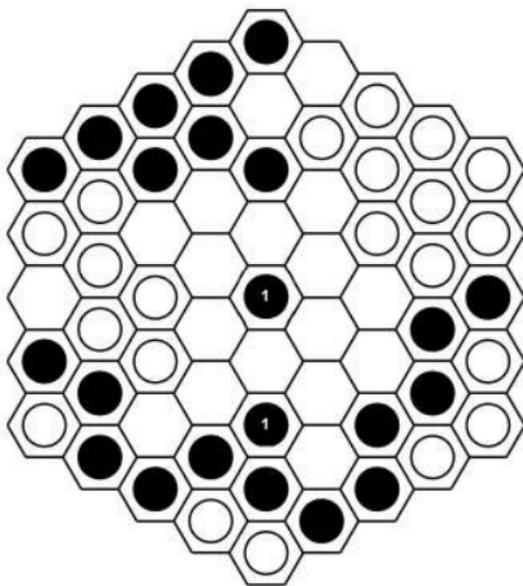
Thus, the make up $13 \times 12 = 156$ pontos contra os $11 \times 14 = 154$ points for Black.

2) The next board shows the moves that guarantee victory:



With the move of the white man, the Dark separate into its final two groups. The move of the black piece prevents the second group of White can grow. As the second white group is small (only four parts), the product of groups of Black will ensure victory.

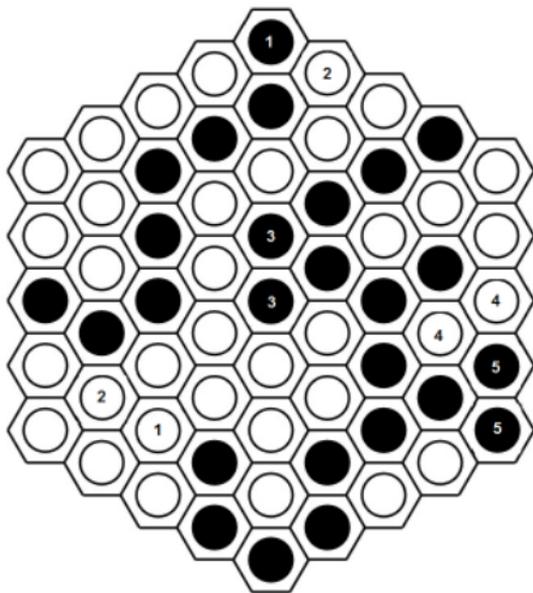
3) The White decide to play two pieces of the opponent.



This type of virtual connection can not be cut with a bid of only two parts. If Black tried to cut the virtual connection to the North or South, the White immediately would make the connection with their consequent double play.

Thus, White ensured that the adversary will only end with a group on the board, ie with zero points. As the White already have more than a single group, this ensures victory.

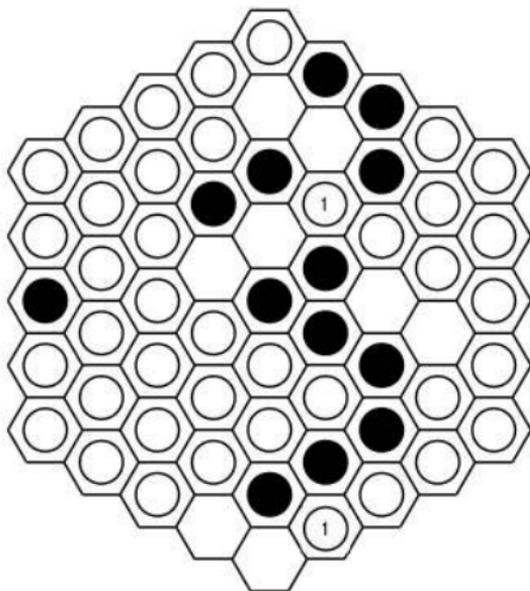
4) This is the typical sequence to the end of the game (there are variations but the outcome of the match is the same).



In the first move Negras were forced to join the two white groups. Otherwise the White, the next bid could connect the two black groups which would result in an immediate defeat (the Black would get zero points). The other black piece was essential to limit the size of the second white group.

In this final position, the Black wins the match with $18 \times 8 = 144$ points, against $20 \times 7 = 140$ points for White.

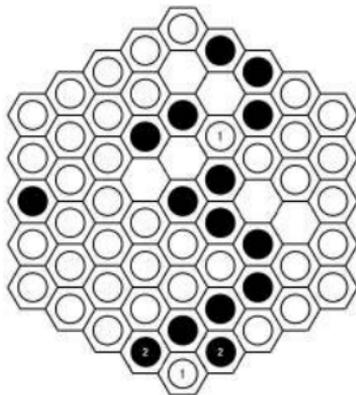
5) Black play two white pieces, as shown in the diagram:



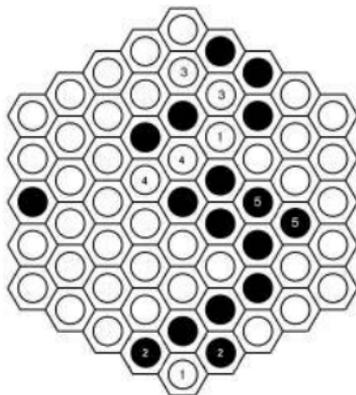
Thus, the need to play White three black parts to prevent the subsequent connection of its only two groups. As they can not do, will lose the game by getting a minimum of zero points.

But to show how subtle play of the Product may be, consider a variation on the following page of this position.

If Black plays his next throw almost equal to the previous? In this case, White would win throwing two black pieces:

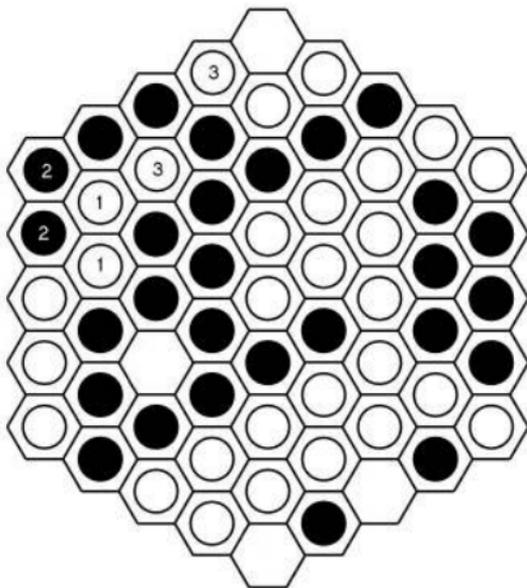


Continuation would be this:



White would get $43 \times 1 = 43$ points against $11 \times 3 = 33$ points for Black!

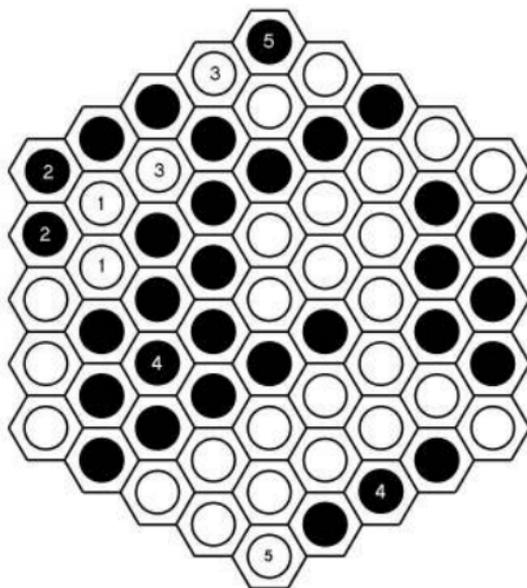
6) The first two moves of the solution are relatively normal. In the 1st move, White threatens to win by the growth of their group left. The largest group has 20 white pieces, and the 2nd group could have 8 pieces (totaling 160 points). Thus, the lance 2, the Black limit the size of group 2 to a maximum of 6 pieces (120 points). The Black in this position have a group of 20 and another of 6 pieces (total 120 points).



However, when playing the lance 3 thus ensure that the White Black do more than 120 points, i.e. 120 points that they now possess White!

But the game can not end tied product as there is a criterion that - in case of equal points - determines who the winner is the player who has *fewer* pieces on

the board. And so, the game has the following outcome:



After the 4th of Black move that guarantees your 120 points, the 5th pitch of White is crucial: the part south guarantees 120 white dots (and we realized that the occasional tie) but his play in the North is a black piece !

The match ended with the full board. If we count the pieces, we have 31 black pieces and 30 white pieces. The White won due to the last move.